Fundamentals of Economics and Finance

(With over 500 exercises and solutions)

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FUNDAMENTALS OF ECONOMICS AND FINANCE

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Acknowledgement

When I did my BA and later MA programs in US I honestly always struggled with finding “perfect book for students”. Some textbooks were focusing on theory too much and with very few application examples while others were too quantitative as if they were written not for students but for professors. Second issue was the absence of a book which covered general economics and finance topics all together. Finance is a part of economics science how can student be a good financier without knowledge of fundamental economics? How can economist be a good economist without knowledge of proper financial system: how it works, how instruments work, and what the main function of each financial institution is? Some textbooks were too quantitative, especially finance books, and with very little solved exercises. Exercises were my biggest problems; first of all they were not solved! No solution or even if book contains solution it was very short, just an answer in many cases, and the question that I always wanted to ask those writers is “do you want to teach students or just torture them?” There were many cases when I solved exercises and looked at back of the book and saw absolutely different answer. Please pay attention, I am not saying solution I am saying “answer”. Yep, and you, poor student will be left there alone with this exercise and believe me, not every student has enough patience to get to the final correct result. Especially I struggled with those authors who liked to “jump” certain step when solving exercises. For example assume for solving certain application example four step is required. Author starts first step and quickly jumps to last instead of showing all the steps. Says for example: “By applying Second order condition we know that this function has two minima”. No I do not know, Sir! How many times I dreamed to shout these words to those authors who liked to cut short. “**I do not know, Sir! Just spare 10 seconds and do all required steps once more because this is a textbook and I am a poor student and please do not make me open three more books to read your book.”** I strongly believe that textbook is “bad” if to read that textbook I need to get another “textbook”. What would have happened if there were economics and finance textbooks with 30-40 exercises in each chapter and with full step-by-step solutions? I probably would have cried from happiness if I had found one of those books when I was student myself. Cried and thanked that author for being “kind” to students. Last thing I never understood is that why exercises are not related to real life out there? The most textbooks contain very low quality exercises. Exercises are vital part of economics sciences: it is heart of economics science! When I started teaching in 2018 I found out that even best economics science majoring students had problems with capturing all of economics science. Finance major student knew very little economics, and economics major student knew very little finance. Insurance major student know little about management and management majoring student knows little about insurance. How can those future graduates be “good” specialists if

they do not know at least fundamentals of each branch of economics? At least fundamentals! My target was to write such a textbook that:

a) Covered all of major economics branches
b) Had a lot of solved exercises which goes from easy to hard
c) With very few theory but with exact explanation
d) High quality questions and solutions

This book is like a challenge, the ones who pass this challenge will be no doubt "well rounded" specialist with both excellent quantitative and qualitative skills. I would not recommend this book for 1st and 2nd year students. This book is for 3rd and 4th year student who has already some idea about economics science but wants to deepen his/her knowledge. This does not mean that 1st and 2nd year students must not open this book and that it is too challenging for them. Instead I would have recommended them to open this book for guidance. Kind of getting idea what he/she must learn to be a better specialist in future. When I was a kid my freestyle wrestling coach always put records of world championships and we watched them. At first we just enjoyed seeing real professionals later on; coach commented and showed us some techniques and trick used by one of masters. 1st and 2nd year students must take this book as guidance. Even if they would not have full capacity to understand what is going on in there, at least by listing chapters they might get an idea what they must learn in near future. Since I wrote this book for 3rd and 4th year students, I already assumed that they know calculus, at least at intermediate level. I have a full right for this assumption because intermediate level of calculus is already in the schedule of all schools. All graphics, tables and numbers used in this book was taken from official internet sites of United Nations, International Monetary Fund, World Bank, Bank for International Settlements, European Central Bank, Federal Reserve System, US Treasury Department, Organization for European Cooperation and Development, and many other open sources. Numbers and tables are for years of 2018, 2019 and 2020 not older than that. I want to take an attention of reader that this is not a research paper but a textbook and all tables are for learning and exercise purposes. Each chapter of this book will touch the most vital and fundamental topics in economics science and will bring many application exercises and solutions together. **First three chapters will cover fundamentals of microeconomics. Chapter 1** will cover supply and demand. I tried to look at these topics from a little bit different perspective and brought many original exercises especially with demand supply curve shifts with different slope. A lot of economics books do not focus on this but I think this is very important and interesting to learn about that. **Chapter 2** will cover cost, revenue and profits. I brought some characteristics of those functions and tried to explain those functions more deeply. **Chapter 3** focused on optimization, classic question of how much to produce and how cheaply to produce. I tried to bring interesting and original questions to make this chapter more appealing. **Next three chapters will cover**

fundamentals of macroeconomics. Chapter 4 is by far the longest and the most original chapter, because I started macroeconomics with explanation of economic indicators. Unfortunately, a lot of textbooks (to be honestly I think all of them) never explain indicators even if they do, they spend so little for this vital topic. I brought the most important indicators with mathematical calculations and tried to explain what, where and why these indicators are used for. It is extremely important for future economist to understand and actually be able to solve those indicators and its economic meaning. Economic indicators are like diagnosis results for doctors. How would doctor help patient if he/she does not understand diagnosis results? How would economist be helpful if he/she does not understand or knows what certain indicators are for? It is impossible. Exercises on this chapter are a little bit harder I admit that, but solutions and explanation is provided to make student's life easy. Giant GDP calculation exercise using Factor income and Spending method will help students in understanding real world economics. **Chapter 5** will talk about topics of growth and development, which is the main focus of macroeconomics science. Solow's growth model was brought in this chapter with precise explanation. This chapter also contains exercises on Solow's growth model which is firstly used here. **Chapter 6** will be talking about fiscal and monetary policies. Efficient tax system, money supply, inflation and interest rate and other vital topics will be covered and explained in great detail how states use those instruments and institutions to provide economy with liquidity. Unique method for solving foreign exchange and inflation related complex problems using vector mathematics was showed here. Also, exercises with non-linear money demand functions were used here firstly. **Next six chapters are dedicated to financial topics. Chapter 7** focuses on banking and its vital place in economic development. All vital calculations used in banking sector were brought and explained with solutions. My target was to give full idea to students about banks and what mechanisms are used in there. **Chapter 8** is about insurance and risk management. Risk averseness of human being is discussed and importance of insurance sector in turning savings into investments is pointed out. Exercises are exactly ones that will future insurance specialist will face. **Chapter 9** will be discussing importance of stock exchanges and its functions. Stock index calculations and fluctuations will be closely analyzed in this chapter. Importance of building "government and public" channel for elimination of panic risk is discussed in this chapter. Panic causes more damage to economy than crisis itself, and to fix that problem states must be in continuous contact with public in any arising financial topic. **Chapter 10** will be talking about derivatives and derivative markets: mechanism of futures market, main objects of options, and many more. All exercises and solutions will be demonstrated with detailed comments. **Chapter 11** focuses on accounting and audit issues. Trust is built through building efficient accounting and auditing system in economy. White collar crime problems such as money laundering, tax evasion, financing of terrorism, etc. and

instruments for combating those crimes are discussed in great detail in chapter. International efforts and importance of international cooperation to fight those crimes are also touched. **Chapter 12** will close this book with discussion of the only input for economic growth and development: human capital. Everything starting from simple machines to the most advanced technology is derived from developed human capital. Human being must be central in all aspect of political economy. Upcoming future dependence problem to technology, problems for developing nations are at the center in this chapter. Not only problems are discussed dully, but some solutions are proved too.

This textbook as any other textbook has strength and weaknesses. One of weaknesses that this book might be criticized is for its short and precise explanations. I strongly believe that economics is application science and calculations must speak lauder than words. This book also does not contain any beautiful attractive pictures and all other interesting stories. I tried to be up to the point with no messing around. Long calculations might scare off somebody I agree with that too. A lot of calculations, a lot of exercises and solutions and some of them really look intimidating. Again, I wrote this book for those who really want to learn something about economics science. This book is not for those who like memorizing definitions and learning some vague ideas but absolutely not interested in application of those ideas. **This textbook is for professionals!** Less words, more actions, a lot of implementations, short and strong philosophy! Since I am not a native English speaker, I sincerely seek an apology from my readers for simple and may be not clearly defined sentences. I tried hard to be precise and up to the point. I also thank by my heart my student Vepayeva Hatyja for her great help in designing of the book and editing grammatical, numerical graphical or any other error left by me. (Now you know whom to blame for errors in this book!).

P.S. I have dreamed and planned to write a textbook like this so much that I have finished it in less than 2 month!

(Start date 09.06.2020 ending date 08.08.2020)

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Shortly, on economics science and essence of finance

Humankind has socially evolved from primitive society, where powerful groups ruled over weak ones, to more complex ones like we see today, where no individual is above the law. You cannot conquer the land just because you liked it anymore. You cannot kill another person or steal his/her property, because these actions are no more considered legal in any country. Speaking of countries, there are over 200 independent countries and over 7 billion people on this planet, history has never witnessed more than that! Each of those countries has their own laws (Constitution), own culture, own history, own heroes, own targets, own economy and own problems, that needs to be solved. Long ago human being understood that solving your own problems by isolating yourself from others is the worst solution of any possible upcoming problem. This social consciousness has created immense energy, which has blasted in two World Wars in mid twentieth century, this was the last outcry of "backward minded" kings and emperors, who tried to solve issues "old ways", by power! Emptiness created in human soul and heart by those tragic wars led to something fundamentally different, something natural: Globalization. World had understood how strongly connected we are. Tragedy in one corner of the world impacts the other corner; the pollution in north impacts the south. No doubt in that! We are all in one ship! In order to answer to any global problem arising from any socio-economic situation, international organizations were created such as UN, WTO, IMF, WB, UNESCO, WHO, etc. Nations are united according to their political or economic targets. This social evolution has led immense change to well-being of people. Actually, some can argue that these sentences could be written vice versa (Economic growth fostered social development). Thus, from primitive barter system our economy of the world has changed, and carries complex transactions: from selling couple of fishes to the neighbor, today billions of dollars of food are sold globally. With evolving societies nations and countries developed to the huge sizes. A couple of hundred years ago there were few banks, now they have over hundred thousand banks all over the world, we have millions of companies, trillions of financial transactions, thousands of billionaires and many much more. Long word short, our world became bigger, richer, wealthier and more connected. These rapid changes increased demand of a science that studied interrelationship of those changes, the name of this science is: Economics. Economics science learns how to use all your inputs: land, labor, resources and technology in the most efficient way.

This science is pretty young, but nevertheless it is the most important one, I will say. Economics science is vital and irreplaceable for future specialists in the area of insurance business, financial sector, state employees, business owners, entrepreneurs and many others. What is the relationship between unemployment and economic growth, education and development, recession and recovery, expansion and inflation, trade and welfare? These are one of the few questions that economists are trying to answer. Finance became a major part of economics science, and any financier who wants to succeed in his job will not be able to do that without knowing fundamental principles of economics. That is the reason I will start my finance chapters with introduction of some core principles of economics. So, what are the main questions financiers are trying to answer? What is the financier's job? Well educated financier will be able to answer to these questions: Where and how much to invest? What is the relationship between investment and risks? How monetary and fiscal policy can impact inflation rates? How to secure my long term investments? How financial inflows and outflows can impact economic growth? Do taxes affect wealth distribution within a country? Since the central topic for any good financier is the money, (interest rate, revenue, profit, loss, debt, loan, expense, budget, etc…) a financier must have excellent mathematical skills. Advanced accounting skills are also vital for future professional. I structured these chapters in such a way that after studying these chapters and doing all exercises, a student will be able to answer to all those questions asked above, showing excellent technical skills both in mathematics and accounting. **I strongly believe that highly qualified financiers carry no less importance to humanity than medical doctors or engineers.**

Chapter 1: Mathematics of supply and demand.

How you ever wondered, what is the purpose of your existence? No, do not worry; I am not going to screw your "extremely busy" brain with one more philosophical question followed with blurry and extremely vague answer that actually clarifies nothing but adds more questions. I am an economist, so my answer is short: You are living to save! Yep, I know, you were expecting some "smart" answer filled with all kinds of technical spirals, but hey, get used to it. This is the world of economics, where answers are short and precise. We are not philosophers (God forbid!): we do not add questions to questions. We bring simple answers! Consumer (yep, that is how we call individuals, you!) tries to maximize its saving! It is in human nature to save, I do not know why and honestly never interested in human psychology that deep. I leave this issue to psychologist and philosophers instead. Consumer earns income (*I*=income) and spends certain part of it (*C*=consumption), saving the rest for tomorrow (*S*=savings).

$$I = C + S$$

Here, we can rewrite above equation as

$$S = I - C$$

As it is said above, consumer wants to maximize its savings. In economics science, one of the biggest assumptions is that a consumer is rational! This means, consumer thinks about his future. Consumer values its future: he wants to buy a new car, better house, take piano classes, visit Morocco, and marry to best girl in class! To do all that, consumer needs money, his savings. That is the reason why he is not consuming everything that he has earned today! He saves some to accomplish his dreams. The more he saves, the more he can accomplish. Income is earned many ways: working at office, from investments, publishing a book, royalties, patents from inventions, consultation services, craftsmanship, etc. Savings, in economics world is equaled to investments. Putting your hard earned money under the pillow is also considered savings. Not a smart savings though! Remember, we agreed on a notion of "rational consumer"? Why would somebody want to keep his savings under the pillow while he can invest this money and get some return? In economics world, we assume (and hope) that consumer invests his savings, making money work for him, rather than making it simply lie under the pillow. Doing this, will benefit both consumer and an economy, a win-win situation I say (we will talk about it in coming chapters!).

Saving = Investment

Consumption is considered everything that consumers buy today to use it in a short period of time: going to restaurant, buying Hawaiian shirts, visiting Manila, going to circus, ordering a box of wine for present, making a wedding, paying lawyer's fees for legal services, paying for rent, getting extra classes for learning Cha-cha-cha dancing, buying a TV, groceries, etc. The reason why I used the word "short period of time" is that, some purchases are made for resale or for business purposes, these are considered as investments: buying a house, buying a hotel, buying restaurant, buying an island, buying office space in business building, purchasing tons of gym equipment for your fitness business, buying an arable land, etc. All of these are entered in inventories accounts in accounting. Education is considered an investment, not consumption in economics world, because it is believed that the main reason people getting higher education is because they want to earn more money in the future. Thus, we can divide goods into two categories in terms of their "time of benefit": *Consumption goods* and *Investment goods. Consumption goods* are those that could be benefitted only for short period of time and do not add anything to our asset, it is recorded as expenses in accounting. *Investment goods* are those that could be benefitted for long period of time and added onto asset side in accounting. Any purchase that was done to resale (or to make another product for sale) or at least has characteristics of resalable is considered *investment purchases.* Going to gym classes for example is considered as an investment in health economics, because fitness strengthens consumers' health, which means he can be active for a longer period of time, which also means that he can earn more money than unhealthy consumer. Money is a median; we use it to purchase things for living but I consider money as an investment good. Consumer can spend income in two ways: purchase consumption goods or investment goods, but he can also do nothing and leave income as it is (deposit it to bank for example) and it will be considered as investment. As it is said above, consumption goods do not add much (nothing!) to our assets while investment goods add, that is the main intuition behind the "saving" behavior[1] of consumers in economics science. Consumers tend to increase their asset values! Simple as it is!

We will talk about investment later on but I have another question, so, from where consumer purchases goods? Consumer purchases goods from suppliers (stores, restaurants, businesses, producers, manufacturers, etc.). Supplier also has a target; she wants to maximize her profits! Supplier sells the goods and earns revenue (*R)* (income!), subtracts costs (*E=costs)* accrued from producing that good (employee salaries, rent, amortization, input costs, transaction costs, taxes, etc.) and makes a profit (Π).

[1] The branch of economics that study consumer behavior is called Behavior Economics.

$$R = \Pi + E$$

Here, we can rewrite above equation as

$$\Pi = R - E$$

Businesses are there for profits! This is one of main assumptions in economics science. Are there entities that are not there for profits? Yes, there are, that is the reason economists do not call them businesses, we call them *non-profit organizations.* Their strategy is not built on a notion of "maximizing profits", rather, they aim at different outcomes. **Suppliers can increase their profits by either increasing their revenue, or decreasing costs. Consumers can maximize savings either by increasing income or decreasing consumption.**[2] Profits are positively correlated with revenue, and negatively correlated with costs. As revenue increases, the profits increase (not always!) and as costs increase profits decrease. As income increases, consumer's savings increase (not always!) and as consumption increases the savings decrease. Supplier and consumer meet at a market to satisfy their needs: consumer wants to purchase goods and supplier to sell the good. Their relationship can be structured in many ways: a) *freely:* which means consumer and supplier decides the price and quantity of goods with no intervention whatsoever. This is called *free market* economy or simply *marked economy* b) *commanded:* when neither supplier nor consumer decision on price and quantity of goods to be produced or purchased is taken into account. State *commands* how much to produce and what price tag to put on those goods. This is called *command economy* c) *mixed:* when price and quantity of goods is determined by consumers and suppliers but state reserves the right to intervene any time it considers right (for example in crises!). As of today (2020), all economies have a structure of *mixed economy,* except North Korea, where *command* structure is preferred. Even though developed economies love boasting about their *absolute free market structure of economy,* global financial crisis of 2008 proved them wrong. All those economies were saved thanks to government intervention through stimulus programs and buyouts. Speaking of price and quantity, let's add it to our consumer and supplier equations shown above. As per consumer's perspective, the quantity of goods to be consumed will depend to the price of that good; and this dependence will have negative correlation. The cheaper the restaurant more often clients go, how often the consumer goes to cinema depends on the price of the ticket, low university fees will attract more students, cheaper medical health services will make patients visit doctors more often, etc. While for suppliers the

[2] Pay attention to sign (negative or positive) in both equations!

quantity of goods they produce also depends on the price of that good in the market, this relationship is positively correlated. Increasing the price of a good in the market will encourage suppliers increase their production. Increasing beverage prices in the market will serve as a signal for supplier to increase his production, increasing ticket prices for rap concert will make promoters to give more concerts, falling prices on tomato will discourage tomato producers, etc. How can we show this relationship mathematically? We can write *consumption* as a *p (*price) times the *q* (quantity) in *consumer function.* Thus:

$$S = I - p * q$$

As you have noticed *savings* have negative correlation with price! The increasing *p* decreases *S.* In *supplier* function we can write *revenue* as a *p* (price) times quantity (*Q,* this quantity is not the quantity of consumer functions, here *Q* is aggregate, total quantity sold, be careful!). Thus:

$$P * Q - E = \Pi$$

Here also, as you have noticed (I strongly hope that you have noticed!), *profit* (Π) is positively correlated with *price!* Increasing *p* increases Π. This is the main explanation why increasing prices "scare off" consumers, because it lowers his savings. Why increasing prices attract more businesses? Because increasing prices the most of the time means increasing profits. Consumers' quantity demanded for the good increases as prices fall, and decreases with increasing prices. Suppliers' quantity produced for those good increases with increasing prices, and falls with falling prices. Economists simplify this even further to:

Consumer equation → Q(demanded) = a – b * P
Supplier equation → Q(supplied) = c * P – d

Mathematically we simplify *consumer and supplier equations* to linear functions where Q means quantity of goods, and P (capital letter) means price, and *a, b, c, d* is just coefficients. (We assume *a, b, c, d* >0)

When trade occurs? **When price is right!** Consumer will buy product if the price is right for him, supplier will be ready to sell the product if the price is right for her. Showing it mathematically:

$$Q(\text{supplied}) = Q(\text{demanded})$$
$$c * P - d = a - b * P$$
$$c * P + b * P = a + d$$
$$(c + b) * P = a + d$$

$$P = \frac{a+d}{c+b}$$

This is the right price! Economists call it an *equilibrium price.* We use *equilibrium price* to find *equilibrium quantity. Consumer equation is termed to a Demand function,* and *supplier equation is termed as a Supply function.* Trade occurs at intersection price of Demand and Supply functions.

Exercise 1: *Find equilibrium price and quantity, draw supply curve, demand curve, and equilibrium price and quantity on the graph. Show volume of the market too.*

$Q_d=2000-10P$

$Q_s=90P-100$

$Q_{demanded}=Q_{supplied}$

$90P-100=2000-10P$

$90P+10P=2000+100$

$100P=2100$

$P=21$

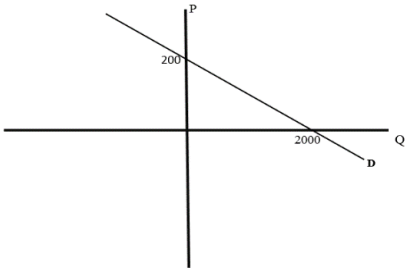
$Q_d=2000-10*(21) =1790$

$Q_s=90*(21)-100=1790$

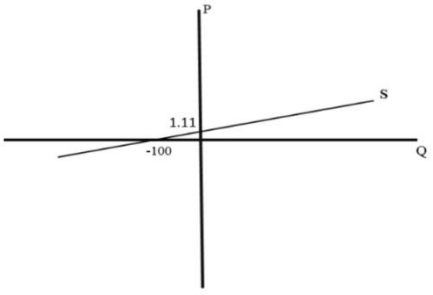
Market volume=1790*21=37590

Our equilibrium price is 21, and equilibrium quantity is 1790! So let's constrain the graph of supply, demand and equilibrium:
First of all, we find intersection of lines on Y and X axes

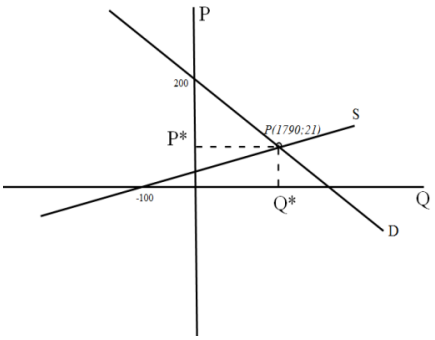
$Q_d=2000-10P$
$Q=0\rightarrow 0=2000-10P$
$10P=2000$
$P=200$
$P=0\rightarrow Q=2000-10*(0)$
$Q=2000$



$Q_S=90P-100$
$Q=0\rightarrow 0=90P-100$
$100=90P$
$P=1.111$
$P=0\rightarrow Q=90*(0)-100$
$Q=-100$



And finally we find equilibrium by matching both (supply and demand functions) into one graph and get:
As far as we know that, there is no negative price and negative quantity, on the following examples we conceal unnecessary sections of the graph (I, III, IV quadrants)



Exercise 2: *Find equilibrium price and quantity, draw supply curve, demand curve, and equilibrium price and quantity on the graph.*

$$Q_d=5000-100P$$

$$Q_s=400P$$

$$Q_{demanded}=Q_{supplied}$$

$$5000-100P=400P$$

$$500P=500$$

$$P=10$$

$$Q_s=400*10=4000$$

$$Q_d=5000-100*(10)=4000$$

Our equilibrium price is 10, and equilibrium quantity is 4000. Again, constrain the graph of supply, demand and equilibrium:
First of all, we find intersection on y and x axes

$Q_d=5000-100P$

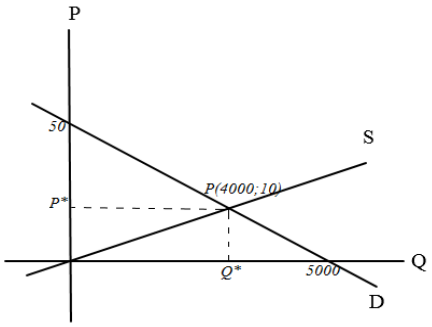
$Q=0\rightarrow 0=5000-100P$

$P=50$

$P=0\rightarrow Q=5000-100*0$

$Q_d=5000$

$Q_s=400P$



P=1→Q=400(1) = 400 (we use iteration method to find the line)*

P=2→Q=400(2) =800*

And get the equilibrium shown above:

Exercise 3: *Let's solve now a different case. Find equilibrium price and quantity, draw supply curve, demand curve, and equilibrium price and quantity on the graph.*

$$Q_d=1000-20P$$

$$Q_s=5P-1000$$

$$5P-1000=1000-20P$$

$$25P=2000$$

$$P=80$$

$$Q_s=5*(80)-1000=-600$$

It is impossible to produce negative quantity! So in this case we are given a wrong equation.

As mentioned above, a household's income breaks down into:

$$I = C + S$$

To C=consumption (expense) plus S=saving. Consequently, expense breaks down into:

$$E=P_1Q_1+P_2Q_2+\ldots+P_nQ_n$$

Basket of certain goods a household buys to consume for certain period of time is called **consumption bundle.**

Exercise 4: *Assume that Erkin wants to consume Gosha Chynar with manty (delivery every day from restaurant) during a week, and he only knows the prices of those goods. How many each of those goods he can consume with his limited income, which is 500 Manat.*

$500=20Q_1+40Q_2$

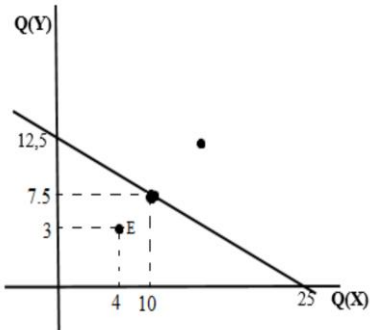
(Q_1 - quantity of Gosha Chynar, Q_2 -quantity of pizza). We write in terms of Q_1 (or Q_2, it doesn't matter):

$$\frac{500-40Q_2}{20}=Q_1$$

$$25-2Q_1=Q_2$$

And we find the graph:

Any point on the line Erkin can buy feasible combination of 2 different goods that fit his income (Points 10;7.5).



Any point below the line shows that he has a savings too, beyond consumed goods.

We can find his saving too. Assume that he consumes 3 Pepsis and 4 pizzas a week (point E):

20*3+4*40=60+160=220 TMT

500-220=280 TMT *(Erkin's savings!)*

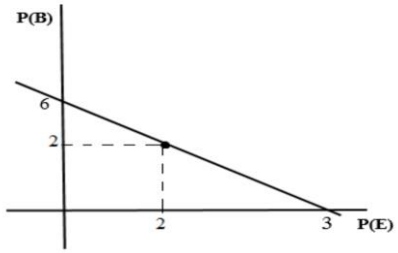
And any point above the line shows his borrowed money to buy these goods.

Exercise 5: *Every week Jennet consumes 6 breads and 12 eggs to keep her protein level on the balance. But she plans to spend for both goods only 36 TMT a week. What must be the price of those goods to satisfy her demand?*

$36=6P_B+12P_E$

$36-12P_E=6P_B$

$6-2P_E=P_B$



Any price on the line is appropriate to get her desired quantity. Say, if she finds both goods at 2 TMT each, she will get her desired level of protein!

What will happen if due to some reasons price changes in the market? Let's take our first example and see it closely.

Q (demanded) =2000-10P

Q (supply) =90P-100

We have found equilibrium price (21) and quantity (1790), and let's assume that price jumped to 25. Then:

Q (demanded) =2000-10*25=1750

Q (supply) =90*25-100=2150

As we have learned, price is negatively correlated with demand and positively correlated with supply. Price jump from 21 to 25 impacted consumers negatively and they demand lesser quantity of goods now. Suppliers seem happier than consumers and for that price they are ready to increase quantity of goods supplied to 2150. So how much goods will be sold and produced? The

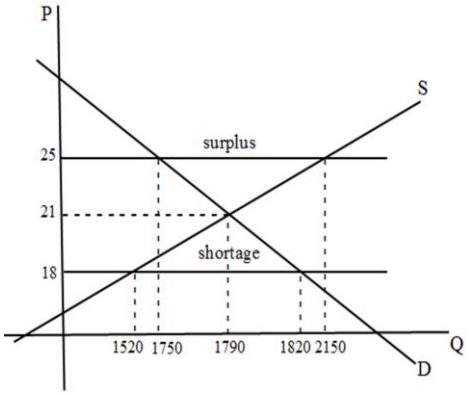
answer depends on demand! At price 25, quantity demanded is only 1750, it means only 1750 pieces will be sold in market. So if suppliers produce more, in our case suppliers are ready to produce 2150, there will be surplus of 2150-1750=400 of goods. **Surplus occurs when more goods were produced than market demand**. 400 unsold goods mean a loss for suppliers. That is why they will not produce that much. At the end of the day, who wants to lose money? If we lower the price instead, you will see reverse effect. Assume prices dropped to 18, then:

$$Q_d=2000-10*18=1820$$

$$Q_s=90*18-100=1520$$

Quantity demanded will increase to 1820, while quantity supplied will drop to 1520. Suppliers are not ready to produce more because they make lesser profits now.

Long word short, suppliers do not want to work hard for low profits. How much will be sold in market? Only 1520, because only that much was produced at price 18 TMT. There will be 1820-1520=300 shortage of good in market. **Shortage occurs when lesser quantity was produced than market demand**.



These price fluctuations create havoc (little or big) in market, creating shortages or surpluses in a short run. This is called **market inefficiency**.

Market inefficiency occurs when prices, due to economic or some other reasons, are above or below the equilibrium price. After a certain time period, market (both consumers and suppliers) gets adapted to a changes and market inefficiency is eliminated. How long can it take to eliminate market inefficiency? Nobody knows. Actually, there are no guarantees that this inefficiency is going to be eliminated at all. Instead, small inefficiencies can grow up to crisis. So waiting for markets to adapt to new situation (new price), might take an eternity. As once one wise man said, "In a long run we are all dead"[3]. Inefficiencies could be created even with good intentions. Government reforms are one good example. Assume that state wants population to be healthy and imposes **price ceiling (maximum price!)** for milk products, requiring suppliers to sell milk for lower price than a market equilibrium price. This creates shortage of milk products

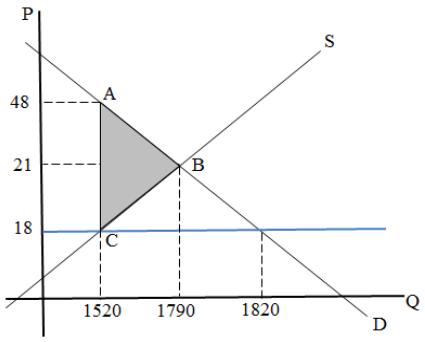
[3] John Maynard Keynes, an American economist.

because producers are unwilling to produce demanded quantity for an imposed price.

Exercise 6: *Let's assume that government imposed price ceiling 18. By taking the function of the first example, find deadweight loss. What is quantity demanded and supplied at that price? What is volume of fishing market?*

Deadweight loss
(triangle area) calculation is 1520=2000-10P → P=48
48-18=30,
height of triangle is 1790-1520=270

then (30*270)/2=4050 *worth of trade was not accomplished due to shortage. What is the price of fish in the market? It is 18 TMT. How much fish is supplied to market? Only 1520. Demanded? 1820! What is the volume of the fishing market? It is:* 1520*18=27360 TMT.



Trade opportunity that was lost due to the price reform in a market (shown triangle area ABC**) is called a deadweight loss.** This market inefficiency will never be eliminated unless state cancels its pricing policy reform of milk products. If state continues its price policy, then shortage of milk products will definitely have impact on health of the population long run. This is a guaranteed health crisis in the future. Deadweight loss will also occur when instead of price, quantity limitations are imposed.

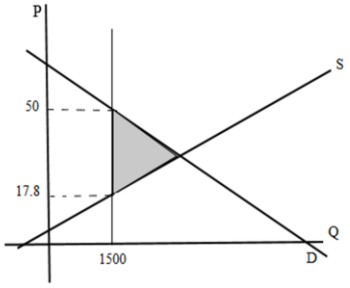
Exercise 7: *Assume for example state wants to control fishing industry against overfishing. State limits fishing to certain quantity, by giving licensed fishermen a quota.*

First of all, let's find what the prices are going to be when quantity of fish supplied to market is limited. In order to do that we have to put 1500 in Quantity in demand function and solve equation:

1500=2000-10P,

500=10P

P=50, *due to limited supply now, scarcity, prices jumped to 50.*



Now let's solve supply function, we do almost the same here too:

1500=90P-100 → 1600=90P

P=17.8

We need to find this price in order to calculate deadweight loss only. What is the price of the fish in the market now? It is 50. How much is produced? 1500 only. What is the volume of the fishing market? 1500*50=75000 TMT.

As a statesman, you must understand that limiting production to certain quantities will help to defend resources from depletion, but at the same time this reform will hike the prices. If state doesn't control resources on a high level, increasing prices will attract illegal activity in the region. In our example, huge price up to 50 TMT will definitely attract illegal fishing.

Do demand and supply remain fixed for all times? No, of course. Nothing stays the same, everything is in motion! Economically, your wage, spending, habits, etc. tend to be in dynamic activity. Those variables either increase or decrease. There might be tons of reasons for that, from macro (economic growth, wars, conflict, political changes, etc.) to micro (marrying, bad health conditions, promotion in your career, demotion, unemployment, etc.). These things impact demand of consumers, change in demand curve impacts supply too.

Exercise 8: *Let's assume that for some reasons, aggregate demand (or just a demand) increased in the market due to some reason (rapid economic growth for example). What will happen to prices and quantity? To answer this question, let's again take our exercise with fishing, where*

Q_d=2000-10P

Q_s=90P-100

We found from here that equilibrium price is 21, and quantity is 1790, market volume is 37590. Let's increase our demand curve now to:

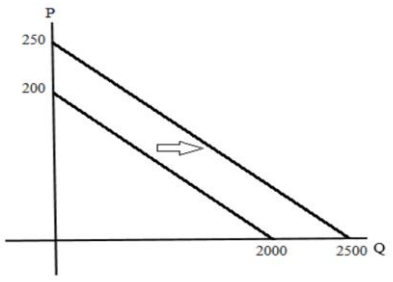
Q_d=2500-10P

Q_d=0, then 0=2500-10P

2500=10P

P=250 (intersects p axis) P=0

Q_d=2500



As we see from graph, our demand function shifted to right. Parallel to old demand curve, because slope of the function did not change!

Let's find out new equilibrium price and quantity using new demand function. Then:

$$Q_d=2500-10P$$

$$Q_s=90P-100$$

$$2500-10P=90P-100$$

$$2600=100P$$

$$P=26$$

$$Q_d=Q_s=2240$$

New equilibrium price is 26 and quantity is 2240. Market volume also increased to, 26*2240=58240.

Exercise 9: *What will happen if demand decreases due to some reasons (financial crisis for example)? Let's answer this question by doing the same exercise. Decrease the demand function to:*

$Q_d=1700-10P$

$Q_d=0$, then $0=1700-10P$

$1700=10P$

$P=170$

$P=0$, then $Q_d=1700$

Let's find new equilibrium price and quantity.

$Q_d=1700-10P$

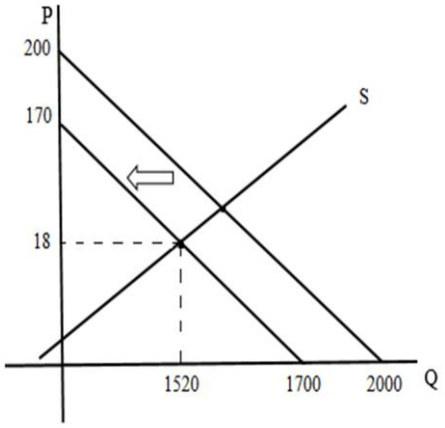
$Q_s=90P-100$

$1700-10P=90P-100$

$P=18$,

$Q_s=Q_d=1520$

Market volume is 18*1520=27360



As we see from graph, our demand function shifted to the left. Parallel to old the demand curve, because slope of the function did not change!

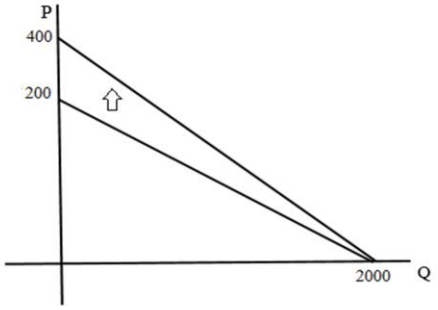
We can also use coefficient of prices while decreasing or increasing demand functions. Let's increase demand function by decreasing coefficient of price; primarily, as always we find intersections:

Q=2000-5P

Q→0 0=2000-5P

P=400

P→0 Q=2000



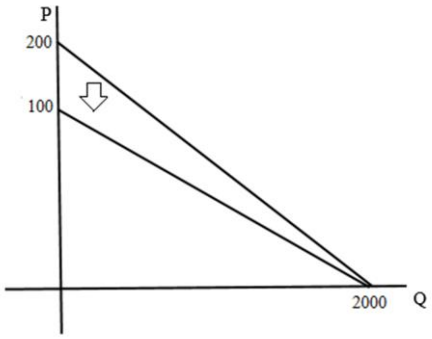
Let's assume that our demand curve decreased due to economic downturn:

Q=2000-20P

Q→0 0=2000-20P

P=100

P→0 Q=2000



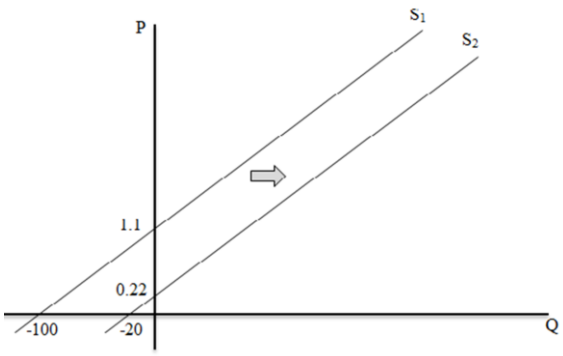
You will see that demand function shifts to the left, not a parallel anymore because we changed the slope of the function, but this shift is to the left anyway.
Let's switch to the supply functions now. Supply increase example:

Q=90p-20

Q=0 0=90P-20

P=0.22

P→0 Q=-20



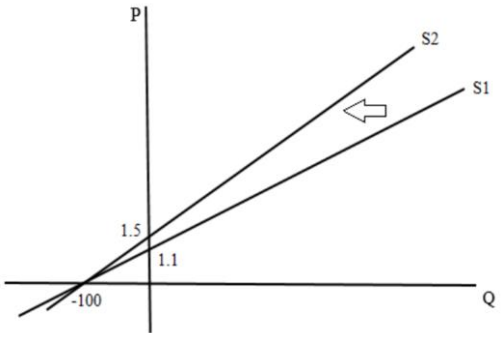
Now, you can see the parallel shift of the new supply function to the right of the old supply function. Decrease of supply function using coefficient:

Q=150P-100

Q=0, then P=1.5

P=0, then Q=-100

You can see the shift of the supply function to the right although this shift is not a parallel to the old supply curve.



These shifts of the demand and supply functions occur due to reasons. In economics science, these reasons are called a **shock**. Shock can be negative (bad for economy!), shifts functions to the left, and positive (good for economy!), shifts the functions to the right. This shock either can impact only one of the functions, demand or supply only, making the shift only in one of them, or can actually impact both of the functions making them move at the same time.

Here are some positive shocks to demand curve (shifts demand curve to right):

1. *Economic growth: Economic growth lowers unemployment and increases wealth of the population. Consumers start buying and investing more. Their demand for goods and services increase.*
2. *Decreasing loan rates: Cheap money makes people borrow more and spend more. This increases demand for goods and services in the market.*
3. *Optimistic future expectations: When consumers know that there are good days coming, it impacts their market decision today. When I know that I will be promoted tomorrow, my spending habits change accordingly.*
4. *Increase of government support: Government subsidies to families and people in needs very positively impacts on their purchasing habits.*
5. *Decreasing income taxes*: *Low taxes always mean more income left to spend, and I spend it.*

Negative shock to demand (shifts demand curve to left):

1. *Economic downturn: Hardship times make people spend less money, this decreases demand for goods and services*
2. *Increasing loan rates: Expensive money makes population take lesser loans, which means their purchasing volume is decreasing.*

3. *Pessimistic future expectations: Bad news about upcoming days impacts my spending habit today.*
4. *Decrease of government support: When government cuts the support, people become more frugal. Demand decreases.*
5. *Increasing income taxes: More taxes means less money left to spend.*

Positive shock to supply (shifts supply curve to the right):

1. *Cheap energy: decreasing petroleum, gas, electric prices lowers supplier's costs. It makes productions cheaper, this encourages productions, supply functions shifts to right.*
2. *Decreasing input costs: Low wages, low rental rates makes production cheaper and more profitable. This encourages production.*
3. *Advanced technology: New methods, new process and new technology increase speed of production, efficiency and decreases waste. Supplier's production increases.*
4. *Decreasing corporate taxes: Low taxes make more money available for expanding production, or investing to a new project. Both cases impact suppliers positively.*
5. *New resources: Opening of new resources always impacts economy positively, especially producers.*

Negative shock to supply (shifts supply curve to left):

1. *Expensive energy: Expensive energy makes products costly, this makes products unattractive, discourages production.*
2. *Increasing input costs: Increasing wages or rental rates makes production costlier, discouraging producers.*
3. *Old technology: Old methods and processes are time consuming compered to new ones, thus with old technology consumers produce less and with lower quality.*
4. *Increasing corporate taxes: Increasing taxes mean less money for innovations, technology and investments.*
5. *Depleted resources: The lower the resources the higher the prices of inputs of production.*

Let's do a little bit deeper analysis of types of goods. What can we say about that good? In economics science, goods are divided into three categories according to their needs:

A. normal (important, strategic, etc.) goods: goods which are needed no matter what the price is. Example: water, gas, bread, sugar, oil, milk, electricity, health services, etc. These goods carry immense importance to people that is why they are price

insensitive, thus, quantity demanded will change at a lower rate than price rate changes. *These goods have inelastic demands!*

B. Luxury goods: these goods are for wealthy part of population, and they do not carry extreme importance to population. You can live without gold jewelry, right? These goods are price sensitive, demand for those goods change with bigger rate compared to price changes. *These goods have elastic demands.*

C. Third category contains goods which have unit elastic characteristics, which means the quantity changes almost with the same rate as price changes. Boring! Let's take our first example and find out what kind of good is that:

$$Q=2000-10P$$
$$Q=90P-100$$

We found equilibrium price and quantity as 21 and 1790. Let's assume that price jumped to 23, what will happen to demand?

Q=2000-10*23=1770. Let's find rates of changes, which mean percentage change of price and quantity:

$$\frac{\Delta Quantity}{\Delta Price} = Elacticity$$

$$\frac{23-21}{21} * 100\% = 9.5\%$$

$$\frac{1790-1770}{1770} * 100\% = 1.1\%$$

When price jumped up to 9.5%, quantity demanded fall only by 1.1%, which shows how vital and important this good is for consumers. This good has inelastic demand functions. Let's finish this up by calculation *price elasticity of demand (PED):*

$$\frac{1.1\%}{9.5\%} = 0.12$$

If price elasticity of demand is between 0 and 1, this is an inelastic good. If price elasticity of demand is above 1, then it is elastic good. If price elasticity of demand equals to 1, it is unit elastic.

Homework:

I. List 10 examples for consumption goods and 10 examples for investment goods.

II. Meret earns 5000 TMT/month. Each month he spends 30% of his income for groceries, 20% for a rent, 5% buying books, 3% present for his mom (he is a mommy's boy!), 1% for his pet dog Tuzik, 8% for utility expenses (gas, electricity, recycling, etc.), and finally 15% for perfume. What is Meret's savings? Would you recommend him to cut expenses or spend more? Cut which expenses? Spend to what? Invest where?

III. Ashyr spends 230 TMT for books, 650 TMT for grocery, 1200 TMT for rent, 460 TMT for buying clothes, 1000 TMT deposits to a bank to his investment account, gives 100 TMT worth of presents to his sister, 320 TMT for his schooling expenses, 90 TMT for transportation expenses, his father always gives him 300 TMT as a present, and finally he has a pet crocodile and he spends 240 TMT each and every month. What are Ashyr's monthly earnings?

IV. If Gorkut saves a 2000 TMT every month. It is 38% of his monthly income. He wants to increase his savings to 50% of his earnings. How much Gorkut wants to save every month?

V. Ayna saves 3000 TMT every month, it is a 54% of her monthly earnings. She was promoted at her job but she kept her habit of saving only 3000 TMT each month, but now, it is only 22% of her promoted new salary. Calculate her previous and new salary.

VI. 12% of total sales is profit (profit margin). If I decrease my total cost by 30000 TMT, without increasing production, then 25% of the sales will be a profit. What is total revenue, profit and total cost?

VII. Ata's monthly income is 4500 TMT, and he spends 85% of his income every month. He got into a car accident where he was guilty. He has to pay 1300 TMT every month from now for 2 years to cover the loss of other party. How much Ata's monthly income must rise to cover all his monthly expenses?

VIII. Company A decides to merge with company B, total revenue of company A is one third bigger than company B's. If total revenue of new company A&B is one billion

TMT, find separately revenues of company A and company B.

IX. Three brothers Guvanch, Begench and Dayanch has created a company GBD, and they share company annual profits such as 20% goes to Guvanch, 42% goes to Begench, and rest is to Dayanch. If total cost is only 9/12 of company revenues, and this year total sales were 2 BLN TMT, how much each brother earned this year?

X. 3/14 of the income is spent on rent, 1/3 of income for groceries, 1/10 of income for education. If total income is 3400 TMT, how much is saved?

XI. A Company's total costs consist of two parts, variable costs (labor wages) and fixed costs (rent only). Assume company has 10 employees, the manager is paid 7000 TMT, accountant is paid 5000 TMT, 5 office employees are paid 3500 TMT and 3 service employees (2 guards and 1 janitor) are paid 2500 TMT each month. Company pays 4000 TMT rent month. If company makes 52000 TMT of profits this month, what is total revenue for that month? If company fires one guard and one office employee, how much cost it would cut? In percentage terms? If the manager's salary is raised to 10,000, how much total cost is going to change? In percentage terms? How much company must make every month in order to just cover the expenses? If company is selling consultation services, each consultation is 100 TMT, how many clients the company must serve to make profit of 70,000 TMT this month?

XII. Q=10000-34P
Q=666P-1000
Find equilibrium price, quantity, market value. What kind of good is it, elastic, inelastic, unit elastic? Prove your words will calculation. What will happen to prices and quantity if demand functions changes to Q=9000-34P? What will happen to prices, and quantity if supply function changes to Q=750P-1000. Show all on graph.

XIII. Q=3600-20P
Q=80P-400
Find equilibrium price, quantity and market value. What kind of good is it: elastic, inelastic or unit elastic?
Prove your words with calculation. How much goods will be sold in the market if price is 50 TMT? Show all in a graph.

XIV. Q=8800-22P

Q=88P-200
Find equilibrium price, quantity, market value. What kind of good is it, elastic, inelastic, unit elastic? Prove your words with calculation. What will happen in market if government limits production to 5000? Calculated deadweight loss and show everything on graph.

XV. Q=2000-5P
Q=995P-100
Find equilibrium price, quantity, market value. What kind of good is it, elastic, inelastic, unit elastic? Prove your words with calculation. What will happen in market if government puts price floor (minimum price) of 4? Calculated deadweight loss and show everything on graph.

XVI. Q=50000-10P
Q=990P
Find equilibrium price, quantity, market value. What kind of good is it, elastic, inelastic, unit elastic? Prove your words with calculation. What will happen in market if government puts price ceiling (maximum price) of 40? Calculated deadweight loss and show everything on graph. How will market change if price floor is 60? Show on graph.

XVII. Q=90000-1000P
Q=9000P-9000
Find equilibrium price, quantity, market value. What kind of good is it, elastic, inelastic, unit elastic? Prove your words with calculation. What will happen in market if government limits production to 70000? Calculated deadweight loss and show everything on graph. What if government limits production to 81000? Show on graph.

XVIII. Q=11000-700P
Q=1300P-9000
Find equilibrium price, quantity, market value. What kind of good is it, elastic, inelastic, unit elastic? Prove your words with calculation. Economic crisis hit the country and it changed slope of demand function 20%. Find all new numbers in market and show on graph.

XIX. Q=78-3P
Q=9P-6
Find equilibrium price, quantity, market value. What kind of good is it, elastic, inelastic, unit elastic? Prove your words with calculation. Government spending increased

and it changed slope of demand function 30%. Find all new numbers in market and show on graph.

XX. Q=5400-700P
Q=2000P
Find equilibrium price, quantity, market value. What kind of good is it, elastic, inelastic, unit elastic? Prove your words with calculation. Increasing energy prices impacted both consumers and producers negatively. While slope of demand function changed only by 20%, the slope of supply functions changed 30%. Find all new numbers in market and show on graph.

XXI. Q=1000P-1000
Supply function is given above. At price of 4, how much is suppliers ready to produce? Market demand is 9000 goods. At what price suppliers will be ready to supply that much? What is the price if demand increases to 13000? What if demand drops to 5000?

XXII. When prices increased by 10%, demand decreased by 3%. What is the price elasticity of demand? What type of good is it?

XXIII. How unit elastic demand function must look like? Write down and graph that function.

XXIV. How must strictly inelastic demand function look like? Write down the function and graph it.

XXV. When prices rose to 10%, quantity demanded dropped by 3000. If we know that this good has unit elastic demand function, then find approximate demand function please.

XXVI. Q=45000-3000P
Q=7000P-5000
Find equilibrium price, quantity, market value. What kind of good is it, elastic, inelastic, unit elastic? Prove your words with calculation. Due to economic boom, government decreased taxes, which in return positively impacted both consumers and suppliers. Slope of demand function changed by 10%, while slope of supply function is about 15%. What are the new numbers in market? Show changes in graph.

XXVII. Write down real demand function for Ferrari please.

XXVIII. Coca-Cola and Pepsi are competitors. Increasing prices of Coca-Cola will make clients choose products of Pepsi. These types of goods are called substitute goods in economy. The goods that can easily be substituted by another one in market. There are also goods which might only be consumed together; chips and soft drinks, electricity and refrigerator, mobile phone and Communication Company, computer and software program, etc. These types of goods are called complimentary goods. Ferrari and Lamborghini are two luxury sport cars that are substitute to each other. What will happen to demand of Ferrari cars if Lamborghini lowers its prices? What will happen to prices of Ice making machines if price of electricity rises up?

XXIX. $P=1000-0.02Q$
$P=0.001Q+5.3$
Which one of them is demand and which one is supply function? Find all needed numbers and plot them on graph.

XXX. $P=Q^2$
$P=Q^2-150Q+5000$
Which one of them is demand and which is supply function? Find all needed numbers and plot them on graph ($P, Q>0$ at all times!)

XXXI. Guljemal has a monthly income of 7000 TMT, and she spends all her money on two goods: Cosmetics and Books. If average price of cosmetics is 50 and average price of books is 30, find and plot Guljemal's income constraint function. How much books she can buy if she decides to spend 2000 TMT on her cosmetics? How many cosmetics she can purchase if she decided not to buy more than 50 books a month? What will happen to her income constraint if she decides to save 3000 TMT each month?

XXXII. Wepa's monthly income is 9000 TMT, and he spends the money on three goods: Books, clothes and shoes. Draw Wepa's income constraint function if you know that he always buys only three pairs of shoes each month. (Average price of books=40 TMT, clothes=80 TMT, shoes=300 TMT). How many books Wepa can buy if he decided to spend only 2400 TMT for clothes this month? What will happen to Wepa's income constraint if his income rises to 11000 TMT? Draw his new constraint if he loaned some money from bank and needs to pay 1000 TMT of interest (income is 11000 TMT).

XXXIII. "Workaholic Advocates" legal consultation company's total cost is 9000 TMT (for office rent and guard wage) each

month (fixed!). If average consultation fees company charges 500TMT, how many clients they must serve in a month so that they start making profits? Plot total revenue and total cost functions on graph. If they can serve no more than one client in one hour, what is their maximum revenue and maximum profit in one month? (They work 9 hours a day, and six days a week! Workaholics!).

XXXIV. Goturlandia has a progressive taxing system, which means that tax rates increase as your revenue increases!
Revenue≤50000, Tax rate is 10% (of profit)
50000<Revenue≤100000, tax rate is 14%
100000<Revenue≤200000, tax rate is 22%
200000<Revenue≤300000, tax rate is 29%
300000<Revenue, tax rate is 45%
"Gedayjackets" company is new company and manufactures jackets and sells for 1000 Gopals (Goturlandia's currency!). If the company has fixed cost of rent 20000 Gopals a month and they spend 500 Gopals to produce each jacket, how many jackets they must produce each month to earn maximum profits if taxes were not imposed? After imposing of revenue taxes? Plot total revenue and total cost functions. What if tax was not imposed on profit, how would the situation changed?

Solutions:

I. Consumption goods: Food, clothes, books, radio, TV, shoes, chocolate, water, a house, a car. Investment goods: Stocks, bonds, derivatives, mutual funds, real estate, jewelry, gold, interest-bearing account, options, shares.

II. 5000*30%=1500 TMT for groceries
5000*20%=1000 TMT for rent
5000*5%=250 TMT for books
5000*3%=150 TMT for Mom
5000*1%=50 TMT for pet Tuzik
5000*8%=400 TMT for utility
5000*15%=750 TMT for perfume.
5000-1500-1000-250-150-50-400-750=900TMT
Meret's savings is 900TMT. I would recommend him invest to the bank.

III. Ashyr's expenses:
230 TMT for books
650 TMT for grocery
1200 TMT for rent
460 TMT for clothes
1000 TMT for investment
100 TMT is for a present
300 TMT is from his father as a present
320 TMT for schooling expenses
90 TMT for transportation
240 TMT for pet crocodile
230+650+1200+460+1000+100+300+320+
90+240=4590
Ashyr's earnings 4590 TMT earnings.

IV. 2000 TMT → 38%
x TMT → 50%

$$x = \frac{2000 * 50}{38} = 2631.58$$

V. 3000 TMT → 54%
x TMT → 100%

$$x = \frac{3000 * 100}{54} = 5555.55\ (Ayna's\ previous\ salary)$$

3000 TMT → 22%
x TMT → 100%

$$x = \frac{3000 * 100}{22} = 13636.36\ (Ayna's\ new\ salary)$$

VI. Π=0.12 TR
TC=0.88 TR
0.88 TR-30000=TC_2
Π=0.25 TR
Π=TR-TC → 0.25 TR=TR-(0.88 TR-30000)
0.25 TR=TR-0.88 TR+30000
0.13 TR=30000
TR=230769.23 TMT
TC= (230769.23*0.88-30000) =173076.92 TMT
Π=TR-TC → 230769.23-173076.92=57692.31 TMT

VII. 4500*85% (Ata's monthly expenses) + 1300 (for car accident) = 5125 TMT

VIII. 1TR=B
1.33TR=A
A+B=1000000
(1 + 1.33)TR =1000000
TR= $\frac{1000000}{(1+1.33)}$ =429184.6 TMT is B's Total Revenue
429184.6 *1.3=570815.5 TMT is A's TR

IX. $TC = \frac{9}{12} * 2000000 = 1.500.000$ TMT
Π=TR-TC=2000 000-1.500.000=500.000 TMT
500.000*20%=100.000 TMT profit of Guvanch
500.000*42%=210.000 TMT profit of Begench
500.000*38%=190.000 TMT profit of Dayanch

X. $\frac{3}{14}$ *3400 = 728,57 TMT for rent
$\frac{1}{3}$ *3400 = 1133.3 TMT for groceries
$\frac{1}{10}$ *3400 = 340 TMT for education
3400 - 728.57 - 1133.33 - 340 = 1198.1 TMT is saved.

XI. 7000+5000+5*3500+3*2500+4000=41000 TMT is Total cost (TC)
Π=TR-TC
52000=TR-41000
TR=52000+41000=93000 TMT
3500+2500=6000 TMT (if 2 employees are fired)
(41000-35000)/41000*100%= 14.6%

TC+3000=44000 TMT new total cost
(44000-41000)/41000*100%=7.3%
In order to just cover the expenses company must make 41000 TMT every month.
70000=TR-41000(previous cost) = 111000 TMT
=1110 clients a month to raise profit to 70000 TMT.

XII. Price Elasticity of quantity demanded at equilibrium price is:

Q=10000-34P

Q=666P-1000

10000-34P=666P-1000

11000=700P

P=$\frac{11000}{700}$=15.7 TMT

Q=10000-34*15.7=9466.2

Market volume: 15.7*9466.2=148619.34 TMT

If demand function changes to:

Q=9000-34P

New equilibrium will be:

9000-34P=666P-1000

10000=700P

P=$\frac{10000}{700}$=14.28 TMT

Q=9000-34*14.28=8514.48

Market volume: 14.28*8514.48=121586.8 TMT

If supply function changes to:

Q=750P-1000

750P-1000=9000-34P

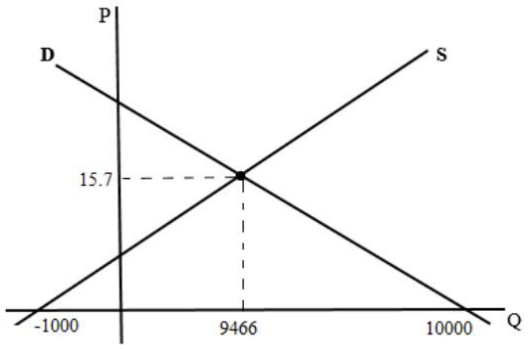
784P=10000

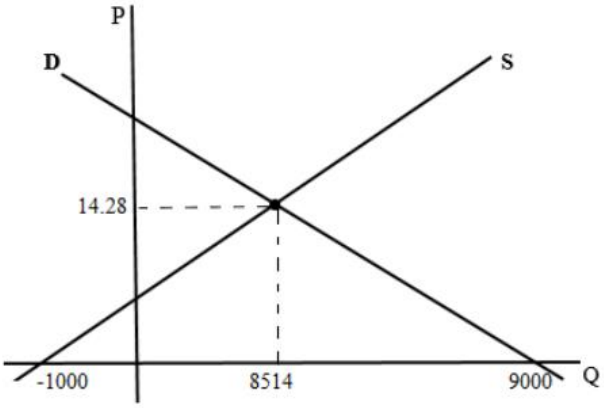
P=$\frac{10000}{784}$=12.75 TMT

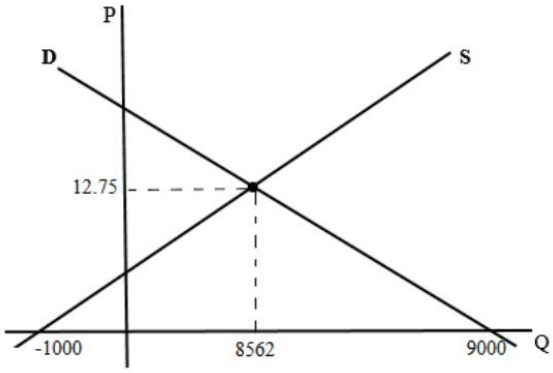
Q=750*12.75-1000=8562

New market volume: 12.75*8562=109165.5 TMT

$$\frac{\partial f(x)}{\partial x} * \frac{P}{f(x)} = \frac{(-34)*15.7}{10000-34*(15.7)} = -0.06 < 1 \; inelastic$$



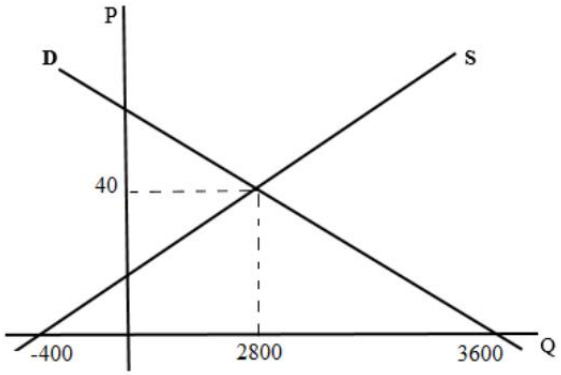




All calculations below will not appear, only graphs!

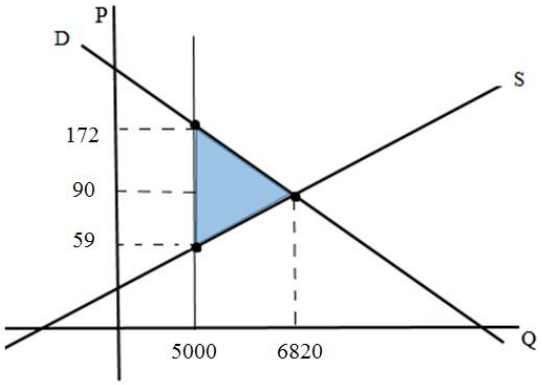
XIII. Market value: 40*2800=112000 TMT
If price of good in the market is 50 TMT, quantity of goods that will be sold:
3600-20*50=2600 units

$$\frac{\partial f(x)}{\partial x} * \frac{P}{f(x)} = \frac{(-20) * 40}{3600 - 20 * 40} = -0.286 < 1 \; inelastic$$



XIV. Market value: 90*6820=613800 TMT
Deadweight loss: 172-59=113
6820-5000=1820
$\frac{1820*113}{2}$=102830

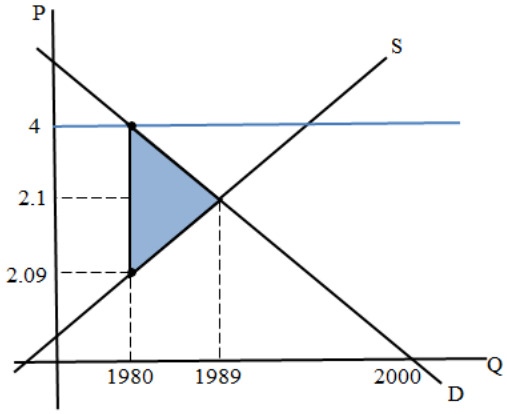
$$\frac{\partial f(x)}{\partial x} * \frac{P}{f(x)} = \frac{(-22) * 90}{8800 - 22 * 90} = -0.29 < 1 \; inelastic$$



XV. Market value: 2.1*1989=4177 TMT
Deadweight loss:
2000-5*4=1980
995*P-100=1980
P=2.09

$$\frac{(4 - 2.09) * (1989 - 1980)}{2} = 8.595$$

$$\frac{\partial f(x)}{\partial x} * \frac{P}{f(x)} = \frac{(-5) * 2.1}{2000 * 5 * 2.1} = -0.005 < 1 \; inelastic$$



XVI. Market value: 50*49500= 2475000

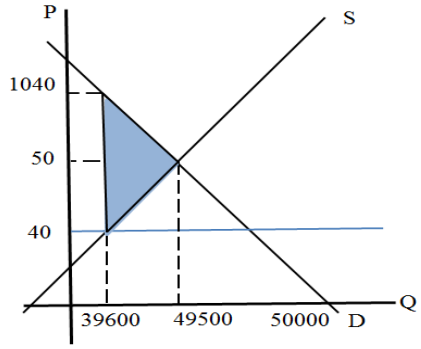
Deadweight loss when price is 40 TMT:

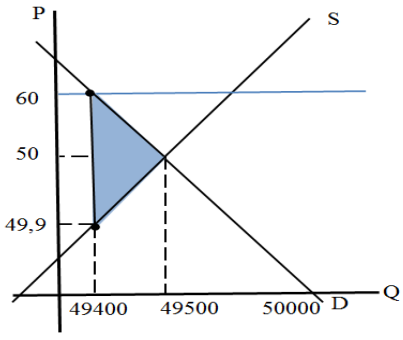
$$\frac{(1040-40)*(49500-39600)}{2}=4950000$$

Deadweight loss when price is 60 TMT:

$$\frac{(60-49{,}9)*(49500-49400)}{2}=505$$

$$\frac{\partial f(x)}{\partial x}*\frac{P}{f(x)}=\frac{(-10)*50}{50000-10*50}=-0{,}01<1\ inelastic$$





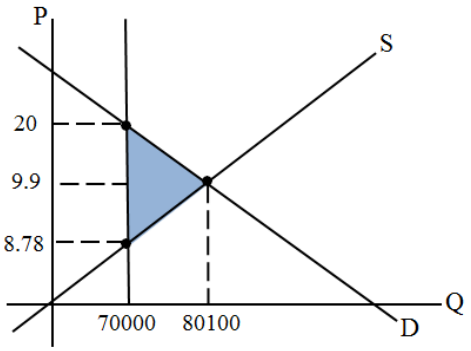
XVII. Market value: 80100*9.9=792990 TMT

When the government limits production to 81000, there will be no deadweight loss because E_q=80100, which is less than 81000.

Deadweight loss when quantity is 70000:

$\frac{10100*11.22}{2}$ = 56661

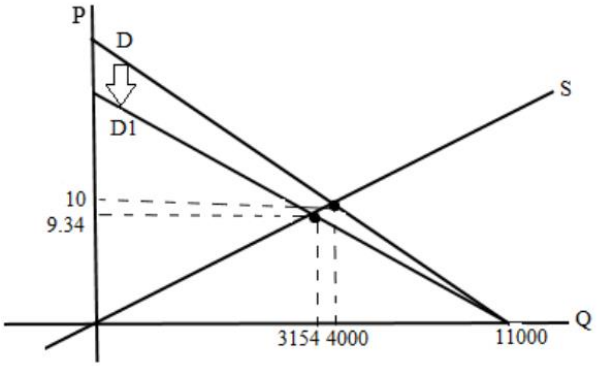
$\frac{\partial f(x)}{\partial x} * \frac{P}{f(x)} = \frac{(-1000)*9.9}{90000-1000*9.9} = -0.124 < 1\ inelastic$



XVIII. Market value: 10*4000=40000 TMT

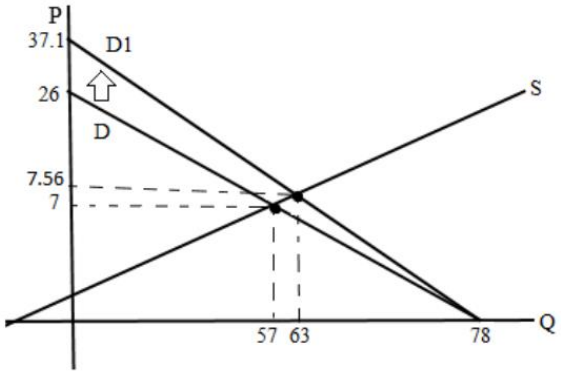
$\frac{\partial f(x)}{\partial x} * \frac{P}{f(x)} = \frac{(-700)*10}{11000-700*10} = -1.75 > 1\ elastic$ (we do not take into consideration the sign)

Old demand function: Q=11000-700P
New slope: -700+ (-700*20%) = -840
New demand function: Q=11000-840P
New equilibrium: 11000-840P=1300P-9000
20000=2140 P
P=9.34



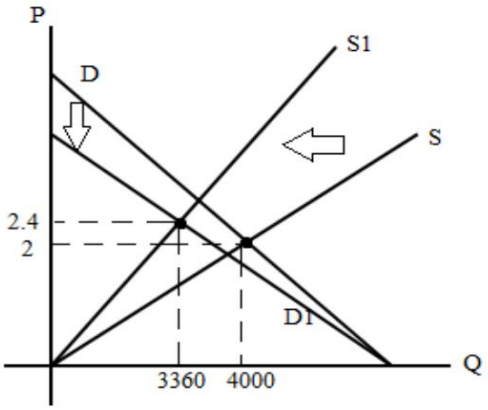
XIX. Market value: 7*57=399 TMT
Old demand function: Q=78-3P
New slope: -3-(-3*30%) =-2.1
New demand function: Q=78-2.1P
New equilibrium: 78-2.1P=9P-6
84=11.1P
P=7.56

$$\frac{\partial f(x)}{\partial x} * \frac{P}{f(x)} = \frac{(-3) * 7}{78 - (-3) * 7} = -0{,}37 < 1 \; inelastic$$



XX. Market value: 2*4000=8000 TMT
Old demand function: Q=5400-700P
New slope:-700+ (-700*20%) =- 840
New demand function: Q=5400-840P
Old supply function: Q=2000P
New slope: 2000-2000*30%=1400
New supply function: Q=1400P
New equilibrium: 1400P=5400-840P
2240P=5400
P= 2.4

$$\frac{\partial f(x)}{\partial x} * \frac{P}{f(x)} = \frac{(-700) * 2}{5400 - 700 * 2} = -0.35 < 1 \; inelastic$$



XXI. Q=1000P-1000
Q=1000*4-1000=3000
9000=1000P-1000
9000+1000=1000P
P=10
13000+1000=1000P

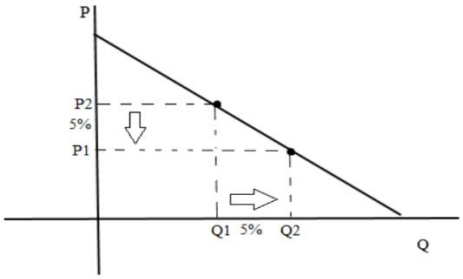
P=14 TMT

5000+1000=1000P

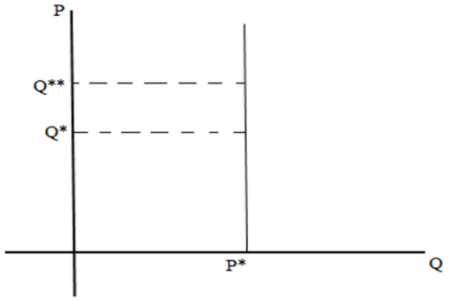
P=6 TMT

XXII. $\frac{\Delta Q}{\Delta P} = \frac{3\%}{10\%} = 0.3 < 1\ inelastic$

XXIII.



XXIV.



XXV. 3000 → 10%

X → 100%

$X = \frac{3000*100}{10} = 30000$

Q=30000

Since unit elastic function has coefficient equal to 1 (b=1),

it has a functional form of: Q=a-P

$\begin{cases} 30000 = a - P \\ 30000 - 3000 = a - 1.1P \end{cases}$

From 1st equation we deduct second:

3000=0.1P

P=30000

Put our finding to 1st equation:

30000=a-30000

A=60000

Q=60000-30000P

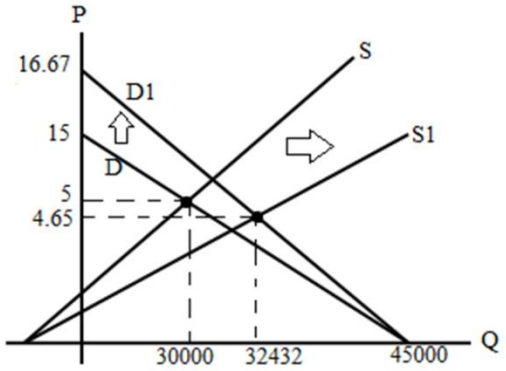
XXVI. Market value:30000*5=150000

Old demand function: Q=45000-3000P

New slope: -3000- (-3000*10%) =2700

New demand function: Q=45000-2700P
Old supply function: Q=7000P-5000
New slope: 7000+7000*15%=8050
New supply function: Q=8050P-5000
New equilibrium: 8050P-5000=45000-2700P
10750P=50000
P=4.65

$$\frac{\partial f(x)}{\partial x} * \frac{P}{f(x)} = \frac{(-3000) * 5}{45000 - 3000 * 5} = -0.5 < 1 \; inelastic$$



XXVII. Q_d(Ferrari)= ↘+b(wage)-c(price)+t(GDP)...

XXVIII. Since Lamborghini is a substitute for Ferrari, if price for Lamborghini decreases demand for Ferrari also decrease. And electricity is complementary good with ice making machine, when the price of electricity rises up, the price of ice making machine also rises. Ice making machines consume electricity and due to increase in electricity price, demand for ice cream machine decreases, because it increases price of ice cream itself thus, decreases demand.

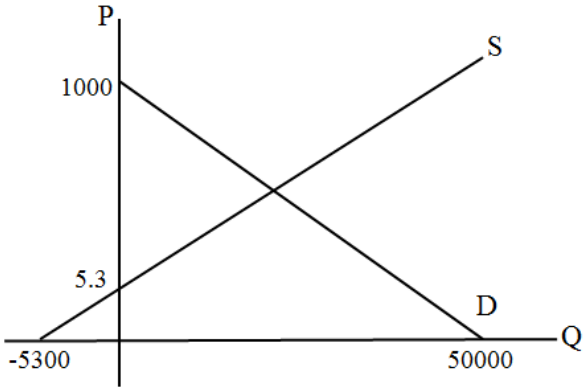
XXIX. P=1000−0.02Q (demand function)
0.02Q = 1000-P

$$Q=\frac{1000-P}{0.02}$$

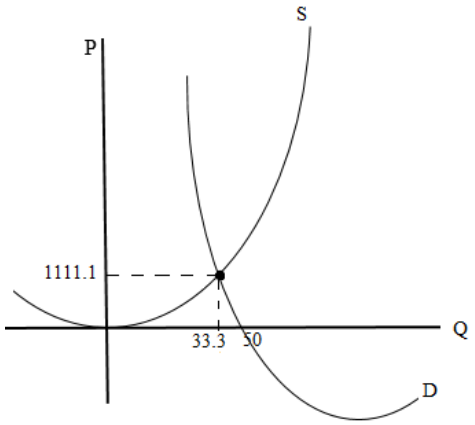
Q_d=50000-50P
P=0.001Q+5.3(supply function)
P-5.3=0.001Q

$$Q=\frac{P-5.3}{0.001}$$

Q_s=1000P-5300

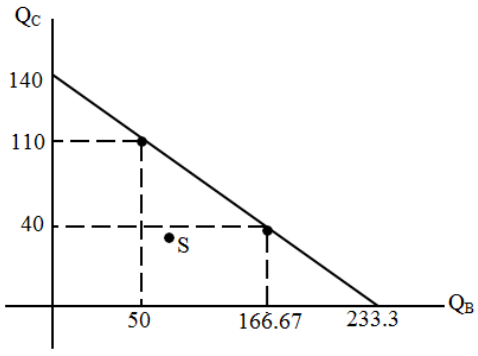


XXX.



XXXI. $7000=50Q_C+30Q_B$

$Q_B=233.3-1.67Q_C$



XXXII. $9000=40Q_B+80Q_C+300Q_S$

$Q_S=3$ always

$8100=40Q_B+80Q_C$

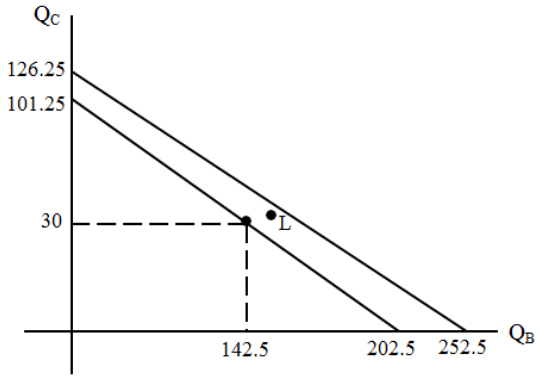
$202.5-2Q_C=Q_B$

If Wepa decides to spend only 2400 TMT for clothes, then he can buy:

$\frac{2400}{80}$ = 30 units of clothes

$9000=40Q_B+300*3+30*80$
$40Q_B=9000-900-2400$

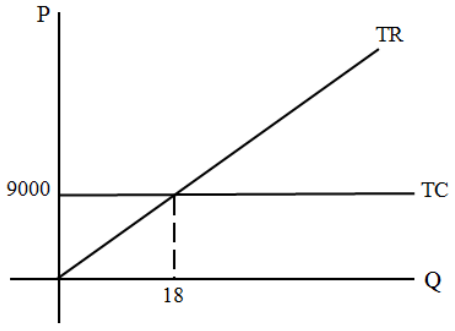
$Q_B=\frac{9000-900-2400}{40}$=142.5 books



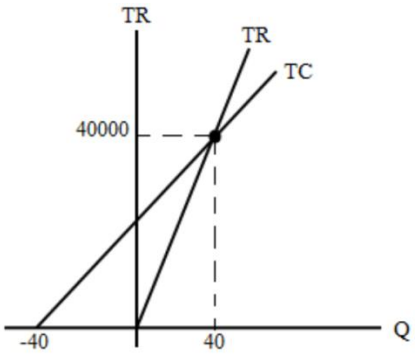
XXXIII. TC=9000

$\frac{9000}{500}$=18 > clients a month to start making profit

TR=9*500=4500 (9 hours a day)
4500*6=27000 (6 days a week)
27000*4=108000=TR (4 weeks in a month)
Π =TR-TC=108000-9000=99000TMT



XXXIV. TC=500Q+20000
TR=1000Q
500Q+20000=1000Q
20000=500Q
Q=40



Q≥40 At least to cover cost we must produce more than (or at least) 40.
If tax is imposed to profit:
TR=1000*40=40000 TMT
Π=TR-TC
Π=40000-(500*40+20000) =40000
40000-40000*10%=36000 TMT (if we produce 40 units)

What if our revenue is 50000<Revenue≤100000, tax rate is 14%:

We must produce: $\frac{100000}{1000}$ = 100 jackets

Π=TR-TC =1000*100-(500*100+20000) =30000 TMT

After imposing tax: 30000-30000*14%=25800 TMT

What if our revenue is 100000<Revenue≤200000, tax rate is 22%:

We must produce: $\frac{200000}{1000}$ = 200 jackets

Π=TR-TC=1000*200-(500*200+20000) =80000 TMT
After imposing tax: 80000-80000*22%=62400 TMT

What is our revenue is 200000<Revenue≤300000, tax rate is 29%:

We must produce: $\frac{300000}{1000}$ = 300 jackets

Π=TR-TC=1000*300-(500*300+20000) =130000 TMT

After imposing tax: 130000-130000*29%=92300 TMT

What is our revenue is 300000<Revenue, tax rate is 45%, say 400000 TMT:

We must produce: $\frac{400000}{1000}$ = 400 jackets

Π=TR-TC=1000*400-(500*400+20000) =180000 TMT

After imposing tax: 180000-180000*45%=99000 TMT

By looking at these calculations we can say that even if we use progressive tax system, as we increase quantity produced, our profit also increases.
If tax is not imposed on profit, the company must produce as much as possible.

Chapter 2: Essential Microeconomics 2

Now, after we had some salad to begin with, we are going to have entrée. I strongly recommend starting to study economics from fundamentals of microeconomics. Not a vice versa! Microeconomics is the field of economics where consumer's and producer's consumption, production, savings, market structure and other interrelated behaviors (such as wages, optimal price, moral hazard, adverse selection, etc.) are studied. If student understands this, the rest is not a problem (Theoretically!)! Let's begin with market structures.

As it was mentioned in Chapter 1, markets are theoretically divided into three categories, despite the fact that only one type exists: mixed economy. These are a) *free market economy* where only consumers and suppliers decide how much to produce and what to charge for the product without any interference, even state. This type of market structure exists only in theoretical world of economics! History does not have a single example where absolute *free market* economy existed. Advocates of this *fairy tale,* excuse me please, *free market economy (liberals, neo-liberals, laissez-faire fanatics, Santa Clause believers, Yeti witnesses, Greenpeace activists and etc. Yep, I put them all in one line!)* always tried to downplay the role of the government as a regulator of the market. History showed that, no matter how hard all those people tried, governments always proven to be the main resort of trust and reliance when things go out of control in markets. *Free market economy* means that markets reach efficient results without any intervention, which in fact means that we do not need governments! This is not true. You can believe to any idea you want, but without facts and implementation examples, this idea is going to exists as an *idea* only.

In second category we had *command economy*, which is absolute opposite of *free market economy,* in this market structure consumers and suppliers (actually, supplier is government here) does not have any voices at all. Prices and quantities are set by government. Take it or leave it! Although there were couple of such type of economies decades ago: Soviet Unions, People's Republic of China, Vietnam, North Korea and others, the structure proven to be inefficient and many of those economies restructured (and some of them dissolved, Soviet Union) themselves to more efficient structure, *mixed economy.* In *command economy,* central government decided how much to produce and what price to put, this created huge inefficiency due to lack of information on markets, immense bureaucracy, wrong strategic choices and corruption, eventually "hitting the wall" of market failure, which created millions of poor, unhealthy and unemployed societies. *Mixed economy*, divides priorities and roles of government and private markets. Through laws and regulations, the interference of

government to the market is limited, but regulatory role is expanded. Today, governments use *fiscal and monetary policies* to regulate the markets, and also *environmental laws* to regulate the ecology. *Budget spending* is also vital element of government regulatory instruments. *Defense of population* and *security* is also on governments, without which no economy can survive. While market is given right and power (by law!) to make decisions on price and quantity of production, governments regulate the safety and security of those products and services. I strongly believe that only *mixed economic* structure can lead to efficient results for humanity[4]. In this book, we will work with *mixed economy* structure and leave *fairy tale worlds* to philosopher and literature critics. In *mixed economy,* markets are left to consumers and suppliers; unless it threatens the welfare of society, where states intervene immediately (they have to, by law!). Consumers demand goods and services, and suppliers try to satisfy them. This creates a competition. Goods and services that satisfy consumer demand the best, becomes more popular, which leads to higher profits for suppliers. Those suppliers that could not satisfy the needs of consumers eventually end up in bankruptcy and leave the market. Society evolves and develops and its needs and demands increase and evolve too. Suppliers try to catch up with that "social evolution" making money on the go, if they succeed of course. Consumers got born and die, same is for companies. Millions of people born and millions die, millions of companies created and millions perish in time without achieving anything. In this "hustling and bustling" world, the smart, the strong, and the fast ones survive. This competition for profits, for big money, pushes companies to create new products, improve quality of goods and services, work more efficiently (low cost high profits!) and invest to the future. Consumers benefit from this "competition of suppliers" because thanks to their competition (or greed), consumers get better, safer, new and advanced products every year. Competition creates magic! The most of goods and high quality services that you see around is due to competition. Those businesses want to make more money, want to survive, they want to attract more customers that is why they are working hard and being nice to you. Will your hairdresser be nice to you if she was the only one around? How good the service quality would have been if there was only one Cab Company in your city? Thanks to competition consumers get lower prices and better quality of products. The problem is, the markets are never stationary! The markets are dynamic! Companies created, some survive, they got bigger and richer, and others fail. The ones that succeed get bigger and richer and slowly attain some power and reputation in market. There are thousands of beverage companies in the world but we

[4] Throughout this book I will put forward many questions for advocates of free market and command economy. I will be happy to get rationally satisfying answers for them.

know only few (Coca-Cola, Pepsi, etc.). This power and reputation changes the rules of game of the market. The size and wealth of a company hugely impacts its behavior in the market. In the market where all suppliers have almost the same size and power will create a *perfectly competitive* market. When certain companies get bigger and wealthier (thanks to their hard work!) certain companies die (fall out of market due to unsuccessful competition!). Let's take beverage business and assume that newly 50 companies enter the business. At the beginning, beverages market will have *perfect competition!* Later on, thanks to hard work and correct market strategy certain companies will start winning the competition, while others will start losing it. Slowly, number of companies will drop to 10 (assumption!). These 10 companies now will be the main players in beverage business. This type of market structure is called *oligopoly*, in economics science. If those 10 companies continue fiercely competing, their numbers will decrease, and if we assume even further, only one will be left in market. The market where only one supplier exists is called *monopoly.* Do all monopolies are created through competition? No of course! Some sectors of economies carry strategic importance to leave it in hands of entrepreneurs that is why *monopolistic* structure is preferred. This is called *natural monopoly.* Let's take a newly created company from beginning and analyze its structure as it evolves to monopolist.

"Bling-bling Diamond Mining Company" (BBDMC) is a newly created diamond mining company. Assume that diamond industry is highly competitive at the beginning (as always!) and that there are hundreds of other diamond mining companies similar to BBDMC. High competition puts pressure on prices, which means that there are many miners and because each wants to sell mined diamonds as soon as possible, they charge the lowest price possible. If miners put high prices, consumers simply will choose another supplier. That is why all miners go with prices that are on the market right now. In *perfect competition, prices are exogenous (given), suppliers do not have any control over prices. They would not be able to make sales if they charge high prices, because there are so many sellers out there.* Producer of tomatoes cannot charge any price he wishes, he charges the price that is around the market price, or otherwise he would not be able to make sales. Same applied to almost all goods and services provided in economy. Take for example beauty salons for women. All of them up and down charge almost the same price for services they provide. Why do they do that? They do it because otherwise clients will choose another salon. For the sake of our example let's assume that right now, the price of 1 carat of raw diamond is about 10,000 Manat in the market.

$$\text{Revenue } (R) = P*Q = 10{,}000*Q$$

Here, as we have learned in previous chapter, revenue (R) is equal to quantity of goods and services sold (Q), multiplied by its price (P). In *perfect competition*, since prices are known and dictated by market, in our case it is 10,000 Manat per carat of diamond, the only unknown variable is quantity. Total revenue of the company will depend on a quantity of diamonds it mines only, because prices are exogenous, given, and the company cannot do anything about it. Again, in *perfect market* competition, companies do not have power over prices; prices are dictated by market demand only.

BBDMC's total cost function consists of these costs: labor wage, (w=2000 Manat*)*, rentals of equipment, (r=7000 Manat) and fixed land rent (20000 Manat) and cost of refining (cleaning and cutting diamond) of each carat of diamond is 1000 Manat. Since BBDMC is a new company, they only have labor force of 10 people and they rented 3 mining equipment.

$$\text{Total Cost (TC)} = 1000*Q+2000*10+3*7000$$
$$TC=1000*Q+41000$$

Here, (1000*Q) is a *variable cost*, which means that cleaning and cutting costs depends on quantity of diamonds mined, thus, this costs will vary according to quantity. Second half of the total cost equation is called *fixed costs*, this cost does not depend on company's production and company will have to pay it no matter how much diamonds it mined. *Fixed cost* does not depend on quantity produced that is why it is called fixed cost. BBDMC will have to pay wages even if it mines zero diamonds. Rentals of equipment and land fees are also must be paid no matter company's situation. Those things are put on contract (labor contract, rental contract, etc.) that is why it is obligatory. Those costs are *"fixed by law".*

How much profit company will earn? We know from previous chapter that:

$$\text{Profit}=\text{Total Revenue}-\text{Total Cost}$$
$$\Pi=TR-TC$$
$$\Pi=10000*Q-(1000*Q+41000)$$
$$\Pi=10000*Q-1000*Q-41000$$
$$\Pi=9000*Q-41000 \text{ (}\textit{profit function!}\text{)}$$

From this profit function we understand that in order to *break even*, which means to produce as much as to cover all your costs and make zero profits, BBDMC must at least mine:

$\Pi=0 \leftrightarrow$ Total revenue=Total cost

(*Break even condition*!)

$\Pi=9000*Q-41000=0$

$9000*Q=41000$

$$Q = \frac{41000}{9000}$$

Q=4.5 carats of diamonds.

In order to cover all costs (*break-even*) BBDMC needs to mine 4.5 carats of diamond.

How much the company needs to mine to start making profit? It needs to mine more than 4.5 carats.

$\Pi>0$ ↔ Total revenue>Total cost
(*Profit condition*!)

$\Pi=9000*Q-41000>0$

$9000*Q>41000$

$$Q > \frac{41000}{9000}$$

Q>4.5 carats of diamonds.

When company is going to be in loss? When it produces less than 4.5 carats of diamond. When BBDMC produces lesser than 4.5 carats of diamond, the earned revenue is not even covering costs.

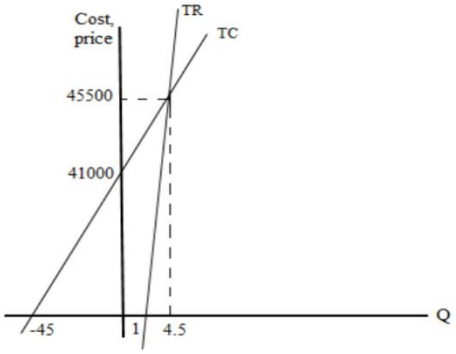
$\Pi<0$ ↔ Total revenue<Total cost
(*Loss condition*!)

$\Pi=9000*Q-41000<0$

$9000*Q<41000$

$$Q < \frac{41000}{9000}$$

Q<4.5 carat of diamond.



How much the company must mine to earn maximum profit? **It depends on functional form of a cost function**! In this particular example we have linear cost function

(TC=1000*Q+41000):

As you can see from the graph, at Q=4.5 company breaks even (revenue and cost function intersection!), makes no profits. As company increases mining, the profits grow (gap between revenue and total cost function grows as Q goes to infinity!).

$$\lim_{Q\to\infty} \Pi(Q) = \infty$$

$$\lim_{Q\to\infty} (9000 * Q - 41000) = \infty$$

The company's profits will go as it mines more! More mining, more profits. Let's put some numbers and see:

Assume company mines 100 carats, then profits will be:

$$\Pi=9000*Q-41000$$

$$\Pi=9000*100-41000=859000$$

What if company mines 200 carats?

$$\Pi=9000*Q-41000$$

$$\Pi=9000*200-41000=1759000$$

What if company mines 1000 carats?

$$\Pi=9000*Q-41000$$

$$\Pi=9000*1000-41000=8959000$$

Our finding is true: as mining increases, profits increase![5]

When the company makes maximum loss? **It also depends on a functional form of cost function!** Let's look at our *loss condition;* we found above that *loss condition* is:

$\Pi=9000*Q-41000<0$

$Q<4.5$ carats of diamonds

This means: the smaller the Q, the bigger the loss! Let's put some number and see, let's assume the company mines only 3 carats of diamonds, then:

$$\Pi=9000*Q-41000<0$$

$$\Pi=9000*3-41000$$

$$\Pi= -14000$$

The company will make a loss of 14000 Manat. Let's decrease Q and make it 2, then:

$$\Pi=9000*Q-41000<0$$

[5] Always check your findings by putting numbers to your equation. Make it a habit!

$$\Pi=9000*2-41000$$

$$\Pi= -23000$$

The company will make a loss of 23000 TMT. Let's decrease Q and make it 1, then:

$$\Pi=9000*Q-41000<0$$

$$\Pi=9000*1-41000$$

$$\Pi= -32000$$

The company will make a loss of 32000 TMT. What if company does not mine anything, thus Q=0, then:

$$\Pi=9000*Q-41000<0$$

$$\Pi=9000*0-41000$$

$$\Pi= -41000$$

If company does not mine anything, it will acquire 41000 TMT of loss. This is the maximum loss the company can acquire **in this condition.**

Exercise 1: *"Pinocchio's brothers" furniture company is manufacturing handmade armchairs. Making of one armchair cost 100 TMT (and company has 7000 TMT fixed cost/month). If company makes 200 armchairs a month;*
a) What must be the price to break-even?
b) What must be the price to make 20000 TMT profit?
c) Graph revenue, total cost and profit functions

Break-even condition⟶ TR=TC

a). TC=100Q+7000
Q=200

P*200=100*200+7000

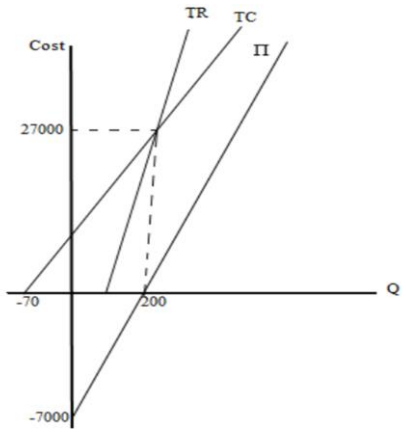
P=135

b) Π=TR-TC

20000=P*200-27000

P=235

c) Graph:



Maximum and minimum losses and profits that can companies acquire depends **only** on their functional forms of **cost function**! Cost functions can be of any form: linear, exponential, logarithmic, quadratic, cubic, constant, etc. but they all must fulfill one very important condition:

***Price* and *Quantity* cannot be negative at all times!!!**

Minimum P=0

Maximum P=∞

Minimum Q=0

Maximum Q=∞

Technically speaking (mathematically), **Price (*P*) and Quantity (*Q*)** ϵ **R**+, Price and Quantity are elements of positive real numbers. This definition is used in economics science where Price and Quantity is concerned. In real life though, we have to agree that when $\boldsymbol{P} \epsilon \mathbf{R}+$ (we can have prices such as 3.40 USD, 9.99 EURO, or even 2.04 RUPEES) while we never speak about manufacturing or buying a 7/6 portion or 34/245 portion of good, thus Quantity is element of only positive integer numbers. Thus, $\boldsymbol{Q} \epsilon \mathbf{Z}+$.

Keeping in mind this condition, we can make some analysis of ***revenue, cost*** and ***profit functions***.

Revenue function (P*Q):

Since P and Q cannot be negative, revenue function cannot be negative at all times! As per formula, revenue depends on quantity demanded and price! We know from our demand function is that price and quantity demanded have negative correlation. Higher the price, the lower the quantity demanded. Lower the price, higher the quantity demanded. Q in our revenue function represents quantity demanded, shortly demand function!

$$\lim_{P\to\infty} Revenue = 0$$

$$\lim_{P\to 0} Revenue = 0$$

$$\lim_{Demand\to 0} Revenue = 0$$

$$\lim_{Demand\to\infty} Revenue = \infty$$

$$\lim_{Supply\to\infty} Revenue = 0$$

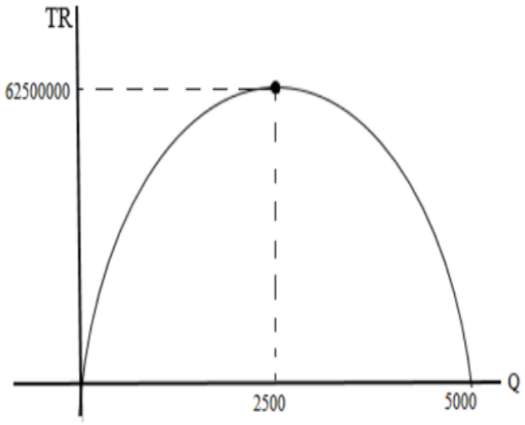
$$\lim_{Supply\to 0} Revenue = \infty$$

A. Revenue function is never negative, because price and quantity cannot be negative!

Since P, Q ≥ 0, then Revenue ≥ 0 (always!)

B. Revenue functions have parabolic (quadratic) structures! Find P in terms of Q in above demand equation and let's solve it, see if anything changes:

Let's for the sake of study, assume that demand function for diamonds in market is



$Q=5000-0.1*P$

$Revenue=P*Q$

$Revenue=P*(5000-0.1*P)$

$Revenue=5000*P-0.1*P^2$

Let's solve this equation:

$Revenue=5000*P-0.1*P^2$

When Revenue=0, then

$$0=5000*P-0.1*P^2$$

$$0=P*(5000-0.1*P)$$

$$P_1=0$$

$$0=5000-0.1*P_2$$

$$5000=0.1*P_2$$

$$50000=P_2$$

$$Q=5000-0.1*P$$

$$Q-5000=-0.1*P$$

$$50000-10*Q=P$$

$$Revenue= (50000-10*Q) *Q$$

$Revenue= 50000*Q-10*Q^2$ (again, quadratic form!)

Revenue=0, then

$$0= (50000-10*Q) *Q$$

$$Q_1=0$$

$$50000-10*Q=0$$

$$Q_2=5000$$

As it could be seen, revenue function has parabolic (quadratic) structure.

C. Revenue function has only one maximum! As it could be seen from above graphs, revenue function both in terms of Quantity and Price has only one maximum point. According to given demand function, maximum revenue is achieved when P=25000, and Q=2500;

$$Revenue=5000*P-0.1*P^2$$

First order condition for finding maxima and minima is:

$$\frac{\partial Revenue}{\partial Price}=0$$

$$5000-0.2*P=0$$

$$P=25000$$

In terms of Q:

$$Revenue= 50000*Q-10*Q^2$$

First order condition for finding maxima and minima is:

$$\frac{\partial Revenue}{\partial Quantity}=0$$

$$50000-20*Q=0$$

$$Q=2500$$

Actually, it is enough to find only maxima point of revenue in terms of any variable P or Q, and simply put it into the demand function. In example above we found maxima of revenue when P=25000, just put it into the demand function:

$$Q=5000-0.1*P$$

$$Q=5000-0.1*25000$$

$$Q=2500$$

Simple!

Now, according to our calculations BBDMC can earn maximum revenue when P=25000 and Q=2500 for given function of market demand. *Be careful, price and quantity depends on market demand!* Any change in demand will definitely impact prices and quantities, and eventually revenues and profits too.

$$Revenue=P*Q$$

$$Revenue=25000*2500=62500000$$

Maximum revenue that can BBDMC can earn for given demand function is 62500000. Let's check answer now (as always!). What if prices are 30000, instead of 25000? Then:

$$Q=5000-0.1*P$$

$$Q=5000-0.1*30000$$

$$Q=2000$$

$$Revenue=30000*2000=60000000$$

At a price of 30000, market will demand only 2000, and thus revenue will be only 60000000, which is less than 62500000! Change price to 20000, then:

$$Q=5000-0.1*P$$

$$Q=5000-0.1*20000$$

$$Q=3000$$

$$Revenue=20000*3000=60000000$$

Again, result is lesser than 62500000! You can try any combination of P and Q but eventually will find out that maximum revenue is earned when P=25000 and Q=2500 for this example.

Marginal Revenue (MR = $\frac{\partial Revenue}{\partial Quantity}$***):*** Used to calculate by how much the revenue will change if one more units of good is produced! *Marginal revenue* calculation is the same for all markets but results change according to market structure! In perfect competition, where prices are fixed and exogenous (suppliers do not have power over prices), *Marginal revenue is equal to price!*

$$Revenue=P*Q$$

$$\boldsymbol{Marginal\ Revenue=\frac{\partial Revenue}{\partial Quantity}=\frac{\partial(P*Q)}{\partial Q}=\frac{R_2-R_1}{Q_2-Q_1}=P}$$

This in fact means that, in perfect competition market, where prices are fixed and exogenous, no matter how much supplier produces, the additional revenue will be equal to price of that good!

What is the marginal revenue of BBDMC in our example (do not forget that we are in perfect completion!)?

Revenue=P*Q
Revenue=10000*Q

$$\boldsymbol{Marginal\ Revenue = \frac{\partial Revenue}{\partial Quantity} = \frac{\partial(P * Q)}{\partial Q} = 10000}$$

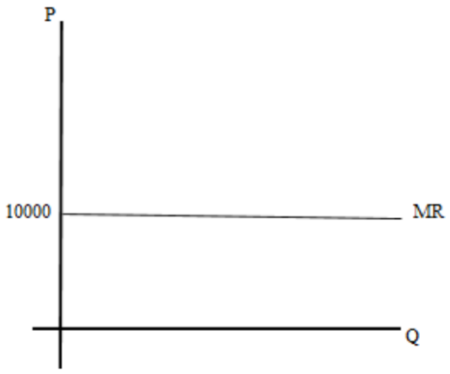
BBDMC will earn 10000 Manat for each carat it mines! (Fixed!)
In numbers, assume that BBDMC is mining 100 carats of diamond. Then:

$Revenue_1$=P*Q
$Revenue_1$=10000*100=1000000

How much revenue will increase if BBDMC decides to mine 101 carats?

$Revenue_2$=P*Q
$Revenue_2$=10000*101=1010000

$$\boldsymbol{Marginal\ Revenue = \frac{\partial Revenue}{\partial Quantity} = \frac{\partial(P * Q)}{\partial Q} = \frac{R_2 - R_1}{Q_2 - Q_1} = P}$$



In imperfect competition (monopoly and oligopoly) environment, suppliers have power over prices because there are only few suppliers. That is why; their Q will depend on market demand (it is actually a market demand). Supplier knows, the more he produces, the prices will go down (oversupply). The less he produces; the prices will go up (because of shortage). Marginal revenue function of monopolist and oligopolies will look like this:

Revenue=P*Q

Q=a-b*P (demand function where a, b>0)

$$P = \frac{a - Q}{b}$$

$$\text{Revenue} = \left(\frac{a - Q}{b}\right) * Q = \frac{a * Q - Q^2}{b}$$

$$\text{Marginal revenue} = \frac{\partial Revenue}{\partial Quantity} = \frac{a-2Q}{b}$$

As we can see, *marginal revenue* is not a constant function here (like in perfect competition) but decreasing linear function. As quantity supplied increases, *marginal revenue* decreases in imperfect competition!

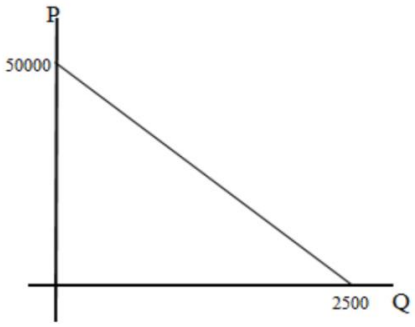
Let's assume, for the sake of example, that diamond industry is in imperfect competition, and BBDMC is the only supplier. Then:

$$Revenue = P*Q$$

$$Q = 5000 - 0.1*P$$

$$P = 50000 - 10*Q$$

$$Revenue = (50000 - 10*Q)*Q = 50000*Q - 10*Q^2$$



$$\text{Marginal revenue} = \frac{\partial Revenue}{\partial Quantity}$$

$$\text{Marginal revenue} = 50000 - 20*Q$$

As we can see, as quantity supplied increase, marginal revenue decreases. Assume BBDMC mines 2000 carats. Its marginal revenue is:

$$\text{Marginal revenue} = 50000 - 20*Q$$

$$\text{Marginal revenue} = 50000 - 20*2000$$

$$\text{Marginal revenue} = 10000$$

The 2000th mined carat of diamond increased revenue of BBDMC by 10000 TMT! How much will add 2001?

$$\text{Marginal revenue} = 50000 - 20*Q$$

$$\text{Marginal revenue} = 50000 - 20*2001$$

$$\text{Marginal revenue} = 9980$$

The 2001st mined carat of diamond increased revenue of BBDMC by 9980 TMT! Marginal revenue is decreasing!

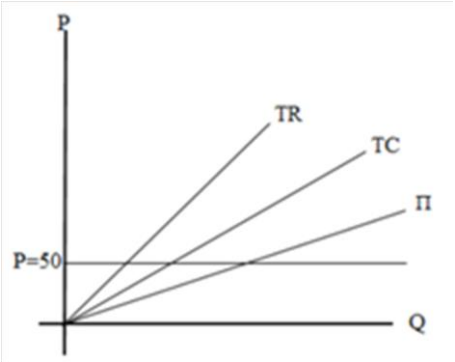
Exercise 2 *"Angry Squirrel" company sells nuts in a very highly competitive market. The company has only a variable cost of 30 TMT per pound of nuts (labor cost for collecting). The company sells them for 50 TMT per pound.*

a) Write down and graph revenue function, cost function and profit function

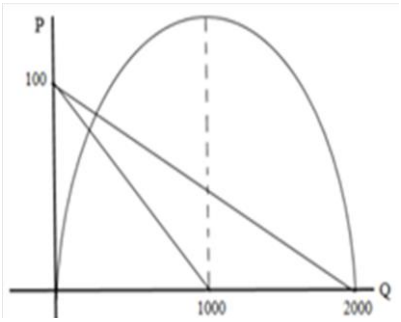
TR=50Q→MR=P=50

TC=30Q→MC=30

Π=TR-TC=50Q-30Q=20Q



b) If we assume that "Angry Squirrel" is monopolist with demand function
Q=2000-20*P,
then graph revenue function, marginal revenue function and find how many pounds of nuts the company must supply in order to earn maximum profit?
Show all functions in one graph. (R, MR, TC, Profit)



c) What is the marginal revenue of 88th pound of nuts for the company?
MR_{88}=100-0.1(88) = 91.2*

Cost function (TC):

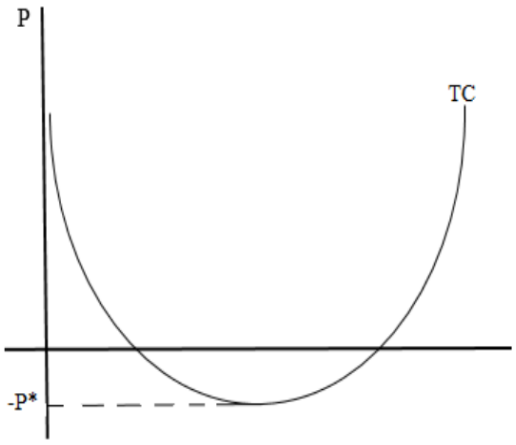
As it was mentioned above, cost functions can be of any form, but there are certain conditions that all of them must satisfy (except the main condition P,Q>0). **Failure of any of these conditions will make function unusable in cost analysis!**

A. **Cost function is never negative, even when Q=0!** It is illogical and unreal having cost function negative. Can you explain what does negative cost function mean actually?

$$\lim_{Q\to\infty} TC(Q) = \infty$$

TC (Q) ≥ 0 (always!)

B. Cost function never intersects Q axis, which means, cost function never has roots! By intersecting Q axis, cost function will break the main rule of economics: P, Q>0. As you can see from graphs, when cost function intersects the Q axis, P becomes negative. So, when cost function of complex form is used in microeconomic analysis, first thing one must do is to check, if this cost function has roots. If it does, then cost function is inappropriate or unusable!



C. If cost function has non-linear form, then this cost function must have only one positive minimum! Let's solve some examples with cost functions. Given:

$$TC=Q^2-40Q+200$$

First of all, we check the conditions of cost function, does it satisfy or not, so first put Q=0 and check if cost function is positive.

$$TC=Q^2-40Q+200$$
When Q=0, then TC=200

This cost function satisfies our first condition; it is not negative even when Q=0. Here, if you have paid attention, cost function consists of two parts: the part that depends on Q, which is (Q^2-40*Q), and the other independent, which is 200. The part of cost dependent to Q is called a *variable cost. Variable costs* include all costs that depend on quantity of production. *Variable cost will increase as production increases, decrease as production decreases.* The volumes of milk or cheese you produce depend on number of cows. Quantity of apple harvest depends on number of trees you have. Quantity of meat you produce depends on cows and feeding, etc. Fixed part of the cost function is called *fixed cost* (surprisingly!). Fixed cost does not depend on quantity of production; fixed cost does not depend on production at all. These are the costs that no matter what supplier must acquire to keep up production. These costs are: office rents, salaries of employees, gas prices, etc. For employees, it does not matter how much you produce and sell, they are on a contract, so they will get what they ought to get! Office building owner does not care about your profits or losses, because he knows you will pay your rent anyway. As it can be seen from our cost function, supplier will pay 200

TMT even if produces zero product. Unfortunately, this cost function fails even our first condition because first condition says that total cost function must be positive at all times! When we put Q=10:

$$TC=Q^2-40Q+200$$

When Q=10, then TC=-100

Cost function cannot be negative at any Q! So, this function even fails our first condition. Also, you will see that TC<0, when 6≤Q<35. This function does not satisfy us as a cost function. You can trash this function immediately but for the learning purposes we will continue to the second condition.

Secondly, check the next condition: does it have roots? To answer, it is enough to find if

$$\sqrt{b^2-4ac}<0$$

If it satisfies, then our total cost function is appropriate for economic analysis. If it does not, it means our total cost function have roots, which makes our cost function incorrect, thus useless. In our total cost function:

$$\sqrt{b^2-4ac}=\sqrt{(-40)^2-4*1*200}=\sqrt{800}>0$$

It does not also satisfy our second condition! Given total cost function has roots. This is not correct cost function and that is why cannot be used in economic analysis. Let's change our cost function to

$$TC=Q^2-10Q+200$$

Check first condition:

Q=0, then TC=200>0

Try couple of numbers for example:

$$TC=Q^2-10*Q+200$$

When Q=1, then TC=191

When Q=5, then TC=175

When Q=10, then TC=200

When Q=100, then TC=9200

When Q=1000, then TC=990200

It all looks good; first condition is satisfied! Go for second condition now:

$$\sqrt{b^2 - 4ac} < 0$$
$$\sqrt{(-10)^2 - 4 * 1 * 200} = \sqrt{-700} < 0$$

Second condition is satisfied too! Go to third condition now; check, does it have minimal or maxima? First order condition for finding minimum or maximum points of a function is to take first derivative of a function and equalize it to zero:

$$\frac{\partial Total\ Cost}{\partial Quantity} = 0$$

How do we know did we find maxima or minima? To check this, we need to take second derivative of the function, and if it is negative we found maxima. If second derivative is positive, we have found minima. The logic behind is this: first derivative of a function shows the change of the function when moved from point to another. At peak points (bottom or top), change in independent variable a bit, will makes almost zero changes. Top or bottom of a function is like a line, slope is zero, and change between two points is zero. That is why first order condition of finding maxima or minima is always equals to zero. Now, how do we know if we found maxima or minima? We take second derivative to find rate of change of a function at peak points. If this rate of change is negative, it means "we were on a top point and now moving downwards". If the rate of change is positive, it means "we were on a bottom and now we started moving upwards".

$$\frac{\partial^2 Total\ Cost}{\partial^2 Quantity} < 0\ \ (Maximum)$$

$$\frac{\partial^2 Total\ Cost}{\partial^2 Quantity} > 0\ (Minimum)$$

In our example:

$$TC=Q^2-10Q+200$$

$$\frac{\partial Total\ Cost}{\partial Quantity} = 2Q - 10$$

$$\frac{\partial^2 Total\ Cost}{\partial^2 Quantity} = 2 > 0$$

This proves that our total cost function has one minimum and this minimum is positive!

At what quantity of production we have the minimum cost?

$$\frac{\partial Total\ Cost}{\partial Quantity} = 0$$

$$2Q - 10 = 0$$

$$Q = 5$$

Minimum cost will be at Q=5, we can check it;

$$TC=Q^2-10Q+200$$

When Q=5, then TC=175

When Q=10, then TC=200

When Q=3, then TC=179

When Q=0, then TC=200

When Q=100, then TC=9200

As you can see, minimum total cost is when the company produces only 5.

Is it so important to have positive minimum point? Yes indeed. Assume we had a total cost function in form of:

$$TC=Q^2+10Q+200$$

This function totally satisfies our first ($TC>0$ at all $Q>0$) and second conditions (Have no roots! $\sqrt{b^2-4ac}<0$) (Check it yourself!). But, fails our third condition: this function has one minimum but this minimum is negative! Having negative minimum point is economically irrational. Imagine you invest tons of money to build cement factory, with huge fixed costs and labor contracts signed already. Is it economically feasible not to produce anything or at least something for you? If you do not produce anything you will pay huge fixed cost and your cost will be higher than you produce at least something to cover portion of your costs. In real life, if companies do not produce anything they will have higher costs than if they produce at least something. Let's take two cost functions and make analysis:

$$TC_1=Q^2-10Q+200$$

$$TC_2=Q^2+10Q+200$$

If both companies produce nothing, their total loss will be 200 (Q=0). When production starts, you will see that first company's cost goes down and reaches minimum at Q=5, while second company's cost only rises. If companies had cost function with minimum negative points, then at first hardship times they would

have stopped production, because they lose minimum money when they do not produce anything. Actually, it is never true. Companies do not shut doors at first hardship, actually for them it is better (less costly!) to keep on producing. At least to cover variable costs! Cover wages! Companies lose more money when they stop production, and lose lesser when they keep on going, at least till they can cover variable costs. That is one of the main reasons why I prefer using cost quadratic costs functions in the form of:

$$TC=aQ^2-bQ+c \ (a,b,c>0)$$

Because they have positive minimum, which is close to real life!

Marginal cost (MC): This calculation shows how much my total cost will increase if one more unit of good is produced.

$$\frac{\partial Total\ Cost}{\partial Quantity} = MC$$

In our example,

$$TC=Q^2-10Q+200$$

$$\frac{\partial Total\ Cost}{\partial Quantity} = MC = 2Q - 10$$

How much my total cost increases when I produce 50th good? My total cost increases approximately by:

$$MC = 2Q - 10 = 2 * 50 - 10 = 90$$

Let's check the answer; what is my total cost when I produce 49th goods?

$$TC=Q^2-10Q+200$$
$$When\ Q=49,\ TC=2111$$
$$When\ Q=50,\ TC=2200$$

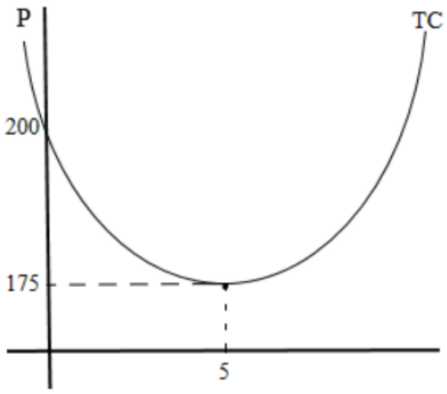
$TC_{50}-TC_{49}=89$ (almost 90!)

How much my total cost increases when I produce 90th good? My total cost increase approximately by:

$$MC = 2Q - 10 = 2 * 90 - 10 = 170$$

Let's check the answer; what is my total cost when I produce 89th goods?

$$TC=Q^2-10Q+200$$
$$When\ Q=89,\ TC=7231$$
$$When\ Q=90,\ TC=7400$$
$$TC_{90}-TC_{89}=169\ (almost\ 170!)$$



Exercise 3: *"Wild Rabbit Carrot Jam Company" produces high quality carrot jams. The economist, Bagul, of this company mistakenly came up with three different cost functions; Help economist Bagul to choose which one is appropriate cost function and explain why. Then graph these cost function, find marginal cost and find minimum cost.*

$TC_1=9Q^2+100Q+2000$

Firstly, check TC when Q=0→TC=2000> 0

Secondly, we check roots: $\sqrt{100^2 - 4 * 9 * 2000} = \sqrt{-62000} < 0$

Thirdly, we check whether it has only one maximal or minimum?

$$\frac{\Delta TC}{\Delta Q} = 18Q + 100 = 0$$

Q= -100/18 = -5,55 < 0 max

We get the minimum cost when quantity is negative

$TC_2=10Q^2-100Q-2000$

I. Q=0→TC = -2000 < 0 this is inappropriate TC function

$TC_3=8Q^2-90Q+1000$

a) Q=0→TC=1000

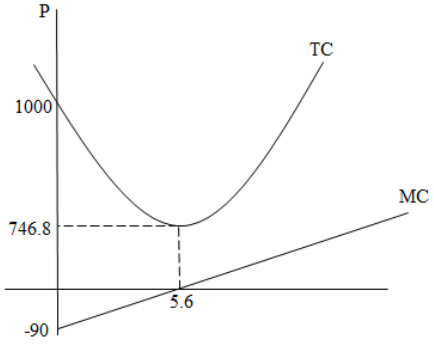
b) Roots: $\sqrt{(-90)^2 - 4 * 8 * 1000} = \sqrt{-23900} < 0$

d). $\frac{\Delta TC}{\Delta Q} = 16Q - 90 = 0$

Q=5.6 is a minimal

TC (5.6) =746.88 is a price!

Only third Total cost function is appropriate.



Profit function (П)

Profit function is found by subtracting total cost function from revenue function:

Profit=Revenue-Total cost

Π=R-TC

A. If profit function is non-linear, then it has only one positive maximum point! Let's see it in example; take cost function we used above and assume that price is 30 TMT per unit:

$$TC=Q^2-10Q+200$$

$$Revenue=P*Q=30Q$$

$$\Pi=30Q-(Q^2-10Q+200)$$

$$\Pi= -Q^2+40Q-200$$

Now, how much the company must produce to earn maximum profit? The answer is, same as we did with revenue and cost functions: We have to take *first order condition!*

$$\frac{\partial Profit}{\partial Quantity} = 0$$

And *second order condition,* for determining did we find maximum or minimum. Let's continue with our profit function:

$$\frac{\partial Profit}{\partial Quantity} = -2Q + 40 = 0$$

Second order condition:

$$\frac{\partial^2 Profit}{\partial^2 Quantity} = -2 < 0 \text{ (Maximum!)}$$

The company will earn maximum profit when it will produce:

$$-2Q+40=0$$
$$Q=20$$

Maximum profit is:

$$\Pi=-Q^2+40Q-200$$

$$Q=20, \Pi=200 \text{ TMT}$$

$$Q=10, \Pi= 100 \text{ TMT}$$

$$Q=30, \Pi=100 \text{ TMT}$$

$$Q=50 \ \Pi=-700 \text{ TMT (loss!)}$$

As you can see from above results, indeed the company earns maximum profit when it produces 20 units of good.

***Exercise 4**: "Fighting Giraffe" boxing promotion company is placing fighting event every Saturday night. Average ticket price they charge is 1000 TMT. The company has total cost function:*

$$TC=2Q^2-2000Q+600000$$

A). Check the equation:

When $Q=0\rightarrow TC>0$

$$\sqrt{(-2000)^2-4*2*600000}=\sqrt{-800000}<0$$

$$\frac{\Delta TC}{\Delta Q}=4Q-2000=0$$

$Q=500$ is a minima;

*B). Π=TR-TC→P*Q-TC*

$$\Pi=1000*Q-2*Q^2+2000*Q-600000$$

$$\frac{\Delta \Pi}{\Delta Q}=3000-4Q=0$$

$$Q=750$$

$$\Pi=1000*750-2*750^2+2000*750-600000=525000$$

$$\Pi\ (700)=1000*700-2*700^2+2000*700-600000=520000$$

$$\Pi\ (800)=1000*800-2*800^2+2000*800-600000=520000$$

Prices in perfect and imperfect market competition!

I just want to add a little thing here, important thing: In perfect competition, prices are known; it is exogenous, given, so suppliers produce according to existing prices. In imperfect competition, prices are not given, it will depend on supply! The more monopolists produce, the lower the price will get. So, when calculating maximum profits one must pay attention to the structure of market. Let's see it in example:

Assume company A is functioning in perfectly competitive environment where price of a good is given as 30 TMT and company's cost function is:

$$TC=Q^2-20Q+120$$ (check cost function!)

How much company must produce to earn maximum profit in perfectly competitive environment?

$$\Pi = \text{Revenue-Total Cost}$$

$$\Pi = P*Q-(Q^2-20Q+120)$$

$$\text{We know the price } P=30$$

$$\Pi = 30Q-(Q^2-20Q+120)$$

$$\Pi = 30Q-Q^2+20Q-120$$

$$\Pi = -Q^2+50Q-120$$

First order condition for finding maximum and minimum:

$$\frac{\partial Profit}{\partial Quantity} = 0$$

$$\frac{\partial Profit}{\partial Quantity} = -2Q + 50 = 0$$

$$Q = 25$$

Company A must produce 25 units of good. What will be the maximum profit?

$$\Pi = -Q^2+50Q-120$$

$$Q=25, \Pi=505 \text{ TMT}$$

Always check answer!

$$\Pi = -Q^2+50Q-120$$

$$Q=20, \Pi=480 \text{ TMT}$$

$$\Pi = -Q^2+50Q-120$$

$$Q=30, \Pi=480 \text{ TMT}$$

I always recommend putting one below and one above number of the result! (Here, it is 20 and 30).

So, only 25 units of good will be produced in the market? No. Company A will produce 25 units of goods, and there are hundreds of other companies like Company A in market. Assume that demand function is:

$$Q=2000-20*P$$

How many goods will be produced in total? In total:

$$Q=2000-20*30$$

$$Q=1400$$

1400 units of goods will be produced in total in the market for price of 30TMT. How many companies will be there in the market? Approximately:

$$1400/25=56$$

56 companies will be in market, each producing 25 units of goods for 30 TMT a piece.

Now, let's assume that Company A is a monopolist! In perfect competition, companies did not worry about demand function, and honestly they did not care about it, because none of the companies played major role and could not impact prices. Prices are known, and they just calculated their costs and produced accordingly. In the case of a monopolist, price and quantity are not known! Monopolist decides how much to produce and what to charge, with one condition, monopolist knows that demand is negatively correlated with price: lesser monopolist supplies, the higher the price will be, and vice versa. So, how monopolist decides how much to produce to earn maximum profit? Firstly, profit function is constructed:

$$\Pi=\text{Revenue-Total cost}$$

Here, we will assume that technology and process does not change, that is why total cost does not change. We can use the same total cost functions used in perfect competition:

$$TC=Q^2-20Q+120$$

What about revenue function? In perfect competition, it was easy, we knew the price and revenue was simply:

$$\text{Revenue}=P*Q$$

$$\text{Revenue}=30Q$$

In imperfect competition, prices are not known! Prices will depend on demand function! From demand function we find:

$$Q=2000-20P$$

$$P=100-0.05Q$$

We use this price function in revenue for monopolist:

$$\text{Revenue}=P*Q$$

$$\text{Revenue}= (100-0.05Q)*Q$$

$$\text{Revenue}= 100Q-0.05Q^2$$

Then, profit function is constructed:

$$\Pi=\text{Revenue-Total Cost}$$

$$\Pi=100Q-0.05Q^2-(Q^2-20Q+120)$$

$$\Pi=100Q-0.05Q^2-Q^2+20Q-120$$

$$\Pi=-1.05Q^2+120Q-120$$

First order condition for finding maximum and minimum:

$$\frac{\partial Profit}{\partial Quantity}=0$$

$$\frac{\partial Profit}{\partial Quantity}=-2.1Q+120=0$$

$$Q=57.14$$

In order to earn maximum profit, monopolist must produce 57 units of good! (We know that there is no such a thing as 0.14 unit of good). What will be the maximum profit?

$$\Pi=-1.05Q^2+120Q-120$$

$$Q=57,\ \Pi=3309$$

$$Q=50,\ \Pi=3255$$

$$Q=65,\ \Pi=3243$$

Monopolist earns maximum profit when produces 57 units of good as it can be seen from above. Our finding is correct. What about monopolist's price then? We simply use demand function for this:

$$Q=2000-20P$$

$$57=2000-20P$$

$$P=97.15$$

Monopolist will put a price of 97 TMT. Let's check the maximum profit again:

$$\Pi=\text{Revenue-Total Cost}$$

$$\Pi=97.15*57-(57^2-20*57+120)=3309$$

As you can see, the quantities and prices in the market depend on the market structure! Monopolist will charge higher price and produce lower amount of goods. In perfect market competition prices will be lower due to pressure from competition and quantities of the goods will be higher. That is the main reason why countries choose market economy and encourage competition, because it brings more efficient results by increasing population welfare.

What if price changes in perfect market economy? Then quantity produced will change too. Suppliers will adapt to new prices in

market. Let's see this from our example, assume price dropped from 30 TMT to 20 TMT:

$$\Pi = \text{Revenue-Total Cost}$$

$$\Pi = P*Q-(Q^2-20Q+120)$$

$$\text{We know the price } P=20$$

$$\Pi = 20Q-(Q^2-20Q+120)$$

$$\Pi = 20Q-Q^2+20Q-120$$

$$\Pi = -Q^2+40Q-120$$

First order condition for finding maximum and minimum;

$$\frac{\partial Profit}{\partial Quantity} = 0$$

$$\frac{\partial Profit}{\partial Quantity} = -2Q + 40 = 0$$

$$Q=20$$

Company A must produce 20 units of good. What will be the maximum profit?

$$\Pi = -Q^2+40*Q-120$$

$$Q=20, \Pi=280 \text{ TMT}$$

$$Q=10, \Pi=180 \text{ TMT}$$

$$Q=30, \Pi=180 \text{ TMT}$$

Always check answer!

At a new price of 20 TMT, each company will produce only 20 units of goods and maximum each will earn 280 TMT of profit.

Let's increase the price from 30 TMT to 50 TMT now:

$$\Pi = \text{Revenue-Total Cost}$$

$$\Pi = P*Q-(Q^2-20Q+120)$$

$$\text{We know the price } P=50$$

$$\Pi = 50Q-(Q^2-20Q+120)$$

$$\Pi = 50Q-Q^2+20Q-120$$

$$\Pi = -Q^2+70*Q-120$$

First order condition for finding maximum and minimum:

$$\frac{\partial Profit}{\partial Quantity} = 0$$

$$\frac{\partial Profit}{\partial Quantity} = -2Q + 70 = 0$$

$$Q=35$$

Company A must produce 35 units of good. What will be the maximum profit?

$$\Pi = -Q^2+70*Q-120$$

$$Q=35, \Pi=1105 \text{ TMT}$$

$$Q=30, \Pi=1080 \text{ TMT}$$

$$Q=40, \Pi=1080 \text{ TMT}$$

Always check answer!

At a new price of 50 TMT, each company will produce only 35 units of goods and maximum each will earn 1105 TMT of profit. These price fluctuations are due to changing market demand. Question is this: in perfect market competition, does price ever come to stability or do they change all the time? Big profits attract more companies. As you can see from example, increasing prices increase profits, and this attracts more companies. As more companies start to enter the market (as supply increases), the prices go down. Lower the prices go, companies earn less profits and this scares off the suppliers from market. This chaotic motion, due to demand and supply side changes stop one day and market (theoretically!) come to equilibrium. In a long run, prices must come to the stable number, and this number equals to ***minimum average cost (ATC).*** Supplier will continue producing until they earn zero profit! (Imagine you produce waffles. If average cost of waffle is 20 TMT and prices are 20 TMT, what is your profit? Zero). When this occurs? When

$$\text{Price}=\text{Minimum Average Cost}$$

When prices fall below the minimum ATC suppliers start to acquire losses. They stop production there. Let's see what is the long run equilibrium price in our example where:

$$TC=Q^2-20Q+120$$

Let's first find ATC:

$$\text{Average Total Cost} = \frac{Total\ Cost}{Quantity}$$

$$ATC = \frac{Q^2 - 20Q + 120}{Q}$$

$$ATC = Q - 20 + \frac{120}{Q}$$

First order condition for finding maximum and minimum:

$$\frac{\partial ATC}{\partial Quantity} = 0$$

$\frac{\partial ATC}{\partial Quantity} = 0 \rightarrow 1 - \frac{120}{Q^2} = 0$

Q=10.95

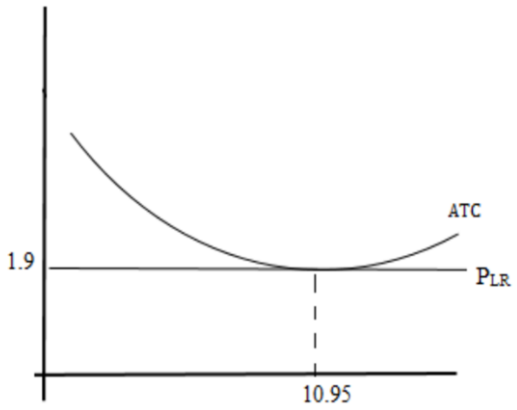
After finding Q,

we put it into the ATC:

$ATC = Q - 20 + \frac{120}{Q}$

Minimum ATC (Q=10.95)

=1.9



In a long run, prices will be equal to 1.9 TMT. At that price long run equilibrium will be attained in market, but until that time, fluctuations in prices will continue.

Oligopoly

Perfect competition environment is never stationary. Market is always in dynamic motion: companies born and die, consumers come and go. Successful companies with successful business strategy eventually start winning the market (increasing share of its products in market). This leads some companies go out of business and while flourishing others. It is rule of nature; nobody can do anything about it. After certain period of time you will see that at any sector there are big players and small players. You cannot keep all companies at the same size or forbid them competing. It is in the rule of competition to have a winner and loser. Companies merge, associations are created, conglomerates are born but at the same time companies are divided. This process never stops. When we have a perfect competition, when prices are known and almost all companies have the same cost structure, there is not much left for strategizing. Same is for the monopolist. When you are the only one supplier, you decide the price and quantity; you do not need genius strategy to make profit. The game starts when there are few strong and big players left in the market. How will they behave? Collude or compete? Unite and be a cartel or compete to death?

These questions are researched a lot in economics science. One of the major fields of economics science is actually totally focuses on this area: Game theory. There are a lot of models, competitions and strategies used for analysis. Generally, almost all of them agree on one thing, which I believe all financier experts must know: **When markets have oligopolistic market structure and companies decide to compete with each other rather than unite, prices will drop to perfect competition levels (zero profit) and quantity also will be produced as if in the perfect competition. Instead, if they choose to unite, prices and quantities produced will be of monopolist.**

Homework:

I. Monopolists MR function is MR=200-0.4Q. Find monopolists demand function. Graph demand and marginal revenue functions.

II. A company has a fixed cost of 90000 TMT. If MC= 10Q-1000. Find TC function. Show MC and TC in the graph.

III. A company has a linear total cost function with fixed cost of 50000 TMT. If the company produces 1000 unit of goods, it's ATC=94 TMT. Find and graph TC function of the company.

IV. A company is functioning in highly competitive environment and it's MR=120. If the company has MC=6Q-900, and FC=50000 TMT. $TC=3Q^2-900Q+50000$. How many goods a company must produce to earn maximum profit?

V. Assume a company is in a perfect market competition has a linear TC function with FC=10000 TMT. Profit 1000TMT when company produces 1000 units. If cost of each product is 4TMT, what is a current market piece?

VI. Q=2000-10P
$TC=Q^2-30Q+300$
Find monopolist's price and quantity. Show all functions in graph (MR, TC, Revenue, profit).

VII. Q=3000-40P
$TC=2Q^2-20Q+60$
Find monopolist's price and quantity. Show all functions in graph (MR, TC, Revenue, profit).

VIII. Q=3000-40P
$TC=2Q^2-20Q+60$
Find supplier's price and quantity in perfect market competition in a long run. Show all functions in graph (MR, TC, Revenue, profit).

IX. $\Pi=-5Q^2+100Q-200$
Profit function of a supplier in perfect market competition is at market price of 50 TMT is given above. How much profit is supplier earning right now?

X. Prove me that in perfect market competition, profit equals zero actually means: MR=MC.

XI. ATC of a supplier in perfect market competition is given below. Find long run equilibrium price.

$$ATC=2Q-70+\frac{1000}{Q}$$

XII. 94^{th} unit of good increases total cost by 182 TMT, 95^{th} unit of good increase total cost by 184 TMT, 96^{th} unit of good increases total cost by 186 TMT. Find quadratic total cost function of a supplier if fixed cost equals to 10 TMT.

XIII. $Q=4500-50P$
$TC=3Q^2-10Q+50$
Find monopolist's price and quantity. Show all functions in graph (MR, TC, Revenue, profit).

XIV. If monopolist produces 1000 units, prices will go down to 29, if monopolist produces 2000, prices go down to 28, if monopolist produces 3000 units, prices go down to 27. Find demand, revenue and marginal revenue functions of a monopolist. Graph them all.

XV. ATC of a supplier in perfect market competition is given below. 200^{th} good will increase my total cost by how much?

$$ATC=Q-90$$

XVI. Revenue $=450Q-0.3Q^2$
$TC=Q^2-100Q+7000$
Do functions shown above intersect? Graph it.

XVII. "Hotel Keloglan" has 2500 rooms of which: 100 rooms of luxury class for 2000 TMT/night, 350 rooms of business class for 1200 TMT/night, 600 double bed rooms for 800 TMT/night and 50 VIP room of extra class for 20000 TMT/night. The rest of the rooms are for 350 TMT/night. Total number of employees working at the hotel is 180, of which 5 is executive employees and paid 9000 TMT per month, 20 of them hold managing positions and paid 6500 TMT per month, 4 chefs are paid 6000 TMT per month, 3 accountants are paid 5500 TMT each. The rest of employees are paid 4000 TMT on average.
a) What is MR of one VIP room?
b) What is the MC of hiring one more chef?

c) What is daily total revenue if all rooms are occupied?
d) What is total cost of hotel?
e) What is MC of hiring one more accountant?
f) If you were to cut the costs, from where would you started?
g) If you were to increase revenue, how would you do that?
h) What is monthly revenue and monthly cost if all rooms are occupied only for 1/3 of the month, and if workers are also paid only for 1/3 of the month?

XVIII. $Q=1300-12*P+0.2*I$
Q=quantity demanded
P=price of the good
I=income

a) How much of the good is demanded with 2000 TMT income and 100 TMT price of the good?
b) By how much the demand will change if income is increased by 10%?
c) How would demand be impacted if prices drop by 30%?
d) Find maximum revenue of supplier (take income fixed as 2000 TMT)
e) Find maximum profit of the supplier if you know that $TC=Q^2-50*Q+700$
f) How profit changes when income rises by 20%?
g) What will happen to demand if we assume that economy plummeted and income is zero (price is still 100 TMT)?

XIX. $TC=Q^2-100Q+7000$
$Q=6300-10P$
Demand and cost functions are given. Find how much would have been produced and price both in monopoly and perfect competition.

XX. $TC=Q^2-100Q+70$
$TC=7Q^2-1.9Q+23$
$TC=-4Q^2+9Q+8$
$TC=50Q^3-Q+55$
$TC=2Q^2+10Q+110$
$TC=-Q^2-7Q-30$
$TC=Q^2-100Q-200$
$TC=Q^2-1090Q$
$TC=3Q^2+50Q+80$
$TC=10Q+6$
$TC=-Q+88$

$TC=100Q$
$TC=800$
$TC=4Q^2$
$TC=120Q-300$
$TC=-30Q-600$
$TC=23Q^3+80$
$TC=Q^7+20Q+50$
$TC=200Q^2-12400Q+970000$
Which one above cost functions are good for using it in cost analysis?

XXI. $TC=Q^2-10Q+135$
Find maximum profit of a supplier in perfect competition at prices 120, 130 and 150 respectively. Graph all needed functions.

XXII. $TC=3Q^2-90Q+1000$
Find maximum profit of a supplier in perfect competition at prices 350, 400 and 500 respectively. Graph all needed functions.

XXIII. $R=300Q-0.03Q^2$
$R=10Q-3Q^2$
$R=Q-Q^2$
$R=-22Q+5Q^2$
$R=100Q^2$
$R=80Q$
$R=1000Q-1000Q^2$
$R=60Q+10Q^2$
$R=10Q+0.00008Q^2$
$R=3400Q-0.0050Q^2$
Which one from above functions is appropriate to use it as a revenue function?

XXIV. $\Pi=Q^2-70Q-120$
$\Pi=-Q^2+300Q-2000$
$\Pi=-Q^2-60$
$\Pi=-Q^2+50Q$
$\Pi= Q^2-20Q$
$\Pi= -Q^2-100Q-90$
$\Pi=5Q^2+35Q+1000$
$\Pi= -3Q^2+Q-400$

$\Pi = -Q^3+67Q-250$
$\Pi = Q^4-10Q^3-49Q^2+300Q-2000$
$\Pi = -8Q^2$
$\Pi = 3900$
$\Pi = 29Q^2-760$
$\Pi = Q+3$
$\Pi = -500Q+5000$
$\Pi = -Q^3+300Q^2-400Q$

Which one of the profit functions above are appropriate to use as a profit function?

XXV. MR of 2000^{th} good is zero for Monopolist. Price drops to 100TMT at that quantity of production. Find the demand function.

XXVI. Monopolist produces **N** amount of good, at that quantity he earns max revenue that coincides with maximum profit. What must be a functional form of the total cost function of a monopolist?

XXVII. Monopolist has an option to choose one of the technologies for production. Each of those technologies impact total cost function of monopolist. Below are given total demand and three total cost functions. Choose the technology that maximizes the monopolist's profit.

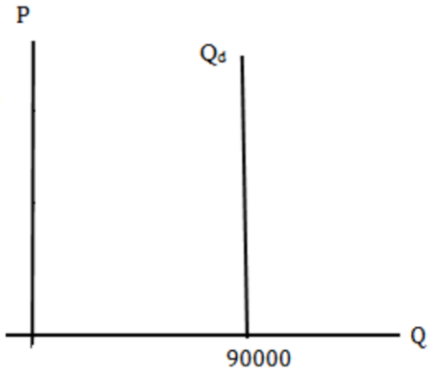
$TC=9Q^2-80Q+500$
$TC=2Q^2-110Q+2000$
$TC=30Q^2-1000Q+9360$
$Q=3000-30P$

XXVIII. Average total cost function of a company in perfectly competitive environment is given below. If prices are right now 200 TMT for that good. Does company make profit or lose money? How prices will change in future?

$$ATC=Q-88+\frac{9090}{Q}$$

XXIX. Construct total cost and revenue functions with tangent critical (maximum and minimum) points.

XXX. With vertical demand function shown below, what is the monopolist's maximum profit?
Q=90000



XXXI. Two companies are the main players in market (oligopolistic market structure), and both have different technology which is reflected in their cost functions below.
$TC_1=10Q^2-50Q+300$
$TC_2=7Q^2-60Q+500$
Instead of competing, both companies decided to unite and share monopolist profits at price of 70 TMT. If they suddenly decide to compete, which company will win over?

Solutions:

I. MR=200-0.4Q

Demand function of a monopolist: Q=a-bP

In terms of P: $\frac{a-Q}{b}$=P

TR=P*Q→$\left(\frac{a-Q}{b}\right)$*Q→$\frac{aQ}{b}-\frac{Q^2}{b}$

MR= $\frac{\alpha TR}{\alpha Q}\rightarrow\frac{a}{b}-\frac{2Q}{b}$

$\frac{a}{b}$=200

$-\frac{2Q}{b}=-0.4Q$

b=5

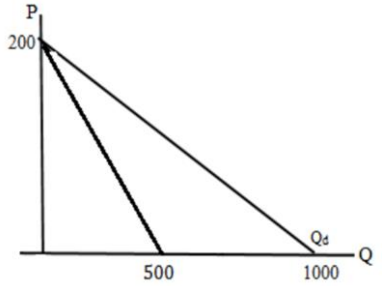
a=1000

Demand function: Q=1000-5P

Demand function in terms of P:

P=$\frac{1000-Q}{5}$

P=200-0.2Q



II. FC=90000

MC=10Q-1000

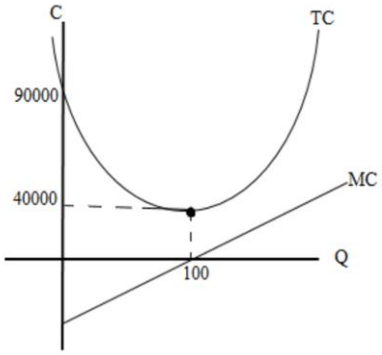
MC = $\frac{\Delta TC}{\Delta Q}$ = (10Q-1000)

$\int$(10Q-1000)dQ=

$\frac{10Q^2}{2}-1000Q=5Q^2-1000Q+C$

C=9000

TC=5Q²-1000Q+90000



III. FC=50000

When Q=1000→ATC=94

TC=aQ+50000

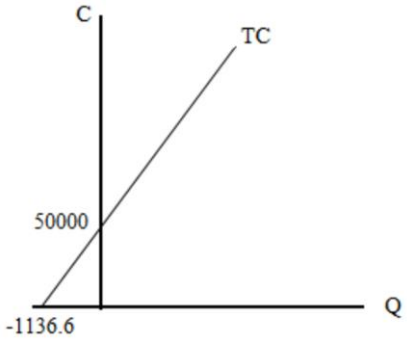
$\frac{TC}{Q}=ATC$

$\frac{aQ}{Q}+\frac{50{,}000}{Q}=94$

a+$\frac{50{,}000}{1000}=94$

a=44

TC=44Q+50000



IV. MR=P=120

MC=6Q-900

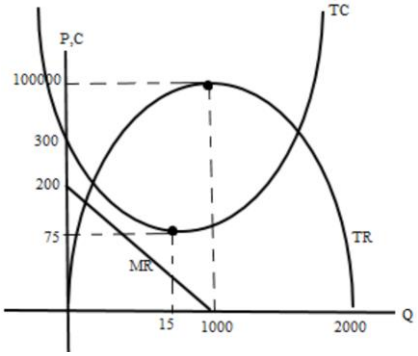
FC=50000

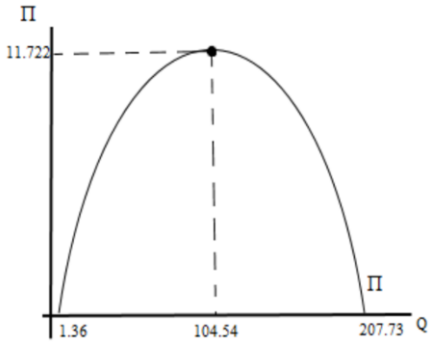
Because company is in highly competitive environment:

MR=MC
120=6Q-900
1020=6Q
$Q = \frac{1020}{6} = 170$

V. FC=10000TMT
TC=aQ+10000
Π=1000TMT when Q=1000
TC=4Q+10000
Π=TR-TC
1000=1000*P-4*1000-10000
15000=1000P
P=15

VI. Q=2000-10P
TC=Q^2-30Q+300
Π=TR-TC
$\frac{2000-Q}{10}$ = P (demand function in terms of P)
P*Q =(200-0.1Q)*Q =TR
200Q-0.1Q^2=TR
Π=TR-TC=200Q-0.1Q^2-Q^2+30Q-300
Π=-1.1Q^2+230Q-300
$\frac{\Delta\Pi}{\Delta Q} = 0 \rightarrow$ -2.2Q+230=0
-2.2Q=-230
Q=104.54
104.54=2000-10*P
P=189.5





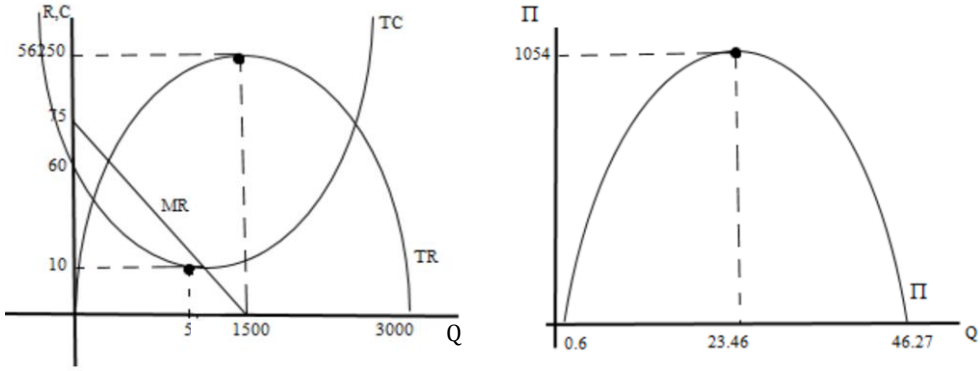
VII. Q=3000-40P
TC=2Q^2-20Q+60
P=$\frac{3000-Q}{40}$ = 75−0.025Q
TR=P*Q=75Q-0.025Q^2

MR=75-0.05Q

Π=TR-TC→ 75Q-0.025Q²-2Q²+20Q-60

$\frac{\Delta\Pi}{\Delta Q} = 0 \rightarrow$ -4.05Q+95=0

$Q = \frac{95}{4{,}05} = 23.46$



VIII. TC=2Q²-20Q+60

ATC=$\frac{TC}{Q}$ = 2Q-20+$\frac{60}{Q}$

$\frac{\Delta ATC}{\Delta Q} = 0 \rightarrow 2 - \frac{60}{Q^2} = 0$

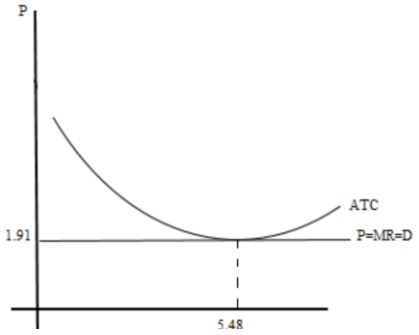
2Q²=60

Q²=30

Q=5.48→ we take only positive quantity

ATC (5.48) = 2*(5.48)-20+$\frac{60}{5.48}$ = 1.91=P_{LR}

In a long run price is equal to minimum average total cost.



IX. Π=-5Q²+100Q-200

Profit is maximized when:

$\frac{\Delta\Pi}{\Delta Q}$=0 → -10Q+100=0

Q=10

Π=500-200=300

X. Π=TR-TC

Profit equals to zero when:

$\frac{\Delta \Pi}{\Delta Q} = 0 \rightarrow \frac{\Delta TR}{\Delta Q} - \frac{\Delta TC}{\Delta Q} = 0$

0=MC-MR

MC=MR

XI. ATC=2Q-70+$\frac{1000}{Q}$

$LR_p = \frac{\Delta ATC}{\Delta Q} = 0$

$2 - \frac{1000}{Q^2} = 0$

$2Q^2=1000$

$Q^2=500 \rightarrow Q=22.36$

$P_{LR}=2*22.36-70+\frac{1000}{22.36}=19.44$

XII. $TC=aQ^2+bQ+c$

$MC=2aQ+b$

$\begin{cases} 2*94a+b=182 \\ 2*95a+b=184 \end{cases} \rightarrow \begin{cases} 188a+b=182 \\ 190a+b=184 \end{cases}$

We deduct from 2nd equation 1st and get:

2a=2

a=1

b= -6

$TC=Q^2-6Q+10$

XIII. Q=4500-50P

$TC=3Q^2-10Q+50$

Q in terms of P→90-0.02Q=P

$TR=P*Q=(90-0.02Q)*Q=90Q-0.02Q^2$

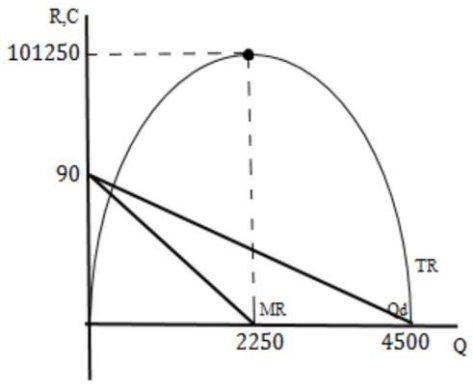
$\Pi=TR-TC \rightarrow 90Q-0.02Q^2-3Q^2+10Q-50$

$\Pi= -3.02Q^2+100Q-50$

$\frac{\Delta \Pi}{\Delta Q} = 0 \rightarrow -6.04Q+100=0$

Q=16.56

90-0.02*16.56=P=89.67



XIV. $Q=a-b*P$

$$\begin{cases} 1000 = a - 29b \\ 2000 = a - 28b \end{cases}$$

$1000=b$

$1000=a-29*1000$

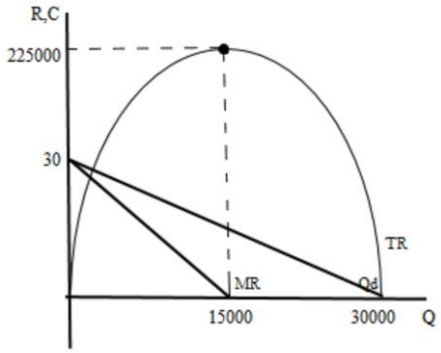
$a=30000$

$Q_d=30000-1000P$

In terms of P$\rightarrow$ $P=\frac{30000-Q}{1000}=30-0.001Q$

$TR=P*Q=30Q-0.001Q^2$

$MR=\frac{\Delta TR}{\Delta Q}=30-0.002Q$



XV. $ATC=Q-90$

$ATC=\frac{TC}{Q}\rightarrow ATC*Q=TC$

$TC=Q*(Q-90)=Q^2-90Q$

$MC=\frac{TC}{Q}=2Q-90$

$MC_{200}=2*(200)-90=310$

XVI. $TR=450Q-0.3Q^2$

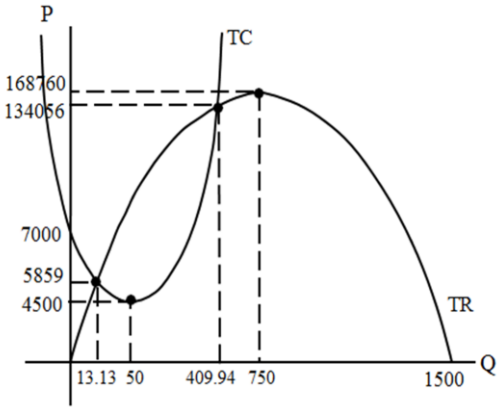
$TC=Q^2-100Q+7000$

$TR=TC\rightarrow 450Q-0.3Q^2=Q^2-100Q+7000$

$1.3Q^2-550Q+7000=0$

$D=\sqrt{(-550)^2-4*1.3*7000}=\sqrt{266100}=515.85$

$Q_{1,2}=\frac{550\pm515.85}{2*1.3}=13.135;\ 409{,}94$



XVII. A) TR=P*Q→20000Q

MR= $\frac{\Delta TR}{\Delta Q}$ =20000

b) MC=6000TMT

c) TR=100*2000+350*1200+600*800+50*20000+ 1400*350=2590000

d) TC=5*9000+20*6500+4*6000+3*5500+148*4000= 807500

e) MC= $\frac{\Delta TC}{\Delta Q}$ =5500

f) From rest of the employees

g) Increasing number of rooms for which demand is high.

h) TR=(100*2000*10)+(350*1200*10)+(600*800*10)+ (50*20000*10)+(1400*350*10)=25900000

TC= $\left(5 * \frac{9000}{3}\right)+\left(20 * \frac{6500}{3}\right) + \left(4 * \frac{6000}{3}\right) + \left(3 * \frac{5500}{3}\right) +$ $\left(148 * \frac{4000}{3}\right) = 269166.67$

XVIII. Q_d=1300-12P+0.2I

a) Q=1300-12*100+0.2*2000=500

b) Q=1300-12*100+0.2*2200=540

c) Q=1300-12*70+0.2*2200=900

d) Q=1300-12P+0.2*2000

Q=1700-12P

P=141.67-0.083Q

TR=P*Q=141.67Q-0.083Q^2

$\frac{\Delta TR}{\Delta Q}$ =0→ 141.67 -0.166Q=0

Q=853

P=141.67-0.083*853=70.87

TR=70.87*853=60452.11

e) TC=Q^2-50Q+700

Π=TR-TC→141.67Q-0.083Q^2-Q^2+50Q-700

$\frac{\Delta \Pi}{\Delta Q}$ =0→ 141.67 -0.166Q-2Q+50=0

Q=88.49

Π=7780

f) Q=1740-12P

P=145.67-0.083Q

TR=145.67Q-0.083Q^2

Π=TR-TC→145.67Q-0.083Q^2-Q^2+50Q-700

$\frac{\Delta \Pi}{\Delta Q}$ =0→ 0.166Q-2Q+50+145.67=0

Q=90.34

Π=8138

g) Q=1300-12P

Q=1300-12*100=100

XIX. $TC=Q^2-100Q+7000$

$Q=6300-10P$

a) We find demand function in monopoly in terms of P:

$P=\frac{6300-Q}{10}=-0.1Q+630$

$TR=P*Q\rightarrow (630-0.1Q)*Q=630Q-0.1Q^2$

$\Pi=TR-TC\rightarrow 630Q-0.1Q^2-Q^2+100Q-7000$

$\Pi=-1.1Q^2+730Q-7000$

$\frac{\Delta\Pi}{\Delta Q}=0\rightarrow -2.2Q+730=0$

$Q=\frac{730}{2.2}=331.82$

$331.82=6300-10P=596.82$

b) In perfect competition, since market is highly competitive, we find long run P and Q when $\frac{\Delta ATC}{\Delta Q}=0$:

$ATC=\frac{TC}{Q}\rightarrow Q-100+\frac{7000}{Q}$

$\frac{\Delta ATC}{\Delta Q}=0\rightarrow 1-\frac{7000}{Q^2}=0$

$1=\frac{7000}{Q^2}$

$Q^2=7000$

$Q=83.67$

$ATC\ (83.67)=83.67-100+\frac{7000}{83.67}=67.3$ (price in a long run)

$Q=6300-10*67.3$ ←market demand

$\rightarrow \frac{5627}{83.67}=67$ ←number of firms in the market in a long run.

XX. $TC=Q^2-100Q+70$ (this function does not satisfy second characteristics of total cost function)

$D<0\rightarrow \sqrt{b^2-4ac}=\sqrt{(-100)^2-4*1*70}=26.8>0$

$TC=7Q^2-1.9Q+23$

a) $Q=0\rightarrow TC=23>0$

b) $D<0\rightarrow \sqrt{b^2-4ac}=\sqrt{(1.9)^2-4*7*23}=-640.39<0$

c) $\frac{\Delta TC}{\Delta Q}=0\rightarrow 14Q-1.9=0$

$Q=\frac{1.9}{14}=0.1$ only 1 minima

$TC=-4Q^2+9Q+8$ (this function does not satisfy second characteristics of total cost function)

$D<0\rightarrow \sqrt{b^2-4ac}=\sqrt{(9)^2-4*(-4)*8}=14.46>0$

$TC=50Q^3-Q+55$ (This function does not satisfy second and third characteristics of total cost function)
This function has more than one minimum; we cannot find discriminant and graph without special mathematical program.
$TC=10Q+6$ since this function is linear, we cannot find discriminant of this function, so we skip it, and check other characteristics:
$\frac{\Delta TC}{\Delta Q} = 0 \rightarrow 10>0$
When $Q=0 \rightarrow TC=6>0$ (acceptable)

$TC=2Q^2+10Q+110$ (This function does not satisfy third characteristics of total cost function)
$\frac{\Delta TC}{\Delta Q} = 0 \rightarrow 4Q+10=0 \rightarrow Q = \frac{-10}{4} < 0$ has a maxima

$TC=-Q^2-7Q-30$ (This function does not satisfy third characteristics of total cost function)
$\frac{\Delta TC}{\Delta Q} = 0 \rightarrow -2Q-7=0 \rightarrow Q = \frac{-7}{2} < 0$ has a maxima

$TC=Q^2-100Q-200$ (This function does not satisfy second characteristics of total cost function)
$D<0 \rightarrow \sqrt{b^2-4ac} = \sqrt{(-100)^2-4*1*(-200)} = 103.9 > 0$

$TC=Q^2-1090Q$ (This function does not satisfy second characteristics of total cost function)
$D<0 \rightarrow \sqrt{b^2-4ac} = \sqrt{(-1090)^2-4*1*0} = 1090 > 0$

$TC=3Q^2+50Q+80$ (This function does not satisfy second characteristics of total cost function)
$D<0 \rightarrow \sqrt{b^2-4ac} = \sqrt{(50)^2-4*3*80} = 39.2 > 0$

$TC= -Q+88$ (This function does not satisfy third characteristics of total cost function)
$\frac{\Delta TC}{\Delta Q} = 0 \rightarrow -1 < 0$

$TC=100Q$ since this function is linear, we cannot find discriminant of this function, so we skip it, and check other characteristics:
$\frac{\Delta TC}{\Delta Q} = 0 \rightarrow 100 > 0$
When $Q=0 \rightarrow TC=0$ (acceptable)

TC=800 This total cost function does not match with all characteristics, but nevertheless this function is applicable, say, house for a rent, apartment for a rent etc. these types of businesses have only fixed cost (cost of the building).

$TC=4Q^2$
a) When $Q=0 \rightarrow TC=0$
b) $D<0 \rightarrow \sqrt{b^2-4ac}=\sqrt{(0)^2-4*4*0}=0$
c) $\frac{\Delta TC}{\Delta Q}=0 \rightarrow 8Q=0 \rightarrow Q=0$ has only one minima

TC=120Q-300 (This function does not satisfy first characteristics of total cost function)
When $Q=0 \rightarrow TC=-300<0$

TC=-30Q-600 (This function does not satisfy third characteristics of total cost function)
$\frac{\Delta TC}{\Delta Q}=0 \rightarrow -30<0$

$TC=23Q^3+80$ (This function does not satisfy second and third characteristics of total cost function)
This function has more than one minimum; we cannot find discriminant and graph without special mathematical program.

$TC=Q^7+20Q+50$ (This function does not satisfy second and third characteristics of total cost function)
This function has more than one minimum; we cannot find discriminant and graph without special mathematical program.

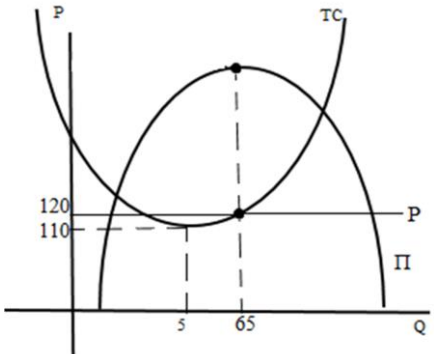
$TC=200Q^2-12400Q+970000$
a) $Q=0 \rightarrow TC=970000>0$
b) $D<0 \rightarrow \sqrt{b^2-4ac}=\sqrt{(-12400)^2-4*200*970000}<0$
c) $\frac{\Delta TC}{\Delta Q}=0 \rightarrow 400Q-12400=0 \rightarrow Q=31>0$ has a minimal

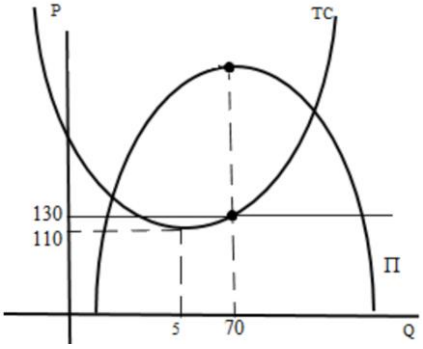
XXI. $TC=Q^2-10Q+135$
$P_1=120 \quad P_2=130 \quad P_3=150$

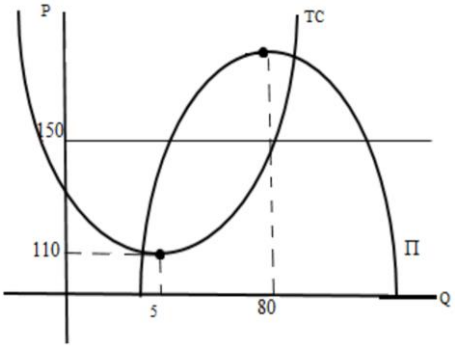
a) $MC=MR=P_1=120 \rightarrow 2Q-10=120 \rightarrow Q=65$
With current price and quantity profit is:
$\Pi=TR-TC \rightarrow 120Q-Q^2+10Q-135=$
$120*(65)-(65)^2+10*65-135=4090$

b) $MC=MR=P_2 \rightarrow 2Q-10=130 \rightarrow Q=70$
With current price and quantity profit is:
$\Pi=TR-TC \rightarrow 130Q-Q^2+10Q-135=$
$130*70-70^2+10*70-135=4765$

c) $MC=MR=P_3 \rightarrow 2Q-10=150 \rightarrow Q=80$
With current price and quantity profit is:
$\Pi=TR-TC \rightarrow 150Q-Q^2+10Q-135=$
$150*80-80^2+10*80-135=6265$







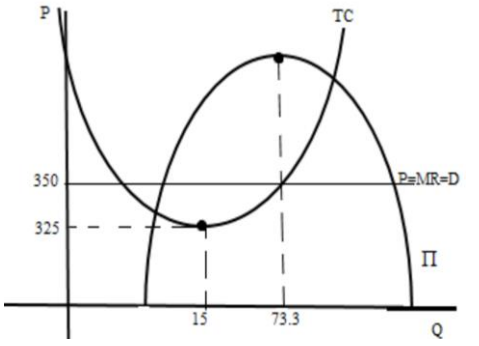
XXII. $TC=3Q^2-90Q+1000$
$P_1=350 \quad P_2=400 \quad P_3=500$

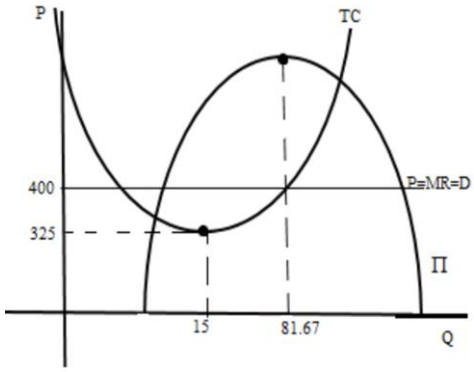
a) $MC=MR=P_1 \rightarrow 6Q-90=350 \rightarrow Q=73.3$
With this price and quantity profit is:

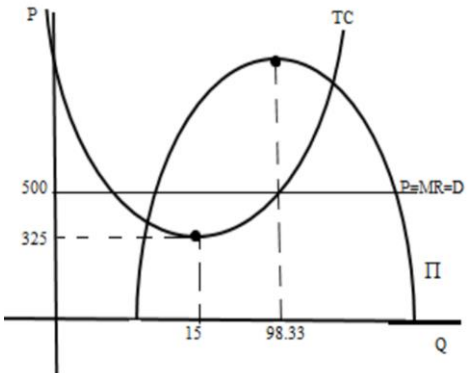
Π=TR-TC $\rightarrow$ 350Q-Q^2+90Q-1000= 350*73-73^2+90*73-1000= 25791

b) MC=MR=P_2 $\rightarrow$ 6Q-90=400 $\rightarrow$ Q=81.67
With this price and quantity profit is:
Π=TR-TC $\rightarrow$ 400Q-Q^2+90Q-1000= 400*81.67-81.67^2+90*81.67-1000=32348

c) MC=MR=P_3 $\rightarrow$ 6Q-90=500 $\rightarrow$ Q=98.3
With this price and quantity profit is:
Π=TR-TC $\rightarrow$ 500Q-Q^2+90Q-1000=500*98.3-98.3^2+90*98.3-1000=47334.11







XXIII. R=300Q-0.03Q^2

a) When Q=0 $\rightarrow$ R=0
Q=5000>0

b) Negative parabolic function

c) MR $= \frac{\Delta TR}{\Delta Q} = 0 \rightarrow$ 300-0.06Q=0

Q=5000>0 one maximum!

$R=10Q-3Q^2$

a) When $Q=0 \rightarrow R=0$

b) Negative parabolic function.

c) $MR=0 \rightarrow 10-6Q=0 \rightarrow Q=1.67>0$ one maximum

$R=Q-Q^2$

a) When $Q=0 \rightarrow R=0$

b) Negative parabolic function

c) MR=1-2Q=0

Q=0.5 >0 one maximum

$R=-22Q+5Q^2$ (This function does not satisfy second and third characteristics of revenue function)

b) Function is positive parabolic

c) $MR=0 \rightarrow -22+10Q=0 \rightarrow Q=2.2 >0$ has a minimum.

$R=100Q^2$ (This function does not satisfy second characteristics of revenue function)

b) Function is positive parabolic

R=80Q (This function does not satisfy second characteristics of revenue function)

b) Function is linear

$R=1000Q-1000Q^2$

a) When $Q=0 \rightarrow R=0$

b) Negative parabolic function

c) $MR=0 \rightarrow 1000-2000Q=0 \rightarrow Q=0.5$ has a maximum!

$R=60Q+10Q^2$ (This function does not satisfy second characteristics of revenue function)

b) Function is positive parabolic

$R=10Q+0.00008Q^2$ (This function does not satisfy second characteristics of revenue function)

a) Function is positive parabolic

$R=3400Q-0.005Q^2$

a) When $Q=0 \rightarrow R=0$

b) Negative parabolic function

c) MR=0 → 3400-0.01Q=0 → Q=340000 has a maximum!

XXIV. Π=Q^2-70Q-120 (This function does not satisfy characteristics of profit function)

a) $\frac{\Delta\Pi}{\Delta Q} = 0 \rightarrow$ 2Q-70=0 → Q=35

b) $\frac{\Delta^2\Pi}{\Delta^2 Q} = 2 > 0$ has a minima

Π= -Q^2+300Q-2000

a) $\frac{\Delta\Pi}{\Delta Q} = 0 \rightarrow$ -2Q+300=0 → Q=150

b) $\frac{\Delta^2\Pi}{\Delta^2 Q} = -2 < 0$ has a maxima

Π= -Q^2-60 (This function does not satisfy characteristics of profit function)

a) $\frac{\Delta\Pi}{\Delta Q} = 0 \rightarrow$ -2Q-60=0 → Q=-30<0 we cannot produce negative quantity to make a profit

Π= -Q^2+50Q

a) $\frac{\Delta\Pi}{\Delta Q} = 0 \rightarrow$ -2Q+50=0 → Q=25>0

b) $\frac{\Delta^2\Pi}{\Delta^2 Q} = -2 < 0$ has a maxima

Π=Q^2-20Q (This function does not satisfy characteristics of profit function)

a) $\frac{\Delta\Pi}{\Delta Q} = 0 \rightarrow$ 2Q-20=0 → Q=10

b) $\frac{\Delta^2\Pi}{\Delta^2 Q} = 2 > 0$ has a minima

Π= -Q^2-100Q-90 (This function does not satisfy characteristics of profit function)

a) $\frac{\Delta\Pi}{\Delta Q} = 0 \rightarrow$ -2Q-100=0 → Q=-50<0 quantity cannot be negative

Π= 5Q^2+35Q+1000 (This function does not satisfy characteristics of profit function)

a) $\frac{\Delta\Pi}{\Delta Q} = 0 \rightarrow$ 10Q+35=0 → Q= -3.5<0 negative quantity

Π= -3Q^2+Q-400

a) $\frac{\Delta\Pi}{\Delta Q} = 0 \rightarrow$ -6Q+1=0 → Q=0.167>0

b) $\frac{\Delta^2\Pi}{\Delta^2 Q} = -6 < 0$ has a maxima

$\Pi = -Q^3+67Q-250$ (This function does not satisfy characteristics of profit function)

a) This function is not parabolic

$\Pi = Q^4-10Q^3-49Q^2+300Q-2000$ (This function does not satisfy characteristics of profit function)

a) This function is not parabolic

$\Pi = -8Q^2$ (This function does not satisfy characteristics of profit function)

a) $\frac{\Delta\Pi}{\Delta Q} = 0 \rightarrow -18Q=0 \rightarrow Q=0$ profit cannot reach its maximum when zero (0) output is produced!

$\Pi=3900$ (This function does not satisfy characteristics of profit function)

a) This function is not parabolic

$\Pi=29Q^2-760$ (This function does not satisfy characteristics of profit function)

a) $\frac{\Delta\Pi}{\Delta Q} = 0 \rightarrow 58Q=0 \rightarrow Q=0$ has a minimum

$\Pi = Q+3$ (This function does not satisfy characteristics of profit function)

a) This function is not parabolic

$\Pi = -500Q+5000$(This function does not satisfy characteristics of profit function)

a) This function is not parabolic

$\Pi = -Q^3+300Q^2-400Q$ (This function does not satisfy characteristics of profit function)

a) This function is not parabolic

XXV. $Q=a-b*P$

Demand function of a monopolist in terms of P:

$P=\frac{a-Q}{b}$

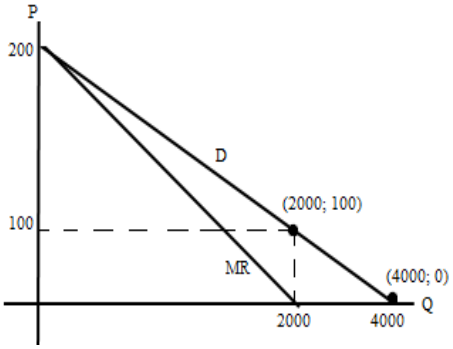
$TR=P*Q \rightarrow \left(\frac{a-Q}{b}\right)*Q = \frac{aQ-Q^2}{b}$

$MR=\frac{\Delta TR}{\Delta Q} = 0 \rightarrow \frac{a}{b} - \frac{2Q}{b} = 0$

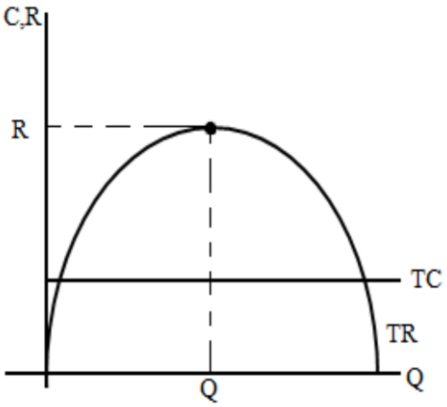
In order MR to be equal to zero, our a-2Q must also to be equal to zero:

$a-2Q = 0$

Since we are given our Q=2000 and P=100:
a-2*2000=0
a=4000
2000=4000-100b
-2000=-100b
b=20
Q=4000-20P



XXVI. Total cost function must be linear, so that the bigger the revenue, bigger the profit. Shown on the graph below:



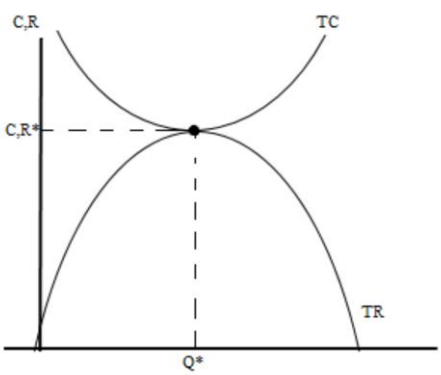
XXVII. Profit is maximized when MR=MC:
Q=3000-30P derive in terms of P→P=100-0.03Q
$TR=P*Q=100Q-0.03Q^2$
$TC_1=9Q^2-80Q+500$
a) $MR=100-0.06Q=MC_1=18Q-80$
18.06Q-180=0
Q=9.97
Π=TR-TC→994-597=397
b) $100-0.06Q=MC_2=4Q-110$
$TC_2=2Q^2-110Q+2000$
4.06Q-210=0
Q=51.72
Π=TR-TC=5091.75-1660.72=3431.03

c) $100-0.06Q=MC_3=60Q-1000$
$TC_3=30Q^2-1000Q+9360$
$60.06Q-1100=0$
$Q=18.31$
$\Pi=TR-TC=1820.9-1107.7=713.2$

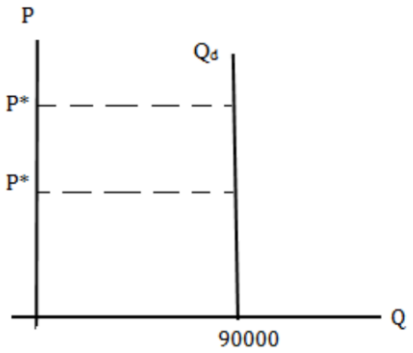
Choose technology of the 2nd TC function because it has minimum cost and maximum profit in comparison with others!

XXVIII. $P=200$
$ATC=\frac{TC}{Q} \rightarrow TC=ATC*Q$
$TC=Q^2-88Q+9090$
As we are in perfectly competitive market MR=MC:
$MR=P=200$
$MC=2Q-88$
$MR=MC \rightarrow 200=2Q-88$
$288=2Q$
$Q=144$
Price in long run will be when $\frac{\Delta ATC}{\Delta Q}=0$:
$ATC=Q-88+\frac{9090}{Q}$
$\frac{\Delta ATC}{\Delta Q}=0 \rightarrow 1-\frac{9090}{Q^2}=0 \rightarrow Q^2=9090 \rightarrow Q=95.34$
$ATC\ (95.34)=95.34-88+\left(\frac{9090}{95.34}\right)=102.684=LR_{price}$
company makes profit because market price is higher than minATC .
Our profit when the price is 200:
$TR=200Q$
$\Pi=TR-TC \rightarrow 200*144-144^2+88*144-9090=11646$
Our profit in a long run:
$\Pi=TR-TC \rightarrow 102.68*95.34-95.34^2+88*95.34-9090=0$
In a long run we make zero profit.

XXIX. Graph:



XXX. Since quantity demanded is fixed at 90000, monopolist will produce only 90000. Since we know quantity to be produced and also that quantity demanded is strictly price inelastic, monopolist will use this opportunity and constantly increase its price. So maximum profit of monopolist depends only on monopolists' price.



XXXI. To know which company will win over we must know which company has minimum cost in production:

$TC_1=10Q^2-50Q+300$

$ATC=\frac{TC}{Q}=10Q-50+\frac{300}{Q}$

$\frac{\Delta ATC}{\Delta Q}=0\rightarrow 10-\frac{300}{Q^2}=0$

$10Q^2=300$

$Q^2=30$

$Q=5.48$

$ATC\ (5.48)=10*5.48-50+\frac{300}{5.48}=59.5$

$TC_2=7Q^2-60Q+500$

$ATC=\frac{TC}{Q}=7Q-60+\frac{500}{Q}$

$\frac{\Delta ATC}{\Delta Q}=0\rightarrow 7-\frac{500}{Q^2}=0$

$7Q^2=500$

$Q^2=71.4$

$Q=8.45$

$ATC\ (8.45)=7*8.45-60+\frac{500}{8.45}=58.3$

Company with 2nd TC function will win over, because its cost is less than 1st firms minimum TC.

Chapter 3: Essential Microeconomics 3

After we have learned about supply and demand, market structures, and behaviors of suppliers in markets, their cost, revenue and profit functions relationships, it is time to move further. It is time we learn about production, input costs, optimum and efficient production. How companies decide how much to produce? How many employees to hire? Invest into capital or just hire new employee? In order to answer all these questions, (no worries kiddies, you will be able to answer to those questions after this chapter I promise!), we need to know what **output is**. **Output (Q in our functions!)** is a final product or service. Every output is a combination of these three things:

a) **Labor:** without human force, employees, labor we cannot produce anything, at least in twenty first century. I do not know how conditions will change in future but nowadays, we still need labor force for producing output.

b) **Capital:** Economic and financial meaning of capital is the same here. Without investment, machinery, inputs, resources, factors of production like land, we cannot produce anything. Hold on! I almost forgot! There is only one thing we can produce without a capital: it is a dream!

c) **Technology:** Here, technology does not mean high tech computers or super cool agent 007 gadgets, in economics world technology means: method, know how, process, knowledge, etc. Can you produce drug against a cancer? Yes, I am talking to you my dear reader, can you? Do not worry about capital (laboratory) it is on me, and you are the labor, so hurry up with production! Are you still there, reading? You must be in a lab mixing chemicals and playing with tubes and so on. Yep, you know why you are not in lab, I know it too: You do not know how to make drugs at all, you do not possess a needed knowledge to produce drugs. You do not know the method, the process; you do not know anything about drugs. Same is applicable to all outputs: products and services such as cakes, space shuttles, military technology, cars, baby foods, haircutting, surgery, etc. all these outputs require technology (the knowledge of method of how to produce those things). Without proper knowledge, experience, training, experiments and education, you will not be able to produce anything of high value. Never forget my friend: More brain means high value, less brain means low value! Drugs require tons of brain and knowledge that is why for some drugs consumers are ready to pay thousands of USDs. You

do not even need to go to school for growing tomatoes, that is why tomatoes are so cheap compared to drugs. Why Lamborghini is much pricier than Toyota cars? Why LeBron James paid millions of USD when you are not? Why Gucci products are much pricier than your textile manufacturing? Why certain restaurants are popular while others are not? Why we hear a lot about Swiss and Dutch made cheese but never heard anything about Russian or Egyptian cheese? Do not they have cows and goats there?

There are many production functions but the best (and the most popular) and the most workable one in economics science is a Cobb-Douglas[6] production function:

$$Q_t = A_0 * K_t^{\alpha} * L_t^{\beta}$$

(We will use $\mathbf{Q=AK^{\alpha}L^{\beta}}$ in book)

Q=output (product or service)

A=technology (method, process, knowledge, know-how, etc.)

K=capital (machinery, equipment, land, resources, etc.)

L=labor (employee, human being!)

(t,0) = shows time variance. Variables with "***t***" are time sensitive, *endogenous*, which means "can change over time". Variables with "***0***" are the ones that are *exogenous,* which means they are given, they cannot be predicted or calculated.

(α, β) = powers or labor and capital. Mathematically, one can say that: "power" of a variable shows the "role" of the variable in function, bigger the "power", bigger the "role" of the variable in function. An economy where labor force is intensively used, labor force still play the main role, is called *labor intensive* economy (developing and underdeveloped economies!). Economies where capital is used intensively, capital has bigger "power" in function, is called *capital intensive* economy.

Production function is considered *increasing returns to scale when (*$\alpha+\beta>0$). It means that output is more than input in an economy. Let's see it with example:

[6] Named after two great economists! I leave it to you to make a research and find out how did they came up with this functional form.

$$Q=AK^{0.6}L^{0.6}$$

Output function (production function) has two independent variables Capital and Labor. Mathematically we express it as:

$$Q\ (K, L) =AK^{0.6}L^{0.6}$$

Question is this: how much my output will increase if I increase my inputs by twice? Answer:

$$Q\ (2K, 2L) =A\ (2K)^{0.6}\ (2L)^{0.6}$$
$$Q\ (2K, 2L) =A\ 2^{0.6}\ K^{0.6}\ 2^{0.6}\ L^{0.6}$$
$$Q\ (2K, 2L) = 2^{0.6}\ 2^{0.6}\ A\ K^{0.6}\ L^{0.6}$$
$$Q\ (2K, 2L) = 2^{1.2}\ A\ K^{0.6}\ L^{0.6}$$
$$Q\ (2K, 2L) = 2^{1.2}\ Q\ (K, L)$$

As you can see from results when $\alpha+\beta>0$, we get increasing returns to our investment. We increased inputs twice but got a return (output!) more than twice ($2^{1.2}$ =2.28 which is bigger than 2!).

When ($\alpha+\beta<0$) then we have *decreasing returns to scale.* Example:

$$Q\ (K, L) =AK^{0.3}L^{0.6}$$
$$Q\ (2K, 2L) =A\ (2K)^{0.3}\ (2L)^{0.6}$$
$$Q\ (2K, 2L) =A\ 2^{0.3}\ K^{0.3}\ 2^{0.6}\ L^{0.6}$$
$$Q\ (2K, 2L) = 2^{0.3}\ 2^{0.6}\ A\ K^{0.3}\ L^{0.6}$$
$$Q\ (2K, 2L) = 2^{0.9}\ A\ K^{0.3}\ L^{0.6}$$
$$Q\ (2K, 2L) = 2^{0.9}\ Q\ (K, L)$$

We increased inputs by twice but got a return (output!) less than twice ($2^{0.9}$ = 1.87 which is less than 2!).

When ($\alpha+\beta=0$) then we have *constant returns to scale.* Example:

$$Q\ (K, L) =AK^{0.3}\ L^{0.7}$$
$$Q\ (2K, 2L) =A\ (2K)^{0.3}\ (2L)^{0.7}$$
$$Q\ (2K, 2L) =A\ 2^{0.3}\ K^{0.3}\ 2^{0.7}\ L^{0.7}$$
$$Q\ (2K, 2L) = 2^{0.3}\ 2^{0.7}\ A\ K^{0.3}\ L^{0.7}$$
$$Q\ (2K, 2L) = 2\ A\ K^{0.3}\ L^{0.7}$$
$$Q\ (2K, 2L) = 2\ Q\ (K, L)$$

We increased inputs by twice and got a twice output.

Now, if we made all clear regarding returns to scale, let's continue working on production function.

***Exercise 1**: Assume we have a production function of a form:*

$$Q(K, L) = 3K^{0.3}L^{0.7}$$

Supplier wants to find the optimal ratio of capital and labor to produce 1000 units of good. So, his production function will look like:

$$1000 = 3K^{0.3}L^{0.7}$$

Supplier can choose any combination of labor and capital to produce 1000 output.

Example:

$$\text{When } L=1000,\ 1000 = 3K^{0.3}(1000)^{0.7},\ K=26$$

If supplier chooses to hire 1000 employees, then he will have to rent a capital of approximately 26 to produce 1000 units of good.

Example:

$$\text{When } L=500 \rightarrow 1000 = 3K^{0.3}(500)^{0.7} \rightarrow K=129$$

If supplier chooses to hire 500 employees, then he will have to rent a capital of approximately 129 to produce 1000 units of good.

Example:

$$\text{When } K=100,\ 1000 = 3(100)^{0.3}L^{0.7},\ L=558$$

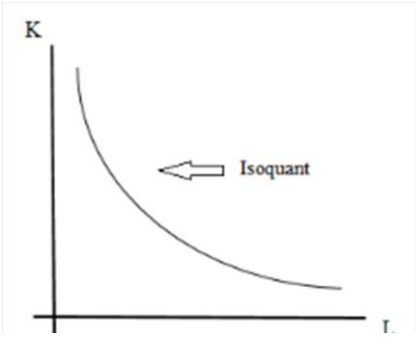
*If supplier chooses to rent a 100 capital, then he will have to hire approximately 558 employees to produce 1000 units of good and so on. This relationship between labor and capital to produce required quantity of output is mathematically formalized and called **Isoquant.** The combination of labor and capital to produce given quantity of goods is called **Isoquant.** By little modification of above given function, we can turn this into **isoquant:***

$$1000 = 3K^{0.3}L^{0.7}$$

$$\frac{1000}{3L^{0.7}} = K^{0.3}$$

$$\left(\frac{1000}{3L^{0.7}}\right)^{3.33} = (K^{0.3})^{3.33}$$

$$\frac{251875672}{L^{2.33}} = K$$



Now, we are coming to million-dollar question: How supplier decides what combination of labor and capital to choose? **Supplier decides the "right" combination according to the cost**

of those inputs! Supplier's target is to maximize profit; another way of saying it is, trying to produce same quantity of output as cheap as possible! Supplier wants to produce 1000 units of good, and he wants to do it as cheap as possible. In order to do that he must choose the best combination of labor and capital so that my total cost is minimum. We cannot do this without cost function. Since in our production function we have only two independent variables, we will use their costs only. Other costs are assumed to be exogenous. Cost of labor is wage, cost of capital is rent. Let's make up some cost function for this example. Assume average wage in labor market is 2000 TMT and rent is about 5000 TMT. Also, let's add that supplier also has fixed cost of 400,000 TMT per month (officer rent, etc.). So, our cost function will look:

$$TC=2000*L+5000*K+400000$$

With little modification, we have:

$$TC - 2000 * L - 400000 = 5000*K$$

$$\frac{TC - 2000 * L - 400000}{5000} = K$$

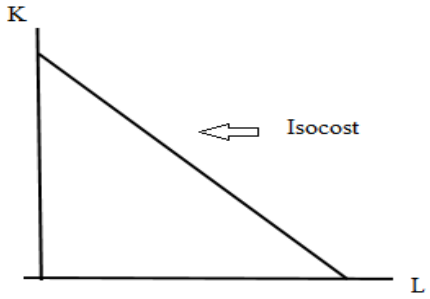
Total cost is just a constant number like 300000 or 500000 TMT that is why we can just write:

$\frac{TC-400000}{5000} = A$, we know that A>0 because TC>400000

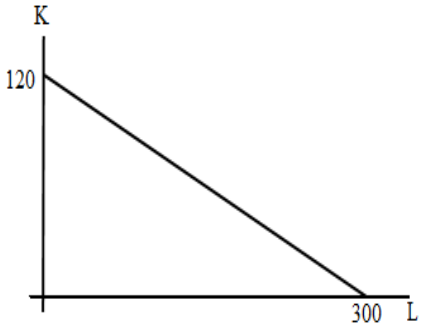
$$A - 0.4*L=K$$

The combination of labor and capital to be within the limits of given cost is called ***Isocost.*** As you can see, we have linear ***isocost*** function here.

Let's play around with *isocost* a little bit. Assume supplier puts a total cost limit as 1000000 TMT. He prefers not to exceed that cost amount in production.



$TC=2000*L+5000*K+400000$
$1000000=2000*L+5000*K+400000$
$1000000-2000*L-40000=5000*K$
$600000-2000*L=5000*K$
$120-0.4*L=K$



According to our *isocost* function, for example if I want to hire 100

employees, I can rent only 80 units of capital to stay within the cost limits of 1000000.

$$120\text{-}0.4*L=K$$

$$120\text{-}0.4*100=K$$

$$K=80$$

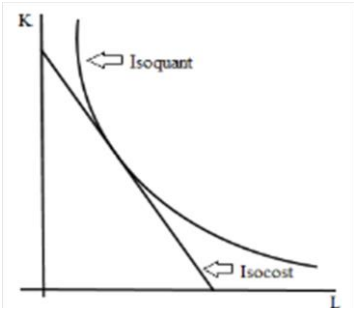
$$TC=2000*L+5000*K+400000$$

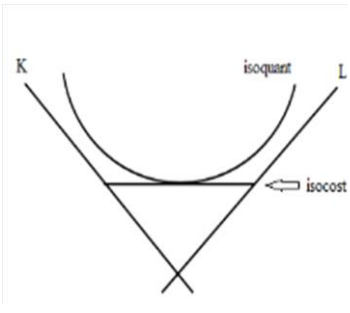
$$TC=2000*(100)+5000*(80)+400000$$

$$TC=1000000$$

Our *isocost* function works, as you can see result from above.

Let's come back to our supplier who wants to produce 1000 units of good. In production process, suppliers first of all always decide how much to produce, according to market demand, then they calculate the minimum costs. When starting restaurant business, you choose the area accordingly, costs come later. When states build military aircrafts, they first consider their strategic points and how many planes do they need, costs come later. Before jumping into drug manufacturing, drug companies research and make analysis about exact number of sick people and put quantity targets accordingly, costs come later. Suppliers always first decide how much to produce, and later choose minimum cost strategies. Our supplier targets producing 1000 units of good, we already constructed *isoquant* for this, we also constructed *isocost* with costs of inputs, and now we try to produce 1000 units of good as cheap as possible. How to do it? If you look at graph below, you will see both *isoquant* and *isocost* functions. In order to produce 1000 units of good with minimum cost, we must find the tangent point of these two functions. Simple! Tangency point of linear *isocost* function and non-linear *isoquant* function will be the *optimal* combination of labor and capital for producing 1000 units with minimum possible costs. We can use the notion *efficient,* instead of *optimal* here, because they have almost the same meaning: producing with minimum costs! So, where is that tangency point? How to find it? Mathematically, tangency points of these two functions are found by taking derivatives and equalizing these two functions. If you turn graph to 45 degrees to the left, you will see that tangent point of *isoquant* is actually minimum point of *isoquant*!





And we know the slope of the linear *isocost*, it is 0.4. Long word short, to find the optimum combination of labor and capital with minimum cost we must find minimum point of *isoquant* function and slope of the linear *isocost* function and equalize them. Solve it for one of the variables. Let's do it:

First order condition for maximum and minimum for *isoquant;* L K Isoquant:

$$\frac{251875672}{L^{2.33}} = K$$

$$\frac{\partial Isoquant}{\partial L} = (-2.33) * 251875672 * L^{-3.33}$$

Slope of the *isocost:*

$$A - 0.4*L=K$$

$$\frac{\partial Isoquant}{\partial L} = -0.4$$

Point of tangency of *isoquant* and *isocost:*

$$(-2.33) * 251875672 * L^{-3.33} = -0.4$$

Solution:

$$(-2.33) * 251875672 * L^{-3.33} = -0.4$$

$$251875672 * L^{-3.33} = 0.17$$

$$\frac{251875672}{L^{3.33}} = 0.17$$

$$\frac{251875672}{0.17} = L^{3.33}$$

$$1481621600 = L^{3.33}$$

$$(1481621600)^{0.3} = (L^{3.33})^{0.3}$$

$$564 = L$$

Let's check the numbers as always, first, put it into the *isoquant:*

$$\frac{251875672}{L^{2.33}} = K$$

$$\frac{251875672}{(564)^{2.33}} = K$$

$$98 = K$$

Double check with production function:

$$Q = 3K^{0.3}L^{0.7}$$

$$Q = 3(98)^{0.3}(564)^{0.7}$$

$$Q = 1000$$

All good with production function. Let's find the total cost now:

$$TC = 2000*L + 5000*K + 400000$$
$$TC = 2000*(564) + 5000*(98) + 400000$$
$$TC = 2018000$$

1000 units of output for given production and cost functions could be produced for 2018000 TMT. This is the minimum cost! Let's check it too:

Assume we disagree with our economist and try different combinations while keeping quantity produced fixed at 1000 units. Question is what if I hire a little bit more employees? Would not the costs go down? Let's try different combinations:

When $L = 590$

$$\frac{251875672}{L^{2.33}} = \frac{251875672}{(590)^{2.33}} = K = 88$$

$$TC = 2000*L + 5000*K + 400000$$

$$TC = 2000*(590) + 5000*(88) + 400000$$

$$TC = 2020000$$

As you can see, cost is higher than we found with L=564 combination!

From the cost function one can see that capital is very costly (rent=5000), compared to labor (w=2000). One might decide that decreasing capital ratio in production might lower down the costs. Let's try this hypothesis:

When $K = 10$

$$\frac{251875672}{K} = L^{2.33}$$

$$\frac{251875672}{10} = L^{2.33}$$

$$25187567.2 = L^{2.33}$$

$$(25187567.2)^{0.43} = (L^{2.33})^{0.43}$$

$$L = 1522$$

Put number into cost function:

$$TC=2000*L+5000*K+400000$$

$$TC=2000*(1522)+5000*(10)+400000$$

$$TC=3494000$$

As you can see from the results, this way is costlier than our optimal combination of L=564 and K=98.

STEPS IN FINDING OPTIMAL LABOR AND CAPITAL FOR EFFICIENT PRODUCTION!

Step 1: *Find* ***Isoquant*** *using given production function.*

Step 2*: Find* ***Isocost*** *using given cost function.*

Step 3*: Find tangency points of two functions. Use first order conditions, derivatives or any other mathematical technique for this. Solve equation and find optimum labor and capital.*

Step 4: *Check your numbers using* ***isoquant*** *and* ***isocost*** *functions.*

Step 5: *Find minimum cost.*

TIP: When taking Isoquant and Isocost, make sure variables remain at the same side in both functions! In example above labor (L) is on the left side of equations both in Isoquant and Isocost. Capital (K) is on the right side! If you do it otherwise your findings will be incorrect!

What will happen if wages and rents of inputs change? Will it impact production? Total cost? The answer is: Yes, changes in input costs will impact our optimal combination of labor and capital, thus impacting cost function. Let's see it with number:

Assume wage rise to 3000 TMT, all other costs remain the same. Then, new total cost function:

$$TC=3000*L+5000*K+400000$$

We follow the steps above shown. *Step 1*:

$$Q=3K^{0.3}L^{0.7}$$

Since quantity remains the same, supplier still wants to produce the same quantity, Q=1000, our *isoquant* does not change:

$$1000=3K^{0.3}L^{0.7}$$

Isoquant:

$$\frac{251875672}{L^{2.33}} = K$$

We continue with *Step 2:*

$$TC=3000*L+5000*K+400000$$

$$TC-40000-3000*L=5000*K$$

$$\frac{TC - 3000 * L - 400000}{5000} = K$$

$$A-0.6*L=K$$

After we found an *Isocost*, we follow the *Step 3*; point of tangency of two functions:

$$\frac{\partial Isoquant}{\partial L} = (-2.33) * 251875672 * L^{-3.33}$$

$$\frac{\partial Isoquant}{\partial L} = -0.6$$

$$(-2.33) * 251875672 * L^{-3.33} = -0.6$$

$$251875672 * L^{-3.33} = 0.26$$

$$\frac{251875672}{L^{3.33}} = 0.26$$

$$\frac{251875672}{0.26} = L^{3.33}$$

$$968752584 = L^{3.33}$$

$$(968752584)^{0.3} = (L^{3.33})^{0.3}$$

$$496 = L$$

Put this into *isoquant* and check results as per *Step 4:*

$$\frac{251875672}{L^{2.33}} = K$$

$$\frac{251875672}{(496)^{2.33}} = K$$

$$132 = K$$

Double check with production function:

$$Q = 3K^{0.3}L^{0.7}$$

$$Q=3(132)^{0.3}(496)^{0.7}$$

$$Q=1000$$

Our optimal combination is correct. Let's finish this by *Step 5*:

$$TC=3000*L+5000*K+400000$$

$$TC=3000*(496)+5000*(132)+400000$$

$$TC=2548000$$

1000 units of output for given production and cost functions could be produced for 2548000 TMT. This is the minimum cost!

What if supplier decides to increase output to 3000 units (assume the rest remains the same)? Then, *Step 1*:

$$Q=3K^{0.3}L^{0.7}$$

$$3000=3K^{0.3}L^{0.7}$$

Isoquant:

$$\frac{9772372209}{L^{2.33}}=K$$

We continue with *Step 2:*

$$TC=3000*L+5000*K+400000$$

$$TC-40000-3000*L=5000*K$$

$$\frac{TC-3000*L-400000}{5000}=K$$

$$A-0.6*L=K$$

After we found an *Isocost*, we follow the *Step 3*; point of tangency of two functions:

$$\frac{\partial Isoquant}{\partial L}=(-2.33)*9772372209*L^{-3.33}$$

$$\frac{\partial Isoquant}{\partial L}=-0.6$$

$$(-2.33)*9772372209*L^{-3.33}=-0.6$$

$$9772372209*L^{-3.33}=0.26$$

$$\frac{9772372209}{L^{3.33}}=0.26$$

$$\frac{9772372209}{0.26}=L^{3.33}$$

$$37586046959=L^{3.33}$$

$$(37586046959)^{0.3}=(L^{3.33})^{0.3}$$

$$1488 = L$$

Put this into *isoquant* and check results as per *Step 4:*

$$\frac{9772372209}{L^{2.33}} = K$$

$$\frac{9772372209}{(1488)^{2.33}} = K$$

$$396 = K$$

Double check with production function:

$$Q = 3K^{0.3}L^{0.7}$$

$$Q = 3(396)^{0.3}(1488)^{0.7}$$

$$Q = 3000$$

Our optimal combination is correct. Let's finish this by *Step 5*:

$$TC = 3000*L + 5000*K + 400000$$

$$TC = 3000*(1488) + 5000*(396) + 400000$$

$$TC = 6844000$$

3000 units of output for given production and cost functions could be produced for 6844000 TMT. This is the minimum cost! If you noticed it, when only quantity produced changed while keeping all other variables unchanged, *ceteris paribus,* inputs are changed with the same rate. Quantity increased from 1000 to 3000, inputs are increased with same rate; labor from 496 to 1488, and capital from 132 to 496. We found optimal combination of labor and capital to produce any quantity of good using production and cost functions, so if question arise regarding only quantity of production, simply change all inputs the with the same rate. Example: what if supplier decides to cut production by 20%? Then all inputs will be cut by 20% too. Let's check it, Q decreased from 1000 to 800 (20%), then:

$$Q = 3K^{0.3}L^{0.7}$$

We found optimal combination when producing 1000 units as:

$$1000 = 3K^{0.3}L^{0.7}$$

$$1000 = 3(98)^{0.3}(564)^{0.7}$$

$$K = 98,\ L = 564$$

Then, when Q decreases 20%, all inputs decrease by 20%:

$$800 = 3K^{0.3}L^{0.7}$$

K decreased from 98 to 78 (20%) and L from 564 to 451 (20%), check the answer:

$$800 = 3K^{0.3}L^{0.7}$$

$$Q = 3(78)^{0.3}(451)^{0.7}$$

$$Q = 800$$

All holds. Once you have found optimal combination, if variables and variable costs such as wage and rent do not change, then quantity change rates will directly impact the same way the variables!

Optimization using Lagrangian multiplier

We used *isoquant* and *isocost* to find the optimal combination of labor and capital to produce required quantity with minimum cost. That is not a single method though. In economics science, for solving optimization problems, Lagrangian multiplier or Lagrangian function is also used very widely. Let's get into it. It is easy.

First of all, we construct our production and cost functions. Set strategic targets. Let's take our example production and cost function used above:

$$Q = 3K^{0.3}L^{0.7}$$

$$TC = 2000*L + 5000*K + 400000$$

Our strategic target is to produce 1000 units of goods. Next, we construct Lagrangian:

$$Ł = 2000*L + 5000*K + 400000 + \lambda*(Q - 3K^{0.3}L^{0.7})$$

Impose strategic target into Lagrangian:

$$Ł = 2000*L + 5000*K + 400000 + \lambda*(1000 - 3K^{0.3}L^{0.7})$$

In this function the initial part, **(3000*L+5000*K+400000)** is our cost function and this is the main focus of our function. The second part, **$\lambda*(1000-3K^{0.3}L^{0.7})$** is our production function and we used it as a constraint. By looking how Lagrangian function was constructed, we can easily spell what is wanted from this calculation. So here, economist (the one who constructed the function) wants to **minimize costs but wants to keep production at a 1000 units.** Economist could minimize cost simply lowering down the production right? Our target is not to lower down the quantity produced; instead we want to minimize cost at targeted

production. λ (lambda) symbol is called a Lagrangian multiplier and mathematically it shows by how much our total constraint function will change if anything changes in cost function. No worries about this symbol for now, you will understand it later on. So, let's get back to our Lagrangian. We constructed the function we know what we want; **we want to find out the best combination of labor and capital so that at a production quantity of 1000 units we will have the minimum cost!** How do we solve it then? Easy, again, we use our *first order conditions* for finding maximums and minimums. Simple! Take Lagrangian's derivative to labor and equalize it to zero.

$$Ł = 2000*L + 5000*K + 400000 + \lambda*(1000 - 3K^{0.3}L^{0.7})$$

$$\frac{\partial Lagrangian}{\partial Labor} = 0$$

$$\frac{\partial Lagrangian}{\partial Capital} = 0$$

Let's go:

$$\frac{\partial Lagrangian}{\partial Labor} = 2000 - \lambda * 3 * 0.7 * L^{-0.3} * K^{0.3} = 0$$

$$2000 - \lambda * 3 * 0.7 * L^{-0.3} * K^{0.3} = 0$$

$$2000 = \lambda * 3 * 0.7 * L^{-0.3} * K^{0.3}$$

$$\frac{2000}{3 * 0.7 * L^{-0.3} * K^{0.3}} = \lambda$$

$$\frac{\partial Lagrangian}{\partial Capital} = 5000 - \lambda * 3 * 0.3 * L^{0.7} * K^{-0.7} = 0$$

$$5000 - \lambda * 3 * 0.3 * L^{0.7} * K^{-0.7} = 0$$

$$5000 = \lambda * 3 * 0.3 * L^{0.7} * K^{-0.7}$$

$$\frac{5000}{3 * 0.3 * L^{0.7} * K^{-0.7}} = \lambda$$

Solve for lambdas:

$$\frac{2000}{3 * 0.7 * L^{-0.3} * K^{0.3}} = \frac{5000}{3 * 0.3 * L^{0.7} * K^{-0.7}}$$

$$\frac{3 * 0.3 * L^{0.7} * K^{-0.7}}{3 * 0.7 * L^{-0.3} * K^{0.3}} = \frac{5000}{2000}$$

$$\frac{L}{K} = 5.8$$

$$\mathbf{L=5.8K}$$

In order to produce 1000 unit of good with that given production and cost functions, the minimum cost will be attained at optimal combination of labor and capital at approximately **L=5.8K**

How do we check it? Just put this into the production function:

$$Q = 3K^{0.3}L^{0.7}$$

$$1000 = 3K^{0.3}L^{0.7}$$

$$L = 5.8K$$

$$1000 = 3K^{0.3}(5.8K)^{0.7}$$

$$1000 = 3K^{0.3}(5.8)^{0.7}K^{0.7}$$

$$1000 = 10.2*K$$

$$K = 98$$

$$L = 564$$

As you can see, solving optimization problem using Lagrangian function brings the same results as using *isoquant* and *isocost. We can change the structure of Lagrangian function and come to the same conclusion. Let's use our production and cost function:*

$$Q = 3K^{0.3}L^{0.7}$$

$$TC = 2000*L + 5000*K + 400000$$

Construct a Lagrangian:

$$\mathbf{Ł} = 3K^{0.3}L^{0.7} + \lambda*(B-2000*L-5000*K-400000)$$

Now, we want to find the optimal combination of labor and capital, with given production and cost functions, so that I can produce maximum goods with limited cost. Here, B is a constant number. For the sake of an example let's take B=2018000. Supplier wants to know how much good he can maximum produce with limited cost function of 2018000.

$$\mathbf{Ł} = 3K^{0.3}L^{0.7} + \lambda*(2018000-2000*L-5000*K-400000)$$

Now we apply the same principle of maximization and minimization:

$$\frac{\partial Lagrangian}{\partial Labor} = 0$$

$$\frac{\partial Lagrangian}{\partial Capital} = 0$$

Solve for lambdas:

$$\frac{\partial Lagrangian}{\partial Labor} = 3 * 0.7 * L^{-0.3} * K^{0.3} - \lambda * 2000 = 0$$

$$3 * 0.7 * L^{-0.3} * K^{0.3} - \lambda * 2000 = 0$$

$$3 * 0.7 * L^{-0.3} * K^{0.3} = \lambda * 2000$$

$$\frac{3 * 0.7 * L^{-0.3} * K^{0.3}}{2000} = \lambda$$

$$\frac{\partial Lagrangian}{\partial Capital} = 3 * 0.3 * L^{0.7} * K^{-0.7} - \lambda * 5000 = 0$$

$$3 * 0.3 * L^{0.7} * K^{-0.7} - \lambda * 5000 = 0$$

$$3 * 0.3 * L^{0.7} * K^{-0.7} = \lambda * 5000$$

$$\frac{3 * 0.3 * L^{0.7} * K^{-0.7}}{5000} = \lambda$$

Then:

$$\frac{3 * 0.3 * L^{0.7} * K^{-0.7}}{5000} = \frac{3 * 0.7 * L^{-0.3} * K^{0.3}}{2000}$$

$$\frac{3 * 0.3 * L^{0.7} * K^{-0.7}}{3 * 0.7 * L^{-0.3} * K^{0.3}} = \frac{5000}{2000}$$

Does this remind you of anything? Yes, we have seen this equation before:

$$\frac{\mathrm{L}}{\mathrm{K}} = 5.8$$

$$\mathbf{L=5.8K}$$

We have gotten the same result, same optimal combination of labor and capital to produce with minimum cost with given production and cost functions.

On WAGE, RENT and PRICE

We learned how to find optimal combination of labor and capital, for given production and cost functions, to produce efficiently! (Maximum output with minimum cost!) Now, let's get to wage and rent, cost of inputs. How do suppliers know if they are

overpaying employees or rent, or instead underpaying? How wage rates and rents are decided in market? To begin answering, we must first understand what is wage and rent?

Rent is easy and we will not waste a time explaining it. Rent of the capital (machinery, equipment, office, etc.) will depend on return on that capital. Machinery that can package 1000 goods in an hour will definitely be more expensive than the one which can do the same in 2 hours. Higher the return, higher the rent. Easy!

Wage is a payment, monthly or weekly, to labor force compensating his/her time spent at work, instead of leisure. It is a compensation for being involved in production. You could have spent your time elsewhere, but you are at work, so your time and effort must be compensated. Wage rate in an economy depends on market structure. In free market economy, or liberal economy where states have almost no power, wages are determined by labor productivity. The more productive labor will get paid more than less productive. The labor that is involved in high value production where skill is extremely important: chemical industry, medical industry, pharmaceutical industry, petroleum industry, IT industry, financial sector, watchmaking, etc. labor force in those industries will be paid high wages. Here, productivity does not mean quantity of production: factory worker who can package 2000 cabbages in an hour will still get paid much lesser than a good surgeon! Productivity is measured by the value of the product! Mechanic in Toyota Company who is involved in production of regular middle class sedan cars is not paid the same wages as a mechanic working in Lamborghini factory. Professor at Oxford University is not paid the same wage as a professor at your local college, because their output is different in value. So, how do I calculate the wage of an employee? It depends on productivity. So how do I calculate productivity? We simply take the derivative of the production function to the labor:

$$\frac{\partial Production\ function(Q)}{\partial Labor} = MPPL$$

We do the same for calculating rent for the capital:

$$\frac{\partial Production\ function(Q)}{\partial Capital} = MPPC$$

In economics science, it is called *Marginal Physical Product of Labor (*MPPL) and same for the capital, *Marginal Physical Product of Capital* (MPPC). MPPL simply calculates by how much the production increase with hiring one more employee. MPPC calculates by how much production increases with renting one more unit of capital. These calculations only give the productivity of inputs, how this is related to wage and rent? The thing is, the

product that is produced by labor or capital is sold in market. So the price multiplied to quantity of goods produced will give productivity in terms of value. Example: Jeren cuts and cleans a fish in the local market and she can cut and clean 30 fish per day. The productivity of Jeren is 30 fish per day. The productivity in terms of value depends on the price of the fish in the market. Assume price is 50 TMT per fish, then productivity of Jeren in terms of value is 30*50=1500 TMT. Jeren brings 1500 TMT of revenue daily for the boss. In liberal market economy, wage rates must be equal to the revenue that employee brings to her boss. So:

MPPL*Price=Wage

$$\frac{\partial Production\ function(Q)}{\partial Labor} * Price\ of\ good = wage$$

MPPC*Price=Rent

$$\frac{\partial Production\ function(Q)}{\partial Capital} * Price\ of\ good = rent$$

Let's take production function we used above and see, what is fair market wage and rent.

$$Q = 3K^{0.3}L^{0.7}$$

$$\frac{\partial Q}{\partial Labor} * p = 3 * 0.7 * K^{0.3} * L^{-0.3} * p = w$$

We found optimum combination of labor and capital as L=5.8K for efficient production. For producing 1000 units of good L=564 and K=98. We also know wages, it is 2000 per labor and rent, it is 5000 per unit. Let's see what must be the minimum price to satisfy that wage and rental rate. Just put the L and K numbers and wage:

$$\frac{\partial Q}{\partial Labor} * p = 3 * 0.7 * (98)^{0.3} * (564)^{-0.3} * p = 2000$$

$$P=1618$$

$$\frac{\partial Q}{\partial Capital} * p = 3 * 0.3 * (98)^{-0.7} * (564)^{0.7} * p = 5000$$

$$P=1618$$

If you paid an attention, price will be same for both equations! According to our calculations we found that in order to pay 2000 TMT salary and 5000 TMT rent, output price on the market must be approximately 1618 TMT. But this price in not efficient price, because at P=1600 TMT, we incur a loss:

$$Q=1000$$

P=1600 TMT

TC=2018000 TMT

Π=TR-TC

Π=1618000-2018000=-400000 TMT (loss)

The problem with *Marginal Physical Productivity of inputs* calculation is that it does not consider fixed cost! If we only calculate the input costs:

$$L*w+r*K=564*2000+98*5000=1618000$$

$$\frac{Variable\ Cost}{Q} = P \rightarrow \frac{1618000}{1000} = 1618\ TMT$$

Then our price will be correct, but since we have fixed cost too, 400000 TMT, our market price finding by using MMP calculation will be wrong. In order to find the efficient price, we will have to simply divide TC to Q:

Q=1000

TC=2018000 TMT

$$\frac{Total\ Cost}{Q} = P(break-even\ price!) \rightarrow \frac{2018000}{1000} = 2018\ TMT$$

At a price of 2018 TMT, supplier will make zero profit that is why it is called *break-even price!*

So, are we underpaying our employees and are we getting cheap capital? Because:

$$\frac{\partial Q}{\partial Labor} * p = 3 * 0.7 * (98)^{0.3} * (564)^{-0.3} * 2018 = wage$$

$$\text{wage} = 2507\ \text{TMT}$$

$$\frac{\partial Q}{\partial Capital} * p = 3 * 0.3 * (98)^{-0.7} * (564)^{0.7} * 2018 = rent$$

$$\text{rent} = 6183\ \text{TMT}$$

Actually no, we are neither underpaying employees nor we are getting a cheap capital. In order to break even, supplier must charge minimum of 2018 TMT per unit of good. While certain amount of money is used to cover the variable costs, the other parts are used to cover fixed cost.

Homework:

I. $Q = 7K^{0.1}L^{0.6}$
$Q = K^{0.2}L^{0.5}$
$Q = AK^{0.5}L^{0.9}$
$Q = KL$
$Q = 2\sqrt{K^3} * L^{0.9}$
$Q = \sqrt[5]{K^3L^7}$
$Q = K^{0.3}L$
$Q = 9K^{0.1}L^{1.3}$
$Q = \sqrt{\frac{80}{56}}K^{0.5}L^{0.5}$
$Q = \frac{K^{0.4}}{L}$
$Q = K^{0.7} + L^{0.6}$
$Q = K^{0.5} - L^{0.4}$

Show what kind of returns to scale characteristics each production function shown above has.

II. A company has production function of $Q = 9K^{0.3}L^{0.8}$ and cost function of TC=L*w +K*r, where L is labor, K is capital, w is a wage, and r is a rent. If you know that right now r=2w (rent is twice higher than the wages!) in market. Find how many labor and capital must be hired for optimal production of 1000 units of goods.

III. Wage=3000 TMT and rent=3000. Construct such a production function so that optimal combination of labor and capital for efficient production must always equal to L=K.

IV. By doing all calculations you found out that optimal number of labor and capital must be L=850 and K=200 for optimal production of 5000 units of good. If company wants to increase production by 50% (assume all other variables remain constant), how many employees and capital he must hire to be efficient?

V. Production function of a company is $Q = 8L^{0.8}$, labor wages are 5000 TMT, and price of a good is 1000 TMT. What is maximum quantity of labors the company can hire?

VI. Production function of a company is $Q=K^2+L^2$, and TC=1000L+1000K+100000. If company wants to

produce 10000 units, what is my optimal combination and quantity of labor and capital?

VII. A company has an option to choose of following production functions: $Q_1=88K^{0.9}$ or $Q_2=120L^{0.6}$. Which production function the manager must choose to earn maximum profit if price is 1000TMT per unit of good, wage=8000TMT and rent=9000TMT?

VIII. Assume a company has a production function $Q=11K^{0.9}L^{0.7}$, and price in the market is 1000TMT. There 3 cases:
Case 1: w=r
Case2: w=2r
Case3: r=2w
In which case a company earns maximum profit?

IX. With given Isoquant $\frac{9000}{L^{0.5}} = K$, and Isocost= A-0.8L=K, find optimal combination and quantity of labor and capital for efficient production?

X. In a market wage=500TMT and rent=1000TMT. And a company has a production function $Q=K^{0.5}L^{0.5}$. What is a market price?

XI. There are two companies and each of them have their own production function $Q_1=8K^{0.9}L^{0.1}$ and $Q_2=10LK^{0.2}$ and these companies are operating in labor intensive economy where wage=500 TMT and rent for capital=7000. If the price of the good is 100 TMT in markets, what do you think which company will succeed in competition and win over the market?

XII. Newly created mining company in "Faraway village", faces Labor supply function of; $L_s=30W-3000$ where W=wage, L means labor quantity. If company's production function is Q=L, then find and construct profit maximizing relationship between wage and market price in perfectly competitive environment.

XIII. Capital and labor supply and demand functions are given:
$K_s=10r-1000$
$K_d=9000-10r$
$L_s=80w-50$
$L_d=5950-20w$

If your production function looks like this; $Q=7L^{0.9}K^{0.2}$ and your company targets to produce 2000 units of good, this is only possible if minimum of 100 capitals are hired. Find the total cost.

XIV. With production function below:

$$Q = \frac{4000}{L - K}$$

What is my revenue maximizing condition? What is my cost minimizing condition?

XV. In an economy where average wage=100 TMT and rent=2000 TMT, which input strategy you will prefer and why?

XVI. In an economy where labor demand and supply function is:
$L_{s1}=3000w-100$
$L_{d1}=79900-1000w$
In another economy where labor demand function is:
$L_{s2}=1000w-200$
$L_{d2}=239800-200w$
If company has production function of a form; $Q=L^{0.88}$, where this company better move production?

XVII. Human labor is cheap in India and China, because they have huge labor force, but still, those states spend billions of dollars in order to acquire new technology and level up their industries. Why are they doing that?

XVIII. Two neighbor countries have different capital markets;
$K_{s1}=20r-400$
$K_{d1}=10600-90r$
$K_{s2}=40r-200$
$K_{d2}=15000-80r$
Which economy will have capital inflow and why? Which economy will have capital outflow and why?

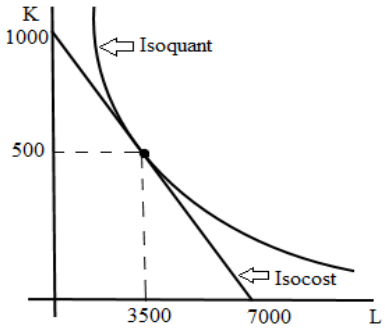
XIX. In an inputs market wage=900TMT and rent=2000TMT. And a company has a production function $Q=2K^{0.8}L^{0.3}$. Find optimal combination and quantity of labor and capital for producing 5000 units of good. Graph isoquant and isocost functions. What will happen to our optimum combination if wages drop to 500 TMT from initial price? What will happen if rent dropped to 1000 TMT from initial price? What will happen if form of the production function changes to

$Q=3K^{0.5}L^{0.4}$ (w=900 and r=2000)? Calculate total cost in all cases above.

XX. Use this isocost function and production function: $Q=3K^{0.3}L^{0.7}$ and K=1250-0.25L and calculate maximum unit of goods that can be produced?

XXI. $MPP_L=1.8*\left(\frac{K}{L}\right)^{0.4}$, wage=800TMT, rent=1800TMT, if the targeted production is 8800 units, how many employees and capital the company must hire? How much profit the company will make?

XXII. Production function has constant returns to scale characteristics. In the graph shown optimal combination of labor and capital to produce 1000 units of goods. Find the production function.

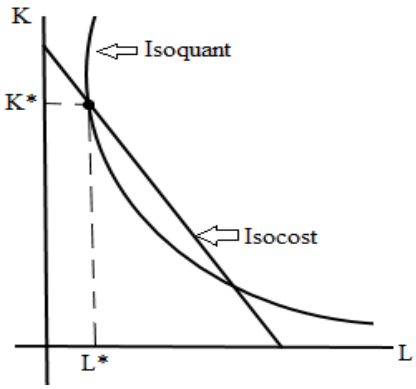


XXIII. If we know that production function: $Q=K^{\alpha}$ has increasing returns to scale characteristics and also Π=0 when r=1000TMT and price=50. Which numbers can alpha and K can take?

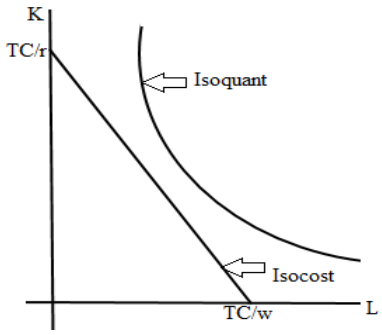
XXIV. Two neighbor countries have different labor markets:
$L_{s1}=100w-40$
$L_{d1}=7460-50w$
$L_{s2}=300w-1000$
$L_{d2}=14000-200w$

Which economy will have labor inflow and why? Which economy will have labor outflow and why?

XXV. Explain the graph. Isoquant and isocost cross.



XXVI. Explain the graph. Isoquant and Isocost does not cross.



XXVII. Q=K+L this is the production function of a pharmaceutical company. Draw the graph of the isoquant if the company targets to produce 2000 units of good.

XXVIII. $MPP_K=\sqrt{\frac{L}{K}}$, optimal combination of labor and capital is L=4K. How many labor and capital company needs to produce 7000 units of goods?

XXIX. If the company wants to stay within the limits of total cost of 2000000TMT at an average wage of 4000TMT and rental rate is at 10000TMT with fixed cost of 500000TMT with constant returns to scale production function, find the relationship function of any of input with power (example $Q=K^{\alpha} L^{1-\alpha}$ then find relationship function between K and α, or L and α)

XXX. Give example of businesses where fixed cost is almost none existent.

XXXI. Give me examples of businesses where variable cost is almost none existent.

XXXII. Give me example of businesses where total cost is minimal compared to other industries.

XXXIII. Give me industries which are associated with huge costs.

XXXIV. Give me the example of labor intensive industries.

XXXV. Give me capital intensive industries.

XXXVI. Medical employees in Switzerland are paid more than medical employees in Kazakhstan. Why is that?

Solutions:

I. $Q=7K^{0.1}L^{0.6}$
$Q(2K;2L)=7(2K)^{0.1}(2L)^{0.6} \rightarrow$
$Q(2K;2L)=7*2^{0.1+0.6}K^{0.1}L^{0.6} =7*2^{0.7}K^{0.1}L^{0.6} \rightarrow$
$2^{0.7}=1.62<2$ decreasing returns to scale.

$Q=K^{0.2}L^{0.5}$
$Q(2K;2L)= (2K)^{0.2}(2L)^{0.5}\rightarrow$
$Q(2K;2L)=2^{0.2+0.5}K^{0.2}L^{0.5} =2^{0.7}K^{0.2}L^{0.5} \rightarrow$
$2^{0.7}=1.62<2$ decreasing returns to scale.

$Q=AK^{0.5}L^{0.9}$
$Q(2K;2L)=A(2K)^{0.5}(2L)^{0.9} \rightarrow$
$Q(2K;2L)=A*2^{1.4}K^{0.5}L^{0.9} \rightarrow 2^{1.4}=2.64>2$ increasing returns to scale.

$Q=KL \rightarrow Q(2K;2L)=(2K)(2L) = 4KL \rightarrow 4>2$ increasing returns to scale.

$Q=2\sqrt{K^3}L^{0.9} \rightarrow Q(2K;2L)= 2(2K)^{\frac{3}{2}} (2L)^{0.9}= 2 * 2^{\frac{3}{2}}K^{\frac{3}{2}}2^{0.9}L^{0.9}=2^{2.4}=5.28>2$ increasing returns to scale.

$Q=\sqrt[5]{K^3L^7} \rightarrow Q(2K;2L)=(2K)^{\frac{3}{5}}(2L)^{\frac{7}{5}} = 2^{\frac{3}{5}}K^{\frac{3}{5}}2^{\frac{7}{5}}L^{\frac{7}{5}} = 2^2 = 4 > 2$ increasing returns to scale.

$Q=K^{0.3}L \rightarrow Q(2K;2L)=(2K)^{0.3}2L =2^{0.3+1}K^{0.3}L \rightarrow 2^{1.3}=2.46>2$ increasing returns to scale.

$Q=9K^{0.1}L^{1.3} \rightarrow Q(2K;2L)=9*(2K)^{0.1}(2L)^{1.3}=$
$9*2^{0.1+1.3}K^{0.1}L^{1.3} \rightarrow 2^{1.4}=2.64>2$ increasing returns to scale.

$Q=\sqrt{\frac{80}{56}}K^{0.5}L^{0.5} \rightarrow$
$Q(2K;2L)=(1.43)^{\frac{1}{2}}*(2K)^{0.5}(2L)^{0.5}=(1.43)^{\frac{1}{2}}2^{0.5}K^{0.5}2^{0.5}L^{0.5} \rightarrow$
$2^{0.5+0.5}=2$ constant returns to scale.

$Q=\frac{K^{0.4}}{L} \rightarrow Q(2K;2L)=(2K)^{0.4}(2L)^{-1}=2^{0.4}K^{0.4}2^{-1}L^{-1} \rightarrow$
$2^{-0.6}=0.66<2$ decreasing returns to scale.

$Q=K^{0.7} + L^{0.7} \rightarrow Q(2K;2L)=(2K)^{0.7}+(2L)^{0.7}=$

$2^{0.7}K^{0.7}+2^{0.7}L^{0.7}\rightarrow 2^{0.7}+2^{0.7}=3.25>2$ increasing returns to scale.

$Q=K^{0.5}-L^{0.4}\rightarrow Q(2K;2L)=(2K)^{0.5}-(2L)^{0.4}=$
$2^{0.5}K^{0.5}-2^{0.4}L^{0.4}\rightarrow 2^{0.5}-2^{0.4}=0.095<2$ decreasing returns to scale.

II. $Q=9K^{0.3}L^{0.8}$

$TC=wL+rK$

$r=2w$

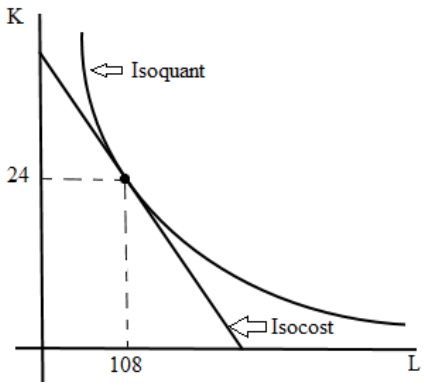
$Q=1000$

$TC=wL+2wK$

$$\frac{TC-wL}{2w}=K$$

$$\frac{TC}{2w}-\frac{1}{2}L=K$$

$$\frac{\Delta K}{\Delta L}=-\frac{1}{2}$$



$1000=9K^{0.3}L^{0.8}$

$$\left(\frac{1000}{9L^{0.8}}\right)^{3.33}=(K^{0.3})^{3..33}$$

$K=6491909*L^{-2.66}$

$$\frac{\Delta K}{\Delta L}=-\frac{1}{2}\rightarrow -2.66*6491909L^{-3.66}=-\frac{1}{2}$$

$(34536956)^{0.27}=(L^{3.66})^{0.27}$

$108=L$

$1000=9K^{0.3}(108)^{0.8}$

$$\left(\frac{1000}{381.1}\right)^{3.33}=(K^{0.3})^{3.33}$$

$24=K$

III. $Ł=3000L+3000K+\lambda(Q-K^{\alpha}L^{\beta})$

$$\frac{\Delta Ł}{\Delta L}=0\rightarrow 3000-\lambda\beta L^{\beta-1}K^{\alpha}$$

$$\frac{\Delta Ł}{\Delta K}=0\rightarrow 3000-\lambda\alpha K^{\alpha-1}L^{\beta}$$

$$\frac{3000}{\beta L^{\beta-1}K^{\alpha}}=\frac{3000}{\alpha K^{\alpha-1}L^{\beta}}$$

$$3000*\beta L^{\beta-1}K^{\alpha}=3000*\alpha K^{\alpha-1}L^{\beta}$$

$\alpha L=\beta K$

L=K

$\boldsymbol{\alpha}=\boldsymbol{\beta}$

$Q = K^{0.5}L^{0.5}$

IV. L=850

K=200

Q=5000

L → 850

K → 200

200L=850K

L=4.25K

If we assume that our production function looks like:

$Q = K^{\beta}L^{\alpha}$

If L=4.25K and $Q = 5000 \rightarrow 5000 = K^{\beta}(4.25K)^{\alpha}$

$5000=4.25^{\alpha}K^{\beta}K^{\alpha}$

$5000=4.25^{\alpha}K^{\alpha+\beta}$

If we increase our quantity produced by 50%:

$7500=4.25^{\alpha}(K_2)^{\alpha+\beta}$

Our (input) K_2 depends on what characteristics have our production function:

5000 → $4.25^{\alpha}K^{\alpha+\beta}$

7500 → $4.25^{\alpha}(K_2)^{\alpha+\beta}$

$5000*(4.25^{\alpha}(K_2)^{\alpha+\beta})=7500*(4.25^{\alpha}K^{\alpha+\beta})$

After cancellations of 4.25^{α} we get:

$5000K_2=7500K$

$\frac{K_2}{K} = \frac{7500}{5000}$

$\frac{K_2}{K} = \frac{3}{2}$

$K_2=1.5K$

If we know that our K=200, then:

$K_2=1.5*200=300$

If production function has increasing returns to scale characteristics ($\alpha+\beta>1$), then $K_2<300$

If production function has decreasing returns to scale characteristics ($\alpha+\beta<1$), then $K_2>300$

If production function has constant returns to scale characteristics ($\alpha+\beta=1$), then $K_2=300$

V. $Q=8L^{0.8}$

w=5000

p=1000

$\frac{\partial Production\ function}{\partial Labor} * p = w$

$8*0.8*L^{-0.2}*1000=5000$

$5L^{0.2}=6.4$

$(L^{0.2})^5=1.28^5$

$L=3$

VI. $Ł=1000L+1000K+100000+\lambda(Q-K^2-L^2)$

$\frac{\Delta Ł}{\Delta K} = 0 \rightarrow 1000 - \lambda\, 2K = 0$

$\frac{\Delta Ł}{\Delta L} = 0 \rightarrow 1000 - \lambda\, 2L = 0$

$2000K=2000L$

$K=L$

$10000=2K^2$

$5000=K^2$

$70.71=K\approx70$

Optimal combination is K=L, then if optimal quantity of capital is 70, then optimal quantity of labor is also 70.

VII. ***Method 1:***

$Q_1=88K^{0.9}$

$P=1000$

$w=8000$

$r=9000$

Since we don't have labor in production function, our total cost will be:

$TC=r*K=9000*K$

$Π=TR-TC \rightarrow 1000*(88K^{0.9})-9000K$

$\frac{\Delta Π}{\Delta K} = 0 \rightarrow \frac{88000 * 0.9}{K^{0.1}} - 9000 = 0$

$88000*0.9=9000K^{0.1}$

$\frac{79200}{9000} = K^{0.1}$

$(8.8)^{10}=(K^{0.1})^{10}$

$2785009760=K$

$Π=TR-TC \rightarrow P*Q-TC=1000*(88*2785009760^{0.9}) - 9000*2785009760=2785009760094$

$Q_2=120L^{0.6}$

Since we don't have capital in production function, out total cost will be:

$TC=w*L=8000L$

Π=TR-TC $\rightarrow$ 1000*(120$L^{0.6}$)-8000L

$$\frac{\Delta\Pi}{\Delta L} = 0 \rightarrow \frac{120 * 1000 * 0.6}{L^{0.4}} - 8000 = 0$$

72000=8000$L^{0.4}$

$$\frac{72000}{8000} = L^{0.4}$$

$(9)^{2.5}$=($L^{0.4}$)$^{2.5}$

L=243

Π_2=TR-TC $\rightarrow$ P*Q-TC=1000*(120*$243^{0.6}$)-8000*243=1296000

The company must choose first production where Q_1=88$K^{0.9}$, because it gets higher profit than with second production function.

Method 2:

Q_2=120$L^{0.6}$

TC=w*L=8000L

MPP_L*P=w

120*0.6$L^{-0.4}$ *1000=8000

$$\frac{72000}{L^{0.4}} = 8000$$

($L^{0.4}$)$^{2.5}$ = $(9)^{2.5}$

L=243

Q_2=120*$243^{0.6}$=3240

Π=TR-TC $\rightarrow$ P*Q-TC =3240*1000-8000*243=1296000

Q_1=88$K^{0.9}$

TC=r*K=9000K

MPP_K*P=r

0.9*88$K^{-0.1}$ *1000=9000

$$\frac{79200}{K^{0.1}} = 9000$$

($K^{0.1}$)^{10} = $(8.8)^{10}$

K=2785009760

Q_1=88*$2785009760^{0.9}$=27850097600

Π=TR-TC $\rightarrow$ P*Q-TC =1000*27850097600-9000*2785009760=2785009760094

VIII. Q=11$K^{0.9}L^{0.7}$

P=1000

Here in order to know in which case we get maximum profit, we must solve all given options:

Case 1:

w=r

Π=TR-TC $\rightarrow$ 1000*(11$K^{0.9}L^{0.7}$)-wL-wK

$$\frac{\Delta\Pi}{\Delta K}=0\rightarrow\frac{11000*0.9L^{0.7}}{K^{0.1}}-w=0$$

$$\frac{\Delta\Pi}{\Delta L}=0\rightarrow\frac{11000*0.7K^{0.9}}{L^{0.3}}-w=0$$

$$\frac{9900L^{0.7}}{K^{0.1}}=\frac{7700K^{0.9}}{L^{0.3}}$$

7700K=9900L

K=1.29L

Π=1000(11*(1.29L)$^{0.9}$L$^{0.7}$)-wL-w(1.29L)

Π=13833L$^{1.6}$-wL-w(1.29L)

$$\frac{\Delta\Pi}{\Delta L}=0\rightarrow1.6*13833L^{0.6}-2.29w=0$$

$$\frac{22132.8L^{0.6}}{2.29}=w$$

9665L$^{0.6}$=w

TC=2.29wL

Π_1=TR-TC $\rightarrow$1000*13833L$^{1.6}$-2.29wL=13833000L$^{1.6}$-2.29wL

Case 2:

w=2r$\rightarrow$r=$\frac{w}{2}$

Π=TR-TC $\rightarrow$ 1000*(11$K^{0.9}L^{0.7}$)$-$wL$-\frac{w}{2}$K

$$\frac{\Delta\Pi}{\Delta K}=0\rightarrow\frac{11000*0.9L^{0.7}}{K^{0.1}}-\frac{w}{2}=0$$

$$\frac{\Delta\Pi}{\Delta L}=0\rightarrow\frac{11000*0.7K^{0.9}}{L^{0.3}}-w=0$$

$$\frac{19800L^{0.7}}{K^{0.1}}=\frac{7700K^{0.9}}{L^{0.3}}$$

7700K=19800L

K=2.57L

Π=1000(11*(2.57L)$^{0.9}$L$^{0.7}$) $-$ wL $-$ $\frac{w}{2}$K

Π=25724L$^{1.6}$-wL-1.29wL

Π=25724L$^{1.6}$-2.29wL

$\frac{\Delta \Pi}{\Delta L} = 0 \rightarrow 1.6 * 25724L^{0.6} - 2.29w = 0$

$41158L^{0.6}=w$

$TC=2.29wL$

Π_2=TR-TC $\rightarrow$ $1000*25724L^{1.6}-2.29wL=25724000L^{1.6}-2.29wL$

Case 3:

$r=2w$

Π=TR-TC $\rightarrow$ $1000*(11K^{0.9}L^{0.7})-wL-2wK$

$\frac{\Delta \Pi}{\Delta K} = 0 \rightarrow \frac{11000 * 0.9L^{0.7}}{K^{0.1}} - 2w = 0$

$\frac{\Delta \Pi}{\Delta L} = 0 \rightarrow \frac{11000 * 0.7K^{0.9}}{L^{0.3}} - w = 0$

$\frac{9900L^{0.7}}{2K^{0.1}} = \frac{7700K^{0.9}}{L^{0.3}}$

$15400K=9900L$

$K=0.643L$

$\Pi=1000*(11*(0.643L)^{0.9}L^{0.7}) -wL-2*(0.643L)*w$

$\Pi=7392L^{1.6}-2.29wL$

$TC=2.29wL$

Π_3=TR-TC $\rightarrow$ $1000*7392L^{1.6}-2.29wL=7392000L^{1.6}-2.29wL$

In all three cases we can see that costs are equal with each other and since we cannot find all unknowns in profit function, to know when we get maximum profit we compare their revenues:

In case 1: $TR=13833000L^{1.6}$

In case 2: $TR=25724000L^{1.6}$

In case 3: $TR=7392000L^{1.6}$

The company earns maximum profit in second case: because when w=2r it has the highest revenue.

IX. $\frac{9000}{L^{0.5}} = K = 9000L^{-0.5}$

$\frac{\Delta K}{\Delta L} = -0.5 * 9000L^{-1.5} = -4500L^{-1.5}$

$K=A-0.8L$

$\frac{\Delta K}{\Delta L} = -0.8$

$-4500L^{-1.5}=-0.8$

$\left(\frac{4500}{0.8}\right)^{0.67} = (L^{1.5})^{0.67}$

L=5625

K=120

L ↗ 5625

K ↘ 120

5625K=120L

L=46.875K

X. ***Method 1:***

w=500

r=1000

$Q=K^{0.5}L^{0.5}$

$MPP_L*P=w$

$$\frac{0.5K^{0.5}}{L^{0.5}} * P = 500$$

$$P = \frac{500L^{0.5}}{0.5K^{0.5}}$$

$MPP_K*P=r$

$$\frac{0.5L^{0.5}}{K^{0.5}} * P = 1000$$

$$P = \frac{1000K^{0.5}}{0.5L^{0.5}}$$

$$\frac{500L^{0.5}}{0.5K^{0.5}} = \frac{1000K^{0.5}}{0.5L^{0.5}}$$

$$250L = 500K$$

L=2K

$$P = \frac{1000K^{0.5}}{0.5(2K)^{0.5}}$$

$$P = 1414$$

Method 2:

$Ł=500L+1000K+\lambda(Q-K^{0.5}L^{0.5})$

$$\frac{\Delta Ł}{\Delta L} = 0 \rightarrow 500 - \frac{\lambda 0.5K^{0.5}}{L^{0.5}} = 0$$

$$\frac{\Delta Ł}{\Delta K} = 0 \rightarrow 1000 - \frac{\lambda 0.5L^{0.5}}{K^{0.5}} = 0$$

$$\frac{500L^{0.5}}{0.5K^{0.5}} = \frac{1000K^{0.5}}{0.5L^{0.5}}$$

250L=500K

L=2K

$$P = \frac{1000K^{0.5}}{0.5(2K)^{0.5}}$$

$$P = 1414$$

XI. $Q_1=8K^{0.9}L^{0.1}$

P=100 TMT

Ł=500L+7000K+λ(Q-$8K^{0.9}L^{0.1}$)

$$\frac{\Delta Ł}{\Delta L} = 0 \rightarrow 500 - \frac{\lambda 8 * 0.1K^{0.9}}{L^{0.9}} = 0$$

$$\frac{\Delta Ł}{\Delta K} = 0 \rightarrow 7000 - \frac{\lambda 8 * 0.9L^{0.1}}{K^{0.1}} = 0$$

$$\frac{500L^{0.9}}{0.8K^{0.9}} = \frac{7000K^{0.1}}{7.2L^{0.1}}$$

3600L=5600K

L=1.5K

Q=$8K^{0.9}(1.5K)^{0.1}$=8.32K

TC=500L+7000K=500(1.5K)+7000K=7750K

Π=TR-TC=P*Q-TC=100*8.32K-7750K=−6918K (first company can't even make a profit when the price is 100 TMT, with that production function)

$Q_2=10LK^{0.2}$

Ł=500L+7000K+λ(Q-$10LK^{0.2}$)

$$\frac{\Delta Ł}{\Delta L} = 0 \rightarrow 500 - \lambda 10 * K^{0.2} = 0$$

$$\frac{\Delta Ł}{\Delta K} = 0 \rightarrow 7000 - \frac{\lambda 10 * 0.2L}{K^{0.8}} = 0$$

$$\frac{7000K^{0.8}}{2L} = \frac{500}{10K^{0.2}}$$

1000L=70000K

L=70K

Q=10*(70K)* $K^{0.2}$=$700K^{1.2}$

TC=500(70K)+7000K=42000K

Π=TR-TC=P*Q-TC=100*$700K^{1.2}$-42000K=$70000K^{1.2}$-42000K (second company's profit is positive with given production function, so second company is winning over the first)

XII. Q=L

L_s=30w-3000 P=1000

Π=TR-TC→P*Q-w*L→P*L-w*L

$\frac{\Delta\Pi}{\Delta L} = 0 \rightarrow P - w = 0$

P=w

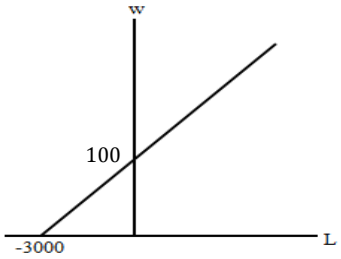
Say, when wage=1000, our labor supplied:

L_s=30*1000-3000=27000

TC=w*L=1000*27000=27000000

Π=TR-TC → P*Q-TC=1000*27000-27000000=0

Indeed, in perfect competition we make zero profit.



XIII. K_s=10r-1000

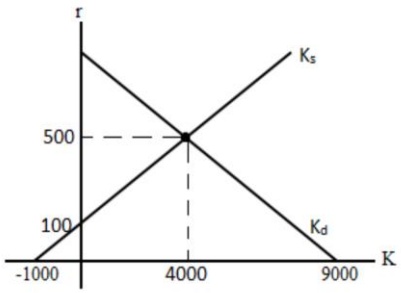
K_d=9000-10r

Equilibrium rent is:

10r-1000=9000-10r

20r=10000

r=500 is equilibrium rental rate



L_s=80w-50

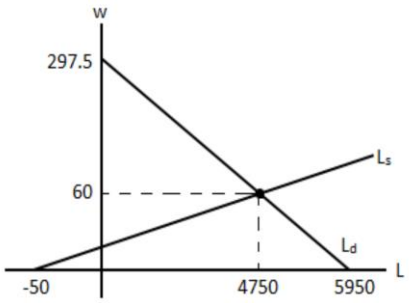
L_d=5950-20w

Equilibrium wage is:

80w-50=5950-20w

100w=6000

w=60



Q=2000

$Q=7L^{0.9}K^{0.2}$

$2000=7*L^{0.9}*100^{0.2}$

$(114)^{1.11}= (L^{0.9})^{1.11}$

192=L

TC=w*L+r*K=60*192+500*100=61520

XIV. $Q = \frac{4000}{L-K}$

Conditions for revenue maximization:

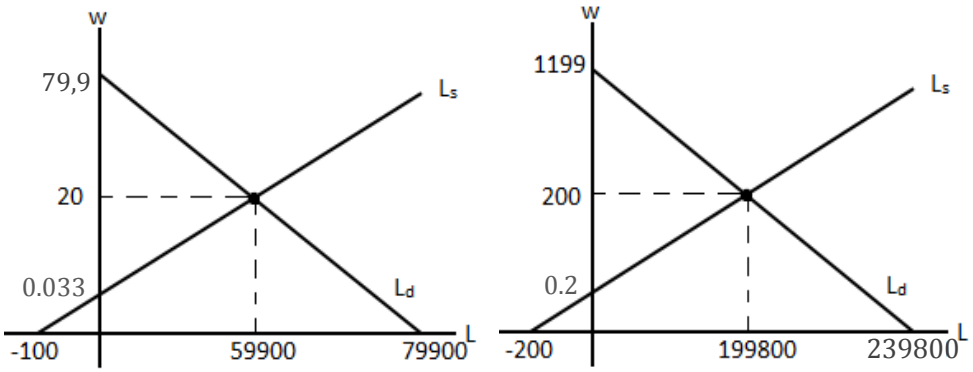
a) L>K; otherwise Q<0
b) Q increase as (L-K) goes down
c) L, K∈N L and K can take any number up to infinity, but when we differentiate them it must be equal to 1:

L-K=1
L=K+1
If above condition (L-K=1) is satisfied, maximum of Q=4000
Conditions minimizing cost:

a) TC=w*L+r*R → both L and K must be as small as possible to minimize the cost.

XV. Input strategy depends on:
1) *Functional form:* if the country has a capital intensive economy, (power of K is higher than L) that country is productive more on rental rate rather than wage rate, even if wage rate is zero, zero benefit from that. And, if the country is labor intensive, if (power of L is higher than K) for that country cheaper labor input is more appropriate.
2) *Business form:* if a certain country gets more revenue from agricultural sector or construction, where more labor (human being) is needed, for that country less costly is to use cheaper labor input because business's main revenue from that sector. And, if specialization is on capital sector where IT, software, programming, car manufacture is much more developed, lower capital input such as rent is appropriate for that country.

XVI. $L_{d1}=L_{s1} \rightarrow 79900-1000w=3000w-100$
$4000w=80000$
$w_1=20$
$L_{d2}=L_{s2} \rightarrow 239800-200w=1000w-200$
$240000=1200w$
$w_2=200$
Since $w_1=20$
The company better move production where labor is cheap, because it decreases cost of the good.
$\Pi=TR-TC \rightarrow P*Q-w*L-r*K$, lower wage rate and rental rate lowers cost of the goods produced.



XVII. Both countries are developing countries where capital is needed, rate of return on capital in those countries are high. Labor is cheap and rate of return for labor force is also cheap. At the beginning of growth countries need capital, it is vital. Overabundance of labor and scarce capital (technology) makes capital very expensive. That is why developing countries invest hugely on technology and education, they aim to increase capital input in economy. Capital increase increases productivity much faster than labor input at the beginning of the development process.

XVIII. $K_{s1}=K_{d1} \rightarrow 20r-400=10600-90r$

$110r=11000$

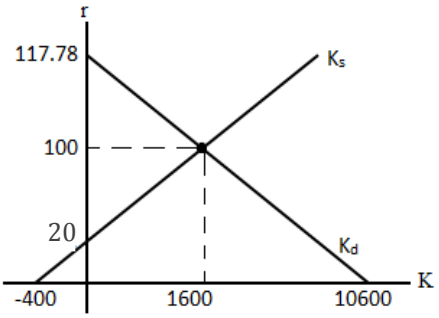
$r=100$

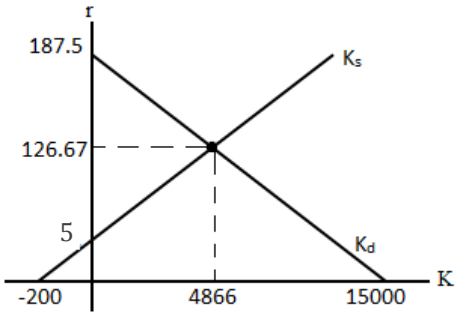
$K_{s2}=K_{d2} \rightarrow 40r-200=15000-80r$

$120r=15200$

$r=126.67$

Economy with K_{s1} and K_{d1} will have capital outflow because when we give a place/equipment or anything else for a rent in return we are getting paid for that, and it is better for us to get higher rental payment.





XIX. a) $w=900TMT$

$r=2000TMT$

$Q=2K^{0.8}L^{0.3}$

$Q=5000$

$TC=900L+2000K$

$$\frac{TC-900L}{2000}=K$$

$A-0.45L=K$

$$\frac{\Delta K}{\Delta L}=-0.45\ (isocost)$$

$5000=2K^{0.8}L^{0.3}$

$$\left(\frac{5000}{2L^{0.3}}\right)^{1.25}=(K^{0.8})^{1.25}$$

$17678L^{-0.375}=K$

$\frac{\Delta K}{\Delta L}(isoquant) = \frac{\Delta K}{\Delta L}(isocost) \rightarrow -0.375 * 17678L^{-1.375} = -0.45$

$\left(\frac{6629.25}{0.45}\right)^{0.727} = (L^{1.375})^{0.727}$

1072=L

K=17678*(1072)$^{-0.375}$

K=1291

TC=1291*2000+1072*900=3546800

b) if wages drop to 500 TMT

TC=500L+2000K

A-0.25L=K

$\frac{\Delta K}{\Delta L}(isoquant) = \frac{\Delta K}{\Delta L}(isocost) \rightarrow -0.375 * 17678L^{-1.375} = -0.25$

$\left(\frac{6629.25}{0.25}\right)^{0.727} = (L^{1.375})^{0.727}$

1644=L

K=17678*(1644)$^{-0.375}$

K=1100

TC=1100*2000+1644*500=3022000

c) If rental rate is 1000TMT

TC=900L+1000K

A-0.9L=K

$\frac{\Delta K}{\Delta L} = -0.9$ (isocost)

$\frac{\Delta K}{\Delta L}(isoquant) = \frac{\Delta K}{\Delta L}(isocost) \rightarrow -0.375 * 17678L^{-1.375} = -0.9$

$\left(\frac{6629.25}{0.9}\right)^{0.727} = (L^{1.375})^{0.727}$

648=L

K=17678*(648)$^{-0.375}$=1560

TC=1560*1000+900*648=2143200

d) Q=3K$^{0.5}$L$^{0.4}$

TC=900L+2000K

A-0.45L=K

$\frac{\Delta K}{\Delta L}(isocost) = -0.45$

$\left(\frac{5000}{3L^{0.4}}\right)^{2} = (K^{0.5})^{2}$

$2777778L^{-0.8} = K$

$\frac{\Delta K}{\Delta L}(isoquant) = \frac{\Delta K}{\Delta L}(isocost) \rightarrow -0.8 * 2777778L^{-1.8} = -0.45$

$\left(\frac{2222222}{0.45}\right)^{0.55} = (L^{1.8})^{0.55}$

4802=L

$K=2777778*(4802)^{-0.8}=3152$

$TC=3152*2000+900*4802=10625800$

XX. $Q=3K^{0.3}L^{0.7}$

$K=1250-0.25L$

$K+0.25L=1250$

$4K+L=5000$

$Ł=3K^{0.3}L^{0.7}+\lambda(5000-L-4K)$

$\frac{\Delta Ł}{\Delta L}=0\rightarrow\frac{3*0.7K^{0.3}}{L^{0.3}}-\lambda=0$

$\frac{\Delta Ł}{\Delta K}=0\rightarrow\frac{3*0.3L^{0.7}}{K^{0.7}}-4\lambda=0$

$\frac{2.1K^{0.3}}{L^{0.3}}=\frac{0.9L^{0.7}}{4K^{0.7}}$

$8.4K=0.9L$

$9.33K=L$

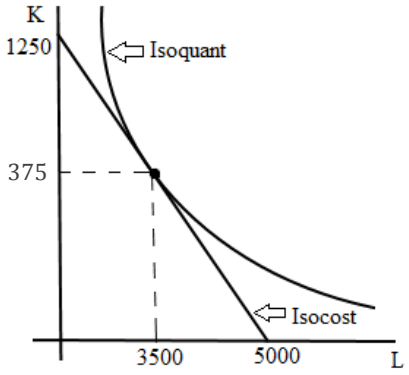
$5000=4K+9.33K$

$K=375$

$L=5000-4*375=3500$

$Q=3*(375)^{0.3}*(3500)^{0.7}$

$Q=5372$



XXI. $MPP_L=1.8*\left(\frac{K}{L}\right)^{0.4}$

$w=800$

$r=1800$

$Q=8800$

$\frac{\Delta Q}{\Delta L}=1.8*\frac{K^{0.4}}{L^{0.4}}$

$Q=AK^{0.4}L^{0.6}$

$1.8=A*0.6$

$3=A$

$Q=3K^{0.4}L^{0.6}$

$TC=800L+1800K$

$A-0.44L=K$

$\frac{\Delta K}{\Delta L}=-0.44$

$8800=3K^{0.4}L^{0.6}$

$\left(\frac{8800}{3L^{0.6}}\right)^{2.5}=(K^{0.4})^{2.5}$

$K=466018914L^{-1.5}$

$\frac{\Delta K}{\Delta L}=-0.44\rightarrow-1.5*466018914L^{-2.5}=-0.44$

$\left(\frac{699028371}{0.44}\right)^{0.4} = (L^{2.5})^{0.4}$

L=4791

K=466018914*$(4791)^{-1.5}$=1405

Now we find price:

$MPP_L*P=w \rightarrow 1.8 * \left(\frac{1405}{4791}\right)^{0.4} * P=800$

P=727

Π=TR-TC →727 * 8800-(800*4791+1405*1800)=35800

XXII. Q=$AK^{\alpha}L^{1-\alpha}$

We assume that technology is constant number 1:

1000=1*$500^{\boldsymbol{\alpha}}$*$3500^{1-\boldsymbol{\alpha}}$

ln1000=αln500+(1-α)ln3500

6.91=6.215α+(1-α) 8.16

6.91=6.215α+8.16-8.16α

8.16α -6.215α=8.16-6.91

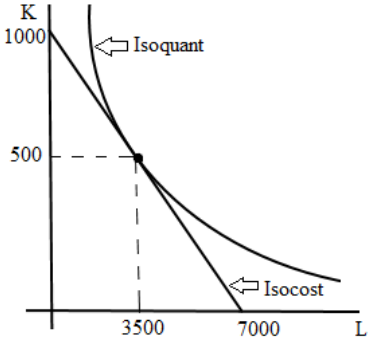
1.945α=1.25

α=0.64

1-0.64=0.36

Q=$AK^{\alpha}L^{1-\alpha}$

Q=$500^{0.64}$*$3500^{1-0.64}$=1007 (due to rounded numbers)



XXIII. Q=$K^{\boldsymbol{\alpha}}$

Π=0 and r=1000

P=50

Π=TR-TC=0 → P*Q-r*K=0

Π=50*$K^{\boldsymbol{\alpha}}$-1000K=0

$K^{\boldsymbol{\alpha}}$=20K

$K^{\boldsymbol{\alpha}-1}$=20

We assume that $\boldsymbol{\alpha}\epsilon R^+$ [$\boldsymbol{\alpha}$>1] and KϵN [0; +∞]

K can't be equal to 1.

When K=2 → $\boldsymbol{\alpha}$ ϵ (5<$\boldsymbol{\alpha}$<6)

When K=3, 4 → $\boldsymbol{\alpha}$ ϵ (3<$\boldsymbol{\alpha}$<4) or 5<K<20 → $\boldsymbol{\alpha}$ ϵ (2<$\boldsymbol{\alpha}$<3)

XXIV. $L_s=L_{d1}$ → 100w-40=7460-50w

150w=7500

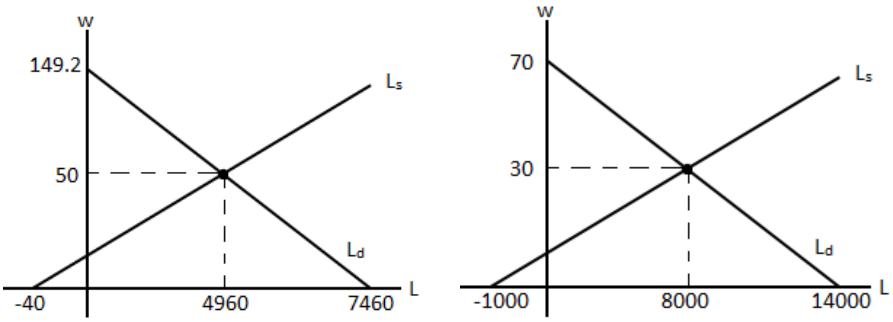
w=50

$L_{s2}=L_{d2}$ → 300w-1000=14000-200w

500w=15000

w=30

Since rate is higher in L_{s1d1} country, that country will have labor inflow. And country with lower wage will have labor outflow, because nobody wants to work for lower wage when there is an opportunity to live better.



XXV. Assume we have following functions to explain the graph better:

Q=100

w=100TMT and r=500TMT

TC=100L+500K→A-0.2L=K

$Q=K^{0.5}L^{0.5}$

$$\left(\frac{100}{L^{0.5}}\right)^2 = (K^{0.5})^2$$

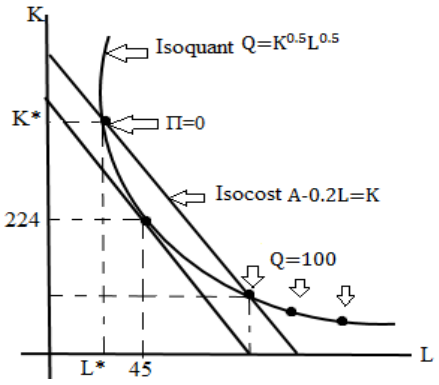
$$\frac{10000}{L} = K$$

$$\frac{\Delta K}{\Delta L} = -0.2 \rightarrow \frac{-10000}{L^2} = -0.2$$

L=224

$100=(224)^{0.5}K^{0.5}$

$44.64=K \approx 45$



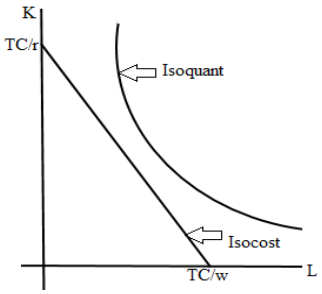
TC=45*500+224*100=44900

44900=500K+100L

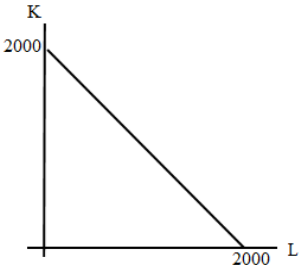
89.8-0.2L=K

Company has inefficient production and total cost function. Efficient production and total cost function must be tangent. Both or at least one of the functions of the company is inefficient. It is either production function, or total cost.

XXVI. With given total cost function, desired output is unattainable.



XXVII. $Q=K+L$
$Q=2000$
$2000=K+L \rightarrow 2000-L=K$



XXVIII. $MPP_K=\sqrt{\frac{L}{K}}=L^{0.5}K^{-0.5}$
$L=4K$
$Q=7000$
$Q=AL^{\alpha}K^{\beta}$
$\beta-1=-\frac{1}{2}$
$\beta=\frac{1}{2}$
$A*\frac{1}{2}=1$
$A=2$
$Q=2L^{0.5}K^{0.5}$
$7000=2*(4K)^{0.5}K^{0.5} \rightarrow 7000=4K \rightarrow 1750=K$
$L=4K \rightarrow L=4*1750=7000$

XXIX. $TC=2000000$
$w=4000$ TMT
$r=10000$ TMT
$FC=500000$ TMT
$Ł=4000L+10000K+500000+\lambda(Q-K^{\alpha}L^{1-\alpha})$
$\frac{\Delta Ł}{\Delta L}=0 \rightarrow 4000-\lambda(1-\alpha)K^{\alpha}L^{-\alpha}=0$
$\frac{\Delta Ł}{\Delta K}=0 \rightarrow 10000-\lambda\alpha K^{\alpha-1}L^{1-\alpha}=0$
$4000=\lambda(1-\alpha)K^{\alpha}L^{-\alpha}$
$10000=\lambda\alpha K^{\alpha-1}L^{1-\alpha}$
$\frac{4000L^{\alpha}}{(1-\alpha)K^{\alpha}}=\frac{10000K^{1-\alpha}}{\alpha L^{1-\alpha}}$
$4000\alpha=10000(1-\alpha)$

$4000\alpha = 10000 - 10000\alpha$
$14000\alpha = 10000$
$\alpha = 0.71$
$Q = K^{0.71}L^{0.29}$

XXX. Businesses where fixed cost is almost nonexistent are the service sectors: for example, lawyer, consulting, audit, designers, barbershop... they pay only rental rate, the rest is their profit.

XXXI. Businesses where variable cost is almost nonexistent: service sector (example hair dressers, massage salons, etc.)

XXXII. Minimal total cost industries: software developers, IT sector, and other service sectors.

XXXIII. Huge cost industries: mining, manufacturing, transportation, car manufacturing etc…

XXXIV. Labor intensive industries are industries of construction business, textile industry, food service, agriculture, and mining.

XXXV. Capital intensive industries: computer, manufacturing, refining, steel production, transportation sectors, IT sector.

XXXVI. The reason why in Switzerland and in Kazakhstan medical employees are paid differently is because those countries have different labor function. Thus, demand for labor and supply of labor is not the same in both countries.

Chapter 4: Essential macroeconomics 1

We have learned about supply and demand, how to find optimal combination of labor and capital so that our product will be as cheap as possible in order to earn maximum profit, how to find maximum profit, how much to produce and many more things but up to this point, everything we have learned were problems (or topics) of micro level. Those were the questions for manufacturers, lawyers, security guards, students and so and so. These are fundamentals and now, it is time to move further. We will now switch to macro questions. The branch of economics that studies macro questions is called macroeconomics. So, what are the macro questions? Let's write it down some of them:

1) Why some countries are developed and wealthy while others are underdeveloped and poor?
2) Why some economies grow faster than others?
3) What is the importance's of institutions in economic growth?
4) What kind of exchange system country must use; fixed or floating?
5) What do we need to fight, unemployment or inflation?
6) What are the impacts of trade?
7) What are the main issues of globalization in financial terms?
8) Which monetary and fiscal policy to implement?
9) Does political system have any impact to economy?
10) What are the impacts of corruption and bureaucracy to transparency of an economy?

…and many more. As you can see, at a first glance those questions might look frightening and even un-answerable, but as we go, you will see that things are not that hard and blurry as it looks. You will be able to answer them if you finish this book (at least I hope you will have an idea how to answer them!).

I strongly believe that macroeconomics must be started from the fundamental measurements of economics such as GDP, GDP per capita, Human Development Index, Rule of Law and so and so, because first of all it is very interesting and secondly, it is easy. By looking at economic indicators and statistics we can actually say almost everything about that economy. What kind of problems it has and what kind of advantages and disadvantages persistent if faced with economic or financial difficulty? Knowing this indexes and being familiar with their calculations makes any economics professional feel like a doctor, who can read the patient and

understands his/her problem. When you visit a doctor, before writing you any pills she always first listens to you. Asks you couple of questions regarding your diet, work and sleep. After that, she sees and compares the symptoms of your illness and sends you for medical checkup and analysis. After blood and other tests are submitted, doctor looks at your results and only after that she puts a diagnosis, and writes you pills and other chemicals which will help you out. Of all this work, the most time consuming and the hardest one is the first part: Talking to a patient and trying to understand the problem than sending to medical checkup for tests. The easiest part is writing a pill and recommendation. If doctor makes wrong decisions and puts wrong diagnosis, then patient will definitely suffer. This irresponsible error of doctor can even lead to tragic consequences. Understanding the problem and putting the right diagnosis is 99% of successful healing! Same is true for any economist. The hardest part of the economist's jobs is to trying to figure out what is the problem. Looking at the numbers, making some calculations, regressions, research, analysis, and coming up with some kind of diagnosis is the most time and brain consuming part of this job. For economist, meeting with the patient and talking to him starts with glancing at macroeconomic indicators. Like Sherlock Holmes, economist tries to understand what the problem is or what is expected to be a problem by simply analyzing these indicators. Putting all these numbers, evidences, stories from past and present is done to find the main clue, the main witness, the main hero or villain. Solving the issue does not take much. **Socio-economic indicators are the best friends of an economist!** All of those indicators are immensely important, and we will study a few of them here.

Gross Domestic Product (GDP)

Let's start from the first question: Why some countries are developed and wealthy while others are underdeveloped and poor? Before answering this question, let me take your attention to another question: How do we know that one country is wealthy and other is poor? Is there any measurement scale? Actually, yes, it is called a GDP, *Gross Domestic Production.* GDP measures annual total output of that economy in money terms. **GDP of India is close to 3 trillion USD (as of 2020)**: this means; that given year total of 3 trillion USD worth of final goods and services were produced by state and private companies in India. GDP of Russia is just over 1.5 Trillion USD (as of 2020), so compared to Indian economy, Russian economy produces two times lesser goods and services. **GDP measures the size of an economy**. According to above data, Indian economy is the twice the size of Russian economy. GDP is good scale to measure "**production might"** of an economy. GDP gives economist information about how much of goods and services are produced in an economy. Without knowing anything about

economies and simply comparing their GDP's one might get an idea in which country number of factories, farms, businesses and labor force exceed. We know India's and Russia's GDP; in which country do you think numbers of factories are more? Number of banks? Number of businesses? If your answer was India, you were correct.

Below is the top 50 countries with their respected GDP's (in millions of USD)[7]:

1	United States	21,427,700
2	China	14,342,903
3	Japan	5,081,770
4	Germany	3,845,630
5	India	2,875,142
6	United Kingdom	2,827,113
7	France	2,715,518
8	Italy	2,001,244
9	Brazil	1,839,758
10	Canada	1,736,426
11	Russia	1,699,877
12	Korea, South	1,642,383
13	Spain	1,394,116
14	Australia	1,392,681
15	Mexico	1,258,287
16	Indonesia	1,119,191
17	Netherlands	909,070
18	Saudi Arabia	792,967
19	Turkey	754,412
20	Switzerland	703,082
21	Poland	592,164
21*	Taiwan	574,905
22	Thailand	543,650
23	Sweden	530,833
24	Belgium	529,607
25	Argentina	449,663
26	Nigeria	448,120
27	Austria	446,315
28	Iran	445,345
29	United Arab Emirates	421,142
30	Norway	403,336
31	Israel	395,099
32	Ireland	388,699
33	Philippines	376,796
34	Singapore	372,063
34*	Hong Kong	366,030

[7] All data presented in this book is from World Bank, International Monetary Fund, OECD, World Gold Council, Transparency International, Bank of International Settlements databases for 2017, 2018, 2019 (worldbank.org, imf.org, data.oecd.org, gold.org, worldjusticeproject.org, bis.org)

35	Malaysia	364,702
36	South Africa	351,432
37	Denmark	348,078
38	Colombia	323,803
39	Egypt	303,175
40	Bangladesh	302,571
41	Chile	282,318
42	Pakistan	278,222
43	Finland	268,761
44	Vietnam	261,921
45	Romania	250,077
46	Czech Republic	246,489
47	Portugal	237,686
48	Iraq	234,094
49	Peru	226,848
50	Greece	209,853

GDP is a total output, like cars, haircuts, consultations, toothpastes, but how come it is expressed in money terms? Because it is easy! What would you say when asked about GDP of your country? Would you start counting quantities of goods and services? Is not it better just to convert all those goods and services produced to one of currencies? (For example to your local currency) How we do it? Just count those goods and services in prices they were sold. Instead of saying that you had sold 200 toothpastes, 20 bags of tea and 30 sodas today, you say I made 2210 TMT revenue today (200*10+20*3+30*5). Would not your boss be happier if you cut it short? Same is true when calculation GDP's. Economies simply calculate **final** goods and services produced annually and multiply it with respected prices. I just want to take your attention to the word final. Why do you think this word is vital when calculating GDP's? Assume for example in one economy you have farmer who grow cottons and sells harvest to nearby domestic textile business (for 800 TMT/ton) who makes fabrics from cotton and also sales ready hand-made textile fabric to local tailor shop (from one ton of cotton 600KG of textile fabric is produced and sold for 3000 TMT). A tailor makes amazing hand-made jackets and sells it to you (from 600KG of textile fabric exactly 10 jackets could be made, each sold for 1000 TMT). What is total output in this particular example? If we add up everything, 800+3000+1000*10=13800, we will make big mistake because value of a final good produced is only 10000 TMT. Final good is 10 jackets for 1000 TMT each. All previous costs are already included in 1000 TMT price tag and if we count all other prices, we will count cotton cost three times and textile fabric costs two times! Baker buys eggs, milk, flour and other ingredients and spends money, but all those expenses are reflected in price of a cake. When I ask how much worth of good produced? You just say the price of the cake, because all the costs are already in there. Economically speaking, price of the final good includes all previous

costs and no need to count them once again. That is why notion *final* good is extremely important.

Exercise 1*: You are a manager of a construction company. You build and sell houses. You bought 10 tons of lumber (wood) for total of 90000 TMT from local lumber manufacturer, you bought 2 tons of steel from local producer for total of 20000 TMT, you also bought 50 tons of cement for 20000 TMT, paints for 5000 TMT, roofing material for 10000 TMT, and finally windows and doors for 15000 TMT. You built this house and sold it for 250000 TMT. What is your and all other suppliers' value of output? Value of total output is 250000 TMT. House is the final good and price of the house includes all other outputs produced by suppliers.*

If we count all outputs separately 90000+20000+20000+5000+10000+15000+250000=410000 TMT *we will make calculation mistake or accounting mistake by counting inputs twice! Price of 250000 TMT already includes all produced inputs (or outputs).*

Let's take our example with jackets, and learn methods of measuring GDP. GDP could be measured in three ways:

1. **Adding up aggregate spending:** Consumers spent 10000 TMT to buy jackets. **When using this method, only spending on final goods are counted!** Spending of textile manufacturer (he spent 800 TMT to buy cotton!) and tailor (she spent 3000 TMT to buy textile fabric) is not counted. All those spending are reflected on price of final good. In our construction example above, the aggregate spending is 250000TMT. The word *"aggregate"* has many meanings and some of them are *final* and *total.*

"Adding up aggregate spending" method is the most widely used method in macroeconomic analysis. Overall, in any economy, spending is divided into four categories:

i. **Consumption spending (C):** In this category all spending by private business and households are included.

ii. **Government spending (G):** In this category all spending by government (budget) is included. Spending on education, healthcare, defense, research, subsidies to private business, etc.

iii. **Investment spending (I):** Spending which is not targeted for consumption today, but rather spending targeted at bringing some returns in the future. Example: Businessman buying

iron mine or gold mine, households spending on higher education, building a hotel, or medical center, etc.

iv. **Spending on Imports (IM):** Businesses and households not only spend money on local goods but they also purchase from foreign economies. Spending on the imported goods is also a vital part of total spending.

Adding all this spending gives us a GDP (we have to subtract spending on imported goods and services from exports because spending on imports are our expenses and exports are income).

GDP=C+G+I+ (Export-Import)

Exercise 2: *Assume that there are only three countries exist, India, Russia and Netherlands. India's consumption spending is three times larger than Russia's and twice larger of Netherland's. Government spending of Russia is twice larger of India's and four times larger than Netherland's. Netherland's* Investment *spending is the same as India's but quarter larger than Russia's. Imported goods to Russia from India are five times less than exported goods to India from Russia. Imported goods to Netherlands from India exceed imported goods to India from Netherlands four times. Russia export earnings to Netherlands equals to its import expenses to Netherlands. Construct GDP's functions of all three countries.*

C_I*=Consumption spending of India*

C_R*=Consumption spending of Russia*

C_N*=Consumption spending of Netherlands*

We write all transactions in terms of India:

$\frac{1}{3}C_I=C_R$

$\frac{1}{2}C_I=C_N$

$2G_I=G_R$

$\frac{2}{4}G_I=G_N$

$I_I = I_N$

$\frac{1}{1.25}I_I=I_R$

Russia $\underset{a}{\overset{5a}{\rightleftharpoons}}$ *India Trade balance =4a*

Netherlands $\overset{b}{\Rightarrow}$ $\overset{4b}{\Leftarrow}$ *India Trade balance =3b;*

Russia $\overset{c}{\Rightarrow}$ $\overset{c}{\Leftarrow}$ *Netherlands balanced trade relationship*

$GDP_I = C_I + G_I + I_I + (3b-4a)$

$GDP_R = \frac{1}{3}C_I + 2G_I + \frac{1}{1.25}I_I + (4a)$

$GDP_N = \frac{1}{2}C_I + \frac{2}{4}G_I + I_I + (-3b)$

***Exercise 3**: Look at the table of GDP's above and find Mexico's GDP. If you know that Mexico's trade balance for last years was about 200 Billion USD (surplus), and that consumption spending makes up of 70% of GDP, and total investment spending last year was equal to only 30 Billion USD, then find how much Mexico's government spent last year.*

GDP_{Mexico}=1.258.287=1.258.287*(70%) +G+30+200
G=1.258.287-(880.800,9-30-200) =377.256,1

2. **Adding up total value of production:** Now the same thing we did in aggregate spending method, we do it a little bit differently, but the result is same. Let's take our cotton example; farmer harvested cotton and sold it for 800 TMT to textile manufacturer. Farmer used his knowledge and land and other resources and turned seeds into beautiful cotton. Seeds worth almost nothing before, it were just a handful of raw biological material. Farmer worked day and night, watering them, adding some chemicals, protecting them from cold and hot and so on. Long word short, farmer turned almost nothing to an 800 TMT worth material. Farmer added value to a cotton seed (we will assume that seed cost is so cheap that close to nothing). How much value? Farmer sold it for 800 TMT so that much value was added to initial product (seeds). Textile manufacturer worked on this raw material (added some chemicals, washed it and pressed it and screwed it and… long word short, applied all the techniques to turn it to textile material!) and turned it to a textile material. Thus, textile manufacturer actually added value to initial raw cotton. From simple raw cotton, it turned to amazing textile material ready to be turned into a jacket! How much value was added to initial product (cotton)? Textile manufacturer sold the material to tailor shop for 3000 TMT, but we know that initial cotton cost is in the price, so, 3000-800=2200

TMT this much value was added to initial product. Product that was bought for 800 TMT became now a product worth of 3000 TMT, textile manufacturer added a 2200 TMT value to initial product. Tailor used her knowledge and did amazing job by giving design, cutting and sewing the textile material and turning this simple textile material into something fashionable. Long word short, tailor added some value to initial textile material. It is not a simple piece of cloth anymore; it is an amazing jacket! Tailor made ten jackets from all textiles bought for 3000 TMT and sold it for 10000 TMT. How much value did she added to initial product (textile material)? She added 10000-3000=7000 TMT value to initial material (textile material). **We just then calculate how much value was added to initial materials to make it a final good: farmer made seeds that worth nothing a cotton product which was sold to 800 TMT, textile manufacturer turned raw cotton material into textile fabric and sold it for 3000 TMT by adding 2200 TMT to initial value, tailor used her talent and turned simple textile fabric into fashionable cloth and sold them for 10000 TMT by adding 7000 TMT to initial product she bought. Total values added=800+2200+7000=10000 TMT.**

Exercise 4: *Diamond miners sold raw diamond for 9000 TMT. All diamonds go through three stages step by step until they find their places in fashionable diamond store: 1) manufacturing, 2) cutting, and polishing. With each stage, raw diamond turns into a gemstone and increases its value. Manufacturing increases value of raw diamond up to 30%, cutting increases the value of manufactured diamond up to 50%, and polishing add 80% value to nicely cut diamond. Generally accepted profit margin for diamond stores is 20%. If our diamond passes through all stages, what will be the final price of that gemstone at store?*

*a) Manufacturing: 9000+9000*30%=11700*
*b) Cutting: 11700+11700*30%=15210*
*c) Polishing: 15210+15210*80%=27378*
*d) Profit margin: 27378+27378*20%=32853.6*
e) Final price: 32853.6

Exercise 5*: You ate an amazing tuna fish at restaurant for 200 TMT. You know that this piece of delicious meat passes through four stages until you ate it: 1) caught by fisherman 2) cleaned by whole sales 3) processed and marinated by restaurant suppliers 4) cooked in restaurant. If you know that each stage does not add more than 10% to initial value of fish, then what was the price that fisherman sold it to whole sales?*

Here we start from the end:
4) *Cooked in restaurant:*

200 = A+A*10% = A*(1+0.1)

A= $\frac{200}{1+0.1}$ = 181.82 TMT

3) *Processed and marinated by restaurant suppliers:*

181.82=A+A*10%=A*(1+0.1)

A = $\frac{181.82}{1+0.1}$ = 165.29 TMT

2) *Cleaned by whole sales:*

165.29 = A+A*10% = A*(1+0.1)

A= $\frac{165.29}{1+0.1}$ =150.26 TMT

1) *Caught by fisherman:*

150.29 = A+A*10% = A*(1+0.1)

A= $\frac{150.26}{1+0.1}$ = **136.6 TMT** *is an initial price!*

3. **Adding up factor income earned by households:** In economics science factors of production means, the tools and resources which without production is impossible. Example: labor, land, resources, capital all these are factors of production. These factors of production cost money. You must pay rent to use a capital, pay salary for labor, pay rent for using land and resources and so on. Economists also use money as a factor of production, money as an investment. The cost of this factor is a dividend or interest rate if loaned it from bank. Labor is a factor of production in factory and gets paid an agreed salary for his work. All equipment was rented and paid a rent. Banker also earns from this factory because he loaned money to factory owner, the bankers payment is an interest rate. What factory owner earns then? He earns a profit. Entrepreneurship is also a factor of production and they earn profits. **Entrepreneurs will not start a business and will not risk if they do not see (expect) profits!** Let's count factor incomes:
    - Salary
    - Profit
    - Interest rate
    - Dividend

- Rent

In third method of calculating GDP, factor incomes are added up. Let's use our cotton example; how much profit farmer earned (we will assume all entrepreneurs has only one cost, resource cost!)? Farmer has no costs, seeds cost nothing, and his profit is:

Revenue-Total cost=Profit
800 TMT-0=800 TMT

What is textile manufacturer's profit? His profit is:

Revenue-Total cost=Profit
3000 TMT-800TMT=2200 TMT

What is tailor's profit? Her profit is:

Revenue-Total cost=Profit
10000 TMT-3000TMT=7000 TMT

Add up all factor incomes:

800TMT+2200TMT+7000TMT=10000 TMT

Total factor income earned is 10000 TMT.

Economists use all three methods to calculate GDP, and all needed statistical numbers are acquired from official resources such as government organizations. The important fact is that all three method results must be equal!

Exercise 6: *Economists of New Zealand calculated that last year total of 67 Billion NZD (New Zealand Dollars) were paid as salary, entrepreneurs earned 10 Billion NZD of profits in terms of rent, 9 Billion NZD from investments (interest) and other profits totaled at 13 Billion NZD. What is New Zealand's GDP?*

Using factor income method:

$GDP_{\text{New Zealand}}$ = 67+10+9+13=99 *Billion (New Zealand Dollars)*

Gross Domestic Product per capita (GDP per capita)

Let's get back to India and Russia. Just by looking at the information about their GDP's, can we say anything about wealth of its citizens? For example, can I say, "India's citizens are twice richer than the Russians"? No. GDP is an indicator of the size of an economy; it does not say anything about wealth and well-being of its citizens. The most appropriate indicator of how wealthy the population lives in a given economy is **GDP per capita**. Economists also use the same indicator when talking about *productivity.*

Productivity in economics science measures how much one unit of factor of production (capital, labor, land, etc.) can produce goods and services in a given time period. Now, how do we calculate this amazing indicator which is used both for representation of labor productivity and well-being of citizens at the same time, named **GDP per capita**? If GDP of any country is divided to population number of that respected country, we will find out that **GDP per capita**. **GDP per capita** is used to measure how much of goods and services were produced per population (labor). GDP per capita of Russia is around 13333 USD, while India's GDP per capita is 2857 USD. Since Indian citizen is producing less than Russian citizen, Indian citizen is twice less productive and poorer compared to Russian citizen. India may be producing more, but Russia is wealthier!

$$\text{GDP per capita of Russia} = \frac{GDP\ of\ Russia}{Population\ of\ Russia}$$

$$\text{GDP per capita of Russia} = \frac{2\ Trillion\ USD}{150\ Million} = 13333\ USD$$

$$\text{GDP per capita of India} = \frac{GDP\ of\ India}{Population\ of\ India}$$

$$\text{GDP per capita of India} = \frac{4\ Trillion\ USD}{1.4\ Billion} = 2857\ USD$$

Below are data on GDP per capita on top 50 countries (in USD)

1*	Macau	129,103
1	Luxembourg	121,293
2	Singapore	101,376
3	Qatar	96,491
4	Ireland	88,241
4*	Cayman Islands (2018)	72,481
5	Switzerland	70,989
6	United Arab Emirates	69,901
7	Norway	66,832
8	United States	65,281
9	Brunei	64,673
9*	Hong Kong	62,375
10	San Marino (2018)	60,750
11	Iceland	60,061
12	Denmark	59,830
13	Netherlands	59,687
13*	Bermuda (2013)	59,483
14	Austria	59,111
15	Germany	56,052
16	Sweden	55,815

17	Belgium	54,545
18	Australia	53,320
19	Kuwait	51,912
20	Canada	51,342
21	Finland	51,324
22	France	49,435
23	Saudi Arabia	48,909
24	United Kingdom	48,710
25	Bahrain	46,892
26	Malta	45,652
27	Italy	44,197
28	New Zealand	43,953
29	Japan	43,236
30	Korea, South	43,029
31	Czech Republic	42,576
32	Spain	42,214
33	Israel	42,194
34	Cyprus	41,254
35	Slovenia	40,657
36	Estonia	38,811
36*	Aruba (2017)	38,442
37	Lithuania	38,214
38	Bahamas, The	37,266
39	Portugal	36,471
39*	Puerto Rico	35,948
41	Poland	34,218
42	Slovakia	34,178
43	Hungary	33,979
44	Panama	32,763
45	Romania	32,297
45*	Sint Maarten (2017)	32,256
46	Latvia	32,205
47	Greece	31,399
48	Seychelles	30,260
49	Croatia	29,973
50	Malaysia	29,526

Exercise 7: *See the table with GDP's and using GDP per capita table shown above calculate each nation's population size. And check your findings from open sources (Skip missing nations)*

Population of United States = $\frac{GDP\ of\ United\ States}{GDP\ per\ capita\ of\ United\ States\ of\ America}$

Population of United States = $\frac{21{,}427{,}700{,}000{,}000}{65{,}281}$ = 328.237.925 people approximately!

№	Countries	Population
1	Singapore	3670129.025
2	Ireland	4404970.479
3	Switzerland	9904097.818
4	United Arabian Emirates	6024835.124
5	Hong Kong	5868216.433
6	Denmark	5817783.721
7	Netherlands	16036657.43
8	Austria	7477591.435
9	Germany	68608256.62
10	Sweden	9508875.952
11	Belgium	9709542.58
12	Canada	33820770.52
13	Finland	5236555.997
14	France	54931081.22
15	Saudi Arabia	16213110.06
16	United Kingdom	58039683.84
17	Italy	45280086.88
18	Japan	117535618.5
19	South Korea	38169211.46
20	Chech Republic	5789388.388
21	Spain	33024968.02
22	Israel	9363866.901
23	Poland	17305628.62
24	Greece	9539206.328

Inflation

Inflation is a price hike. The price hike might be due to many reasons:

- **Loss of the value of local currency due to:**

  - ❖**Overprinting of local currency by central bank:** The easiest way to solve your budget deficit problem is to just print the money. A lot of developing and underdeveloped nations knows this trick very good. And they also very familiar with deadly consequences of this politics. Hyperinflation.
  - ❖**Indebtedness of government:** Huge internal debts or external debts and decreasing revenue push states to pay off the debt to the cost of public by printing money. This leads to inflation.

- **Product shortages:** There are certain strategic products we studied in previous chapters which have strictly price inelastic

demand functions; they carry extreme importance for population. These products are:

- ❖**Energy:** Petroleum, gas, electricity, coal, gasoline, etc. These products are vital. Shortage of these products will create havoc in economy. Prices for all goods and services will "fly over the roof", because energy is the core of not only economy, but a life!
- ❖**Food and water:** Water, flour, sugar, oil, rice, etc. is the on the top of the "the most important items for life" list. Do I need to add what will happen to economy where those products will suddenly be scarce?

- **Sanctions:** In our global world where all states are connected to each other through trade, politics or any other way are actually dependent to each other. Especially big markets (countries with big GDP!) have huge impact on small economies. Economic sanctions of big economies can cause great damage to economy of small economy, which will be reflected in increasing prices.

- **Economic or financial crisis:** Economic turmoil due to any reason is always a problem for a market. First of all, crisis always hits the stability of an economy. Stability means-stable prices!

- **Trade issues:** If an economy is export or import oriented, which means depends huge on exports or imports, in case of trade issues like trade embargo, economies will suffer. Economies depended to import will suffer from shortage of goods which will cause huge price increase of imported goods. Economies depended to export will have huge losses because they cannot sell goods they produced.

- **Increase in aggregate demand:** Sudden inflow of capital or fast growth of economy can cause "overheating" of an economy, which means due to demand increase prices increase. If this continues market will face with "financial bubbles". Prices and income of population does not match: Prices rise faster than income!

- **Economic or financial reforms:** Sudden economic or financial reforms can cause panic in the market. Market and government must have very good communication channels so that each step of reform is well understood by both parties. Bad communication channels added up with distrust of population to government can cause misunderstanding, which will cause price instability. Each step of any reform must be explained well before stepping up for implementation by authorities.

Markets are volatile by nature, and as you can read from above, any information or incorrect step will be punished immediately in terms of rising prices. Inflation is never good, high inflation is deadly. The reason is in savings function of fiat money. Money has many functions, but the most important one is its savings function. Old days people saved in gold and silver, but nowadays people save money. Savings is actually investment. Savings is an airbag of population. Without savings future look grim. But how will population save if inflation is destroying the value of those savings? Imagine annual inflation rate of 10%, what does it mean? It means that the money that you saved for new bike or new house is losing its 10% value. Saying it differently, your dream bike is increasing its price by 10% annually. This makes your dream unattainable. Your income increases, but prices rise faster! High inflation rates distorts population from saving, this kills investment, especially long term investment. No investment, no future! Instead of investing money into future, population prefers consume it today by buying cars, apartments, jewelry, etc. Nobody will want to keep money in hand when inflation rate is high. High inflation rates kill trust of people to local currency. Low inflation rates show that economy is stable, prices are not fluctuating. Stable economy attracts investment, and investment means economic growth. **Inflation is the biggest and the deadliest enemy of an economy and it is duty of every central baker to fight against it.** Inflation is core of distrust between government and population.

So how inflation is calculated? Inflation is calculated by simply calculating the prices of yesterday and comparing it with prices of today. The difference is inflation or deflation:

$$P_{t+1}-P_t = \Psi \quad \text{if} \quad \Psi>0 \text{ inflation}$$

$$P_{t+1}-P_t = \Psi \quad \text{if} \quad \Psi<0 \text{ deflation}$$

Governments create a basket of goods and services that population buy on daily basis, which carry high importance to it and strategic importance for states such as: rice, flour, gasoline, oil, sugar, potato, apple, etc. Let's create our own basket as an example:

1kg of rice = 6 TMT
1kg of flour = 3 TMT
1 liter of vegetable oil = 8 TMT
1 kg of potato = 5 TMT
Rent for 1-bedroom apartment/month = 1000 TMT
1kg of beef meat = 18 TMT
1kg of chicken meat=10 TMT
1 kg of sugar = 7 TMT
10 M^3 natural gas fee = 1 TMT
100 M^3 water supply=3 TMT

The goods that included in basket solely depend on that country's consumption culture. Indian basket will contain spices while American consumer basket will definitely contain beef. Basket content will be nation specific goods, even though majority of goods will still be a common everywhere. Price information on those goods will be collected on regular basis (daily, weekly, monthly, etc.) from all corners of the country; average weight of prices is found and compared to previous price list. Collecting prices from all corners of a country eliminates price bias due to geographic location (Prices in north provinces might be different than south, etc.). Example: List of goods are sent to biggest ten cities in different provinces for gathering of information on prices of the goods and services on the list. After getting all those ten price list information, we take a weighted average (we sum all prices and divide it to 10). In economics, that is how inflation is calculated and the average price list is called ***Consumer Price Index (CPI).*** Depending on change of prices required economic measures will be taken. If change in price is positive, thus prices increased it is inflation, then after analysis of consumer consumption basket authorities will decide which actions to take. Again, if price change is negative we have a deflation, which means currency gained value, purchasing power of currency increased, authorities will decide what to do or what not to do. Let's assume we are living in a very tiny island where only three towns exist. We are interested in inflation rates and want to calculate it. How we do it? First thing we do is to create basket of goods and services that all people on island use:

1 Coconut
1 KG of fish
1 Liter of fresh water
Rental rate of boats for fishing
Haircuts

Now, when basket of goods and services is created send it to three towns for gathering price information. Assume that we did it in below are prices from three towns:

Town1:

1 Coconut = 3 USD
1 KG of fish=5 USD
1 Liter of fresh water=0.1 USD
Rental rate of boats for fishing/day=15 USD
Haircuts=4 USD
Total price=3+5+0.1+15+4=27.1 USD

Town2:

1 Coconut = 3.1 USD

1 KG of fish=4.9 USD
1 Liter of fresh water=0.2 USD
Rental rate of boats for fishing/day=14 USD
Haircuts=5 USD
Total price=3.1+4.9+0.2+14+5=27.2 USD

Town3:

1 Coconut = 3.1 USD
1 KG of fish=5 USD
1 Liter of fresh water=0.15 USD
Rental rate of boats for fishing/day=16 USD
Haircuts=4.5 USD
Total price=3.1+5+0.15+16+4.5=28.75 USD

Now, add all prices and take weighted average:

$$Consumer\ Price\ Index = \frac{27.1 + 27.2 + 28.75}{3} = 27.683$$

Our current CPI is 27.683. Assume we want to know how prices changed in a one month. In order find out price changes in one month, we have to take CPI of previous month. Let's assume that CPI of previous month was 27.420. Then we can calculate monthly price change (monthly inflation rate):

$$Inflation\ rate = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}} * 100 = \frac{27.683 - 27.420}{27.420} * 100 = 0.96\%$$

As you can see, prices have increased almost 1% compared to previous month. Thus, we have monthly inflation rate of 0.96%. We have inflation! What if monthly price difference was negative? Let's assume CPI of previous month was 27.9, then:

$$Inflation\ rate = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}} * 100 = \frac{27.683 - 27.9}{27.9} * 100 = -0.78\%$$

Negative inflation rate means deflation! Prices decreased (dropped, fallen, etc.).

We can use any time frame for inflation rate calculation but the most widely used ones in macroeconomic analysis is annual inflation rate (yearly inflation rate). We simply find current CPI and compared it with previous years CPI (we take CPI of 12 months ago). Assume we are in March of 2020 and we want to calculate annual inflation rate. Current CPI is 27.683 and CPI of last year in month of March is 25.85. Then:

$$Inflation\ rate = \frac{CPI_{03.2020} - CPI_{03.2019}}{CPI_{03.2019}} * 100$$

$$Inflation\ rate = \frac{27.683 - 25.85}{25.85} * 100 = 7.09\%$$

Our annual inflation rate is 7.09%, which means; compared to last year, prices of goods and services had risen by 7.09%.

Below are top 50 countries with highest annual inflation rates:

1	Guatemala	4.43
2	Myanmar	4.57
3	Barbados	4.66
4	Afghanistan	4.97
5	South Africa	5.18
6	Uganda	5.21
7	Lesotho	5.31
8	Tanzania	5.32
9	Tunisia	5.33
10	Algeria	5.59
11	São Tomé n Prín p	5.69
12	Bangladesh	5.78
13	Belarus	6.03
14	Georgia	6.04
15	Mexico	6.04
16	Namibia	6.14
17	Eswatini (Swaziland)	6.22
18	Uruguay	6.22
19	Moldova	6.57
20	Zambia	6.58
21	Tajikistan	7.31
22	Kazakhstan	7.44
23	Sri Lanka	7.7
24	Kenya	8.02
25	Gambia	8.05
26	Rwanda	8.28
27	Madagascar	8.28
28	Guinea	8.92
29	Ethiopia	9.85
30	Iran	10.48
31	Turkey	11.14
32	Malawi	12.18
33	Ghana	12.37
34	Liberia	12.42
35	Azerbaijan	12.91
36	Ukraine	14.44
37	Haiti	14.67
38	Mozambique	15.11
39	Burundi	16.05
40	Nigeria	16.52

41	Sierra Leone	18.22
42	Suriname	22.02
43	Yemen	24.66
44	Argentina	25.67
45	Egypt	29.5
46	Angola	31.69
47	Sudan	32.35
48	Democratic Republic of the Congo	41.5
49	South Sudan	187.85
50	Venezuela	1087.51

Exercise 8: *Assume current CPI of Venezuela is 98776000 VES (Bolivar Soberano is the national currency of Venezuela). Use above annual inflation rate table (current) and find out what was CPI of Venezuela last year.*

$$Inflation\ rate = \frac{CPI_{current\ year} - CPI_{last\ year}}{CPI_{last\ year}} * 100$$

$$= \frac{98776000 - CPI_{last\ year}}{CPI_{last\ year}} * 100 = 1087.51\%$$

$$Inflation\ rate = \frac{98776000 - CPI_{last\ year}}{CPI_{last\ year}} = \frac{1087.51}{100}$$

Inflation rate =98776000-$CPI_{last\ year}$=10.8751$CPI_{last\ year}$ + $CPI_{last\ year}$

98776000=11. 8751 $CPI_{last\ year}$

$CPI_{last\ year}$=8317909

Exercise 9: *If you were an investor, where would you prefer invest your money Egypt or Iran? Use above table to answer question.*

Iran, because there is lower inflation rate (10.48%), than in Egypt (29.5%), which means currency in Iran loses its value less than in Egypt, or in other words price fluctuation in Iran is less than in Egypt.

Exercise 10: *My Surinamese uncle saved some money "under the pillow" last year at an amount of 2000000 SRD (Surinamese Dollar). Use table given above and tell me what current purchasing power of my uncle's savings is?*

Inflation rate in Suriname is 22.02%

2000000-2000000*22.02%=1559600 (Purchasing power of 2000000 SRD)

Unemployment

Unemployed people are those who are at the appropriate age and willing to work but for some reasons cannot find a job. In economics science, reasons of unemployment are generally divided into two categories, although recently many other categories are added: **Structural unemployment** and **Frictional unemployment.** When labor force with certain skills cannot find a job appropriate for his/her skill, structural unemployment is created. You studied economics and 7000 other students also studied economics like you but after graduation how many of those graduates will find job as an economist or at least in economic sector? Especially in developing economies or underdeveloped economies you will meet a lot of people working not in their "specialty". Law graduates working as an accountant, economists working as construction specialists, medical school graduates working in economics sector, bus drivers with construction major background and so and so. Skills are attained but structure of an economy is not eligible to accommodate all specialists. In order to lower down structural unemployment rates economies must grow, and they must grow and develop as fast as possible. Frictional unemployment is created due to job search of an unemployed. When employed person loses his job for some reasons, it takes a time to find another job. For some time period that specialist is unemployed, this is called a frictional unemployment. For lowering down frictional unemployment rates nations must encourage state and private labor agencies. **Unemployment rate is an excellent indicator of how well economies are doing.** Nations with low unemployment rates symbolize growth and development while nations with high unemployment rates represent totally opposite. Economies with high unemployment rates generally face big social problems. **Unemployment is the main cause of all social problems because unemployed person means a person without an income.** Person without income cannot meet his first needs as food and shelter and cannot be counted as a full member of society. Unemployed people for a long time will slowly start losing respect, eventually being isolated from society. It is especially hard for unemployed people with weak or no social connection such as strong relative ties and loved family members. With low or no income at all, isolated from society people generally start either self-destruction (alcoholism or drug abuse) or even worse, start committing crime for living. Joblessness pushes people to commit all kinds of crimes from prostitution; to drug-trafficking that is why all nations do all they can to curb down unemployment rates.

So, how is unemployment rate calculated? Before answering this question, we must learn something about **labor force**. Labor force is the total number of population who are at their legal age, appropriate health condition and eligible for work. All those people that do not satisfy those conditions cannot be counted as labor force of that country (underage kids, physiologically or mentally handicapped, old, unhealthy, incarcerated, etc. these group of people are not included in labor force). **Labor force includes two categories of people: employed and unemployed.** An unemployment rate is calculated by:

$$Unemployment\ rate = \frac{Number\ of\ unemployed\ people}{Labor\ force} * 100$$

Exercise 11: *If you know that total number of unemployed in Gabon is 230000, by using below given unemployment rate table calculate the labor force of Gabon.*

Unemployment rate in Gabon is 20%

$20\% = \frac{230000}{Labor\ force} * 100$

$Labor\ Force = \frac{230000}{20\%} * 100$

Labor force = 1150000

Exercise 12: *Below are given three tables: Unemployment rate, Intentional homicide rate and Incarceration rate tables. How many countries are included in all three tables?*

Nigeria, Iraq, Iran.

Exercise 13: *Countries of which regional zone exceeds others in three table?*

South American Nations and nations in South of Africa are on the top list.

Exercise 14: *Total labor force of Armenia is 1200000 people. What is the number of unemployed in Armenia? Use below table.*

Unemployment rate of Armenia is 18.9%

$$18.9\% = \frac{Number\ of\ unemployed\ people}{1200000} * 100$$

$$\frac{18.9\%}{100} * 1200000 = Number\ of\ unemployed\ people$$

Number of unemployed people=226800

Below are top 50 countries with highest unemployment rate

1	Mayotte (France)	30
2	Kosovo	25.9
3	Palestine	25.3
4	Grenada	24
5	Lesotho	23.4
6	Nigeria	23.1
7	Eswatini	22.1
8	Netherlands Antilles (Netherlands)	21.2
9	Namibia	20.3
10	Gabon	20
11	Colombia	19.8
12	Armenia	18.9
13	Libya	18.6
14	Bosnia and Herzegovina	18.4
15	Botswana	18.2
16	North Macedonia	17.8
17	Philippines	17.7
18	Sudan	16.5
19	Tunisia	15.1
20	Jordan	14.7
21	Greece	14.4
22	Haiti	13.8
23	São Tomé n Prín p	13.4
24	Peru	13.1
25	Rwanda	13.1
26	Yemen	12.9
27	Georgia	12.8
28	Iraq	12.8
29	New Caledonia (France)	12.8
30	Costa Rica	12.4
31	Brazil	12.2
32	Cape Verde	12.2
33	South Sudan	12.2
34	French Polynesia (France)	12.1
35	Albania	11.9
36	Guyana	11.9
37	Algeria	11.7
38	Iran	11.4
39	Somalia	11.4

40	Zambia	11.4
41	Afghanistan	11.1
42	Tajikistan	11
43	The Bahamas	10.4
44	Barbados	10.3
45	Djibouti	10.3
46	Congo	9.5
47	Mauritania	9.5
48	Serbia	9.5
49	Brunei	9.1
50	The Gambia	9.1

Table of Intentional homicide victims per 100000 residents.

1	El Salvador	52.02
2	Jamaica	43.85
3	Lesotho	41.25
4	Honduras	38.93
5	Belize	37.9
6	Venezuela	36.69
7	Saint Vincent and the Grenadines	36.46
8	South Africa	36.4
9	Bahamas	30.9
10	Trinidad and Tobago	30.88
11	Mexico	29.07
12	Brazil	27.38
13	Dominica	25.7
14	Colombia	25.34
15	Guatemala	22.5
16	Saint Lucia	21.44
17	Puerto Rico	21.09
18	Central African Republic	19.76
19	Namibia	17.14
20	Botswana	15.04
21	Guyana	14.25
22	South Sudan	13.9
23	DR Congo	13.55
24	Seychelles	12.74
25	Uruguay	12.06
26	ôt 'Ivo r	11.63
27	Costa Rica	11.26
28	Palau	11.17
29	Grenada	11.1
30	Mali	10.9
31	Uganda	10.42
32	Iraq	10.08
33	Dominican Republic	10.05

34	Mauritania	9.94
35	Nigeria	9.85
36	Iran	9.85
37	Barbados	9.7
38	Eswatini	9.5
39	Panama	9.39
40	Congo	9.32
41	Gambia	9.13
42	Chad	9.04
43	Togo	9
44	Guinea	8.82
45	British Virgin Islands	8.37
46	Russia	8.21
47	Bermuda	8.2
48	Eritrea	8.04
49	Gabon	8.04
50	Guadeloupe	8.01

Table of Top 50 countries by total number of Incarcerations

1	United States of America	2,121,600
2	China	1,700,000
3	Brazil	746,532
4	Russian Federation	523,928
5	India	466,084
6	Thailand	367,993
7	Indonesia	266,584
8	Turkey	264,842
9	Iran	240,000
10	Philippines	215,000
11	Mexico	203,364
12	South Africa	163,015
13	Vietnam	123,697
14	Colombia	123,005
15	Ethiopia	113,727
16	Egypt	106,000
17	Argentina	103,209
18	Myanmar (formerly Burma)	92,000
19	Peru	90,934
20	Bangladesh	88,211
21	Morocco	85,767
22	United Kingdom: England & Wales	83,329
23	Pakistan	77,275
24	Poland	74,130
25	Malaysia	74,000
26	Nigeria	72,627
27	France	70,818

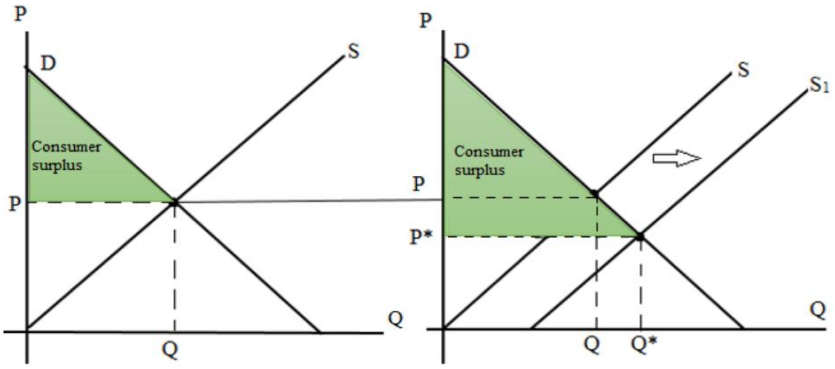
28	Rwanda	65,000
29	Germany	63,851
30	Algeria	63,000
31	Saudi Arabia	61,000
32	Taiwan	60,956
33	Italy	60,769
34	Spain	58,457
35	Cuba	57,337
36	Venezuela	57,096
37	Uganda	55,229
38	Republic of (South) Korea	55,198
39	Ukraine	52,863
40	Kenya	51,130
41	Japan	48,802
42	Iraq	45,000
43	Uzbekistan	43,900
44	Australia	43,032
45	Chile	42,921
46	Ecuador	40,006
47	Canada	39,579
48	El Salvador	38,114
49	Cambodia	36,600
50	Tanzania	35,803

Trade

Trade indicators such as export and import; show the level of economic development and level of diversification of an economy. Which goods and services are being exported and which goods and services are being imported in an economy says a lot about level of development of an economy. If only raw goods such as wood, cotton, iron, aluminum, etc. and agricultural goods are exported, and manufactured goods such as chemicals, pharmaceutical products, high tech goods, cars and machinery, etc. is imported, this indicates that economy is developing or underdeveloped. High tech goods, high end goods, luxury goods, capital intensive goods, high quality goods could only be produced in developed economy because to produce these goods one needs huge investments, high labor skills with great education level, developed economic and financial environment, which is only possible in developed economies. The more brain power and money invested into good, the higher the price of the product. How much brain is needed to grow rice? How much brain is needed for developing a vaccine? That is the reason why vaccines cost huge while rice is cheap. That is the reason why pharmaceutical companies make billions in profit while agricultural companies make only millions. Trade directly impacts economy and to show the impact the notion ***current account*** is used. *Current account* shows the trade balance.

GDP=C+G+I+ (Export-Import)

(Export-Import) is trade. If imports are more than exports, it impacts economy negatively because more money is flowing out of the economy than it inflows. To describe these situations economists, say "***negative current account***". When exports exceed imports, impact on economy is good because more money is flowing in than money flowing out. This situation is called "***positive current account***". A lot of economists believe that **trade liberalization**, minimizing legal barriers for goods and services to enter the country;



will only positively impact the nations because trade liberalization increases population welfare.

Benefits of trade is well known that is why many countries united according to their regional or traditional backgrounds and formed special customs unions and trade blocs for easy trade such as NAFTA (North American Free Trade Agreement), APEC (Asia-Pacific Economic Cooperation), EU (European Union), Mercosur (South American Trade bloc), ASEAN (Association of Southeast Asian Nations), Gulf Cooperation Council (GCC), and many more. International organizations created such as World Trade Organization (WTO), for improving trade relations of nations and solve trade issues. Even within a country special free-trade areas are created to foster and encourage trade. There are hundreds of free trade areas all around the world.

At the same time there are numerous economists that argue against trade liberalization saying that welfare benefit from trade liberalization is a short term phenomena and in a long term developing and underdeveloped economies lose the competition battle to more developed economies. These economists believe that while trade is definitely economically beneficial but the process must be liberalized step by step. This thought of trade is called ***protectionism***. Actually, if you pay attention to trade history, you will find out that all economies used protectionist policies when facing with more developed economies.

Exercise 15: *Use table below and other open sources to answer. Find United Kingdom's current account balance*

Export of UK is 495600 MLN USD

Import expense of UK is 602500 MLN USD

Current account balance= 495600-602500= -106900 MLN USD

Exercise 16: *Which country has biggest positive current account balance?*

Biggest positive current account balance is Germany's: Export-Import→1401000 -1104000=297000 *MLN USD*

Exercise 17: *Which country has biggest negative current account balance?*

The biggest negative current account balance is United States of America's:

Export-Import→1576000-352000 = -776000 MLN USD

Exercise 18: *Which country's current account balance is close to zero?*

Chile's and Finland's current account balance is close to zero: Chile's Export-Import →64510-59920=4590 MLN USD *and Finland's Export-Import*→59600-62100=-2500 MLN USD

Exercise 19*: Use GPD table above and Trade tables below and tell me, which country has biggest positive* $\frac{Current\ account\ balance}{GDP}$ *ratio?*

Germany's current account balance ratio= $\frac{297000}{3845630}$ = 0.077

Exercise 20: *Use GPD table above and Trade tables below and tell me, which country has biggest negative* $\frac{Current\ account\ balance}{GDP}$ *ratio?*

USA's current account balance ratio= $\frac{-776000}{21427700}$ *= -0.036*

Exercise 21: *Find out which goods and services are imported in and exported out of Netherlands.*

Exported goods from Netherlands are: Machinery and transport equipment, chemicals, mineral fuels, food and livestock, manufactured goods.

Imported goods to Netherlands are: Crude petroleum, refined petroleum, broadcasting equipment, computers and packaged medicaments.

Exercise 22: *Find out which goods and services are imported in and exported out of Japan.*

Exported goods from Japan are: Metal, vehicle parts, trucks, LCD's, equipment, cars, engine parts, voltage protection equipment and many others.

Imported goods to Japan are: Crude oil, petroleum gases, phone system devices including smartphones, coal, and fuel made from coal.

Exercise 23: *Find out which goods and services are imported in and exported out of Mexico.*

Exported goods from Mexico are: Vehicles, Electrical machinery, equipment, mineral fuel including oil, technical and medical apparatus, and plastics.

Imported goods to Mexico are: Petroleum oils, vehicle parts, electronic integrated devices.

Exercise 24*: Find out which goods and services are imported in and exported out of India.*

Exported goods from India are drug formulations, biological products, gold and other precious metal jeweler, petroleum products, pearls, precious stones.

Imported goods to India are: Oil, electronics, heavy machinery.

Exercise 25*: Find out which goods and services are imported in and exported out of Thailand.*

Exported goods from Thailand are: Refined petroleum, rubber, rice, raw sugar. Imported goods to Thailand are: Gold, petroleum gas, coal.

Exercise 26: *Find out which goods and services are imported in and exported out of Norway.*

Exported goods from Norway are: Crude oil, petroleum gas, fish, processed petroleum oil.

Imported goods to Norway are: machineries, computers, mineral fuel including oil, equipment, electrical machinery, iron and steel.

Exercise 27: *Find out which goods and services are imported in and exported out of Turkey.*

Exported goods from Turkey are: machinery, equipment, steel, iron, textile, manufactured goods, electrical machinery and other appliances.

Imported goods to Turkey are: Mineral fuel, computers, equipment, and precious metals.

Exercise 28: *Find out which goods and services are imported in and exported out of USA.*

Exported goods from USA are: crude oil, fuel and other petroleum products, civilian aircraft, auto parts, passenger cars, pharmaceuticals, feed, engines and many others.

Imported goods to USA are: machinery including computers, automobiles, vehicles, pharmaceuticals, medical equipment and supplies.

Exercise 29: *Find out which goods and services are imported in and exported out of China.*

Exported goods from China are: Computers, broadcasting equipment, office machine parts, and telephones.

Imported goods of China are: Micro assemblies, crude oil, iron, petroleum gases.

Exercise 30: *Which countries are not full members of WTO?*

There are only 16 countries that are not WTO members: Turkmenistan, Aruba, Curacao, Eritrea, Kiribati, Kosovo, Marshall Islands, Micronesia, Monaco, Nauru, North Korea, Palau, the Palestinian Territories, San Marino, Sint Maarten and Tuvalu.

Exercise 31: *How many free trade zones are there in United Arab Emirates?*

There are 30 free trade zones in UAE and some of them are: Dubai Airport free zone, Dubai Flower center, Dubai Internet city, Dubai.

Exercise 32*: What is OPEC? What are its targets?*

OPEC is an Organization of Petroleum Exporting Countries and its mission is "to coordinate and unify the petroleum policies of its Member countries, bring efficient supply of petroleum and fair return on capital investment".

Table of countries by their export revenue (in millions of USD)

1	China	2,157,000
2	United States	1,576,000
3	Germany	1,401,000
4	Japan	683,300
5	South Korea	577,400
6	France	551,800
7	Netherlands	526,400
8	Italy	600,000
8*	Hong Kong	496,900
9	Mexico	500,000
10	United Kingdom	495,600
11	Canada	433,000
12	Singapore	372,900
13	India	354,800
14	Brazil	349,200
15	Taiwan	344,600
16	Russia	336,800

17	Switzerland	336,800
18	United Arab Emirates	314,700
19	Belgium	309,100
20	Spain	301,500
21	Saudi Arabia	231,300
22	Thailand	228,200
23	Ireland	225,100
24	Australia	224,500
25	Poland	221,400
26	Vietnam	214,000
27	Malaysia	188,200
28	Sweden	169,700
29	Indonesia	168,800
30	Austria	161,800
31	Czech Republic	157,400
32	Turkey	157,300
33	Denmark	113,200
34	Norway	102,800
35	Hungary	100,000
36	Iran	91,990
37	Slovakia	80,570
38	South Africa	78,250
38*	Puerto Rico	73,160
39	Romania	64,920
40	Chile	64,510
41	Philippines	63,230
42	Portugal	62,600
43	Israel	60,600
44	Argentina	59,690
45	Finland	59,600
46	Iraq	56,740
47	Qatar	56,260
48	Kuwait	54,090
49	Kazakhstan	49,290
50	Peru	42,470

Table of countries by their import expenses (in millions of USD)

1	United States	2,352,000
2	China	1,000,000
3	Germany	1,104,000
4	Japan	625,700
5	France	624,900
6	United Kingdom	602,500
7	Hong Kong	558,600
8	South Korea	457,500
9	Canada	443,700

10	Netherlands	435,400
11	India	426,800
12	Italy	426,700
13	Mexico	417,300
14	Spain	333,400
15	Singapore	327,400
16	Belgium	306,100
17	Switzerland	286,700
18	Taiwan	272,600
19	United Arab Emirates	241,300
20	Poland	221,800
21	Australia	215,400
22	Russia	212,700
23	Vietnam	211,100
24	Turkey	196,800
25	Thailand	190,000
26	Austria	168,300
27	Malaysia	163,400
28	Sweden	154,800
29	Brazil	151,900
30	Indonesia	200,000
31	Czech Republic	147,400
32	Saudi Arabia	136,800
33	Ireland	96,030
34	Denmark	94,610
35	Hungary	93,280
36	Philippines	92,840
37	South Africa	80,220
38	Norway	79,900
39	Romania	75,000
40	Slovakia	77,960
41	Portugal	74,320
42	Iran	70,530
43	Israel	66,760
44	Finland	62,100
45	Argentina	60,780
46	Chile	59,920
47	Egypt	53,020
48	Greece	50,230
49	Algeria	49,990
49*	Puerto Rico	49,010
50	Pakistan	48,510

Investment

In our increasingly globalizing world where capital and labor movement across the nations is constantly speeding up, it is very important to follow investment inflows and outflows. Residents of

country A invest in country B, while country B residents invest into country A. One of the reasons why it is done is to secure your money, hedge your assets against all kinds of domestic financial and economic risks. Japanese invest into US economy; US investors invest into Japan, by doing that both investors hedge their assets against domestic financial slowdowns. In financial terms it is called "**diversification of investment portfolio**". If US economy slowdowns for some reasons, US investors who invested portion of their assets abroad will be able to secure their assets. Investors that invested only into their domestic economy will take bigger risk, because they will tie up all their assets to one economy only. This habit of investors, trying to secure their assets, increased the tendency of globalization. Now, a lot of countries are kind of economically and financially tied up to each other. Swiss are investing in Germany, Germans into China, Chinese into US, US into Taiwan, Taiwanese into Netherlands, and so on. This big net, created by investments has both positive and negative effects to the world. Positive effect is that now crisis does not last long thanks to risk sharing, and the proof is the financial crisis of 2008 which started in US and spread to global world. Just in couple of years' economies that suffered already started showing signs of recovery and some are even growth. This fast recovery was due to the created net, investment net, which helped to smooth down hard crash like a car belt. Each and every country had lost something, but without that net, there would have been many countries like Greece. Especially last couple of decade investment is not seen as only an opportunity to earn but rather opportunity to secure your assets. Nations are divided now into two main categories: creditors and debtors. **Creditor nations are those nations whose investments abroad exceed its liabilities (investments in!). Debtors are vice versa. Creditors earn return from their assets, while debtors pay interest.** Investment might be in the form of purchasing government debt obligations (bonds) or purchasing private entity shares or in any other form (direct or indirect!). Investments can be directly government investments, when one government purchases other governments bonds or it can be private investment. Increasing liabilities pose socio-economic risks to any debtor nation. Creditors are also not safe from debtors' default. Default causes huge economic problems not only to debtor, but also to creditor, because debtor nation might need to go through "debt restructuring" which means creditor nation might have to amend portion of debtor's debt. Otherwise debtor nation might not be eligible to pay anything at all.

Assets –Liabilities = Ω, if Ω>0 then Creditor nation
Assets –Liabilities = Ω, if Ω<0 then Debtor nation

Below is the table of countries with their investment account balance (Assets-Liabilities) which is called Net International Investment Position (NIIP) in millions of USDs.

1	United States	-12,057,500
2	Spain	-1,035,453
3	United Kingdom	-732,206
4	Brazil	-731,906
5	Ireland	-671,100
6	Mexico	-655,500
7	France	-633,584
8	Australia	-624,571
9	India	-426,528
10	Indonesia	-338,231
11	Greece	-317,171
12	Turkey	-314,870
13	Poland	-298,193
14	Portugal	-232,561
15	Egypt	-188,020
16	Colombia	-157,132
17	Pakistan	-125,363
18	New Zealand	-115,079
19	Romania	-107,247
20	Sudan	-85,180
21	Peru	-84,332
22	Nigeria	-82,598
23	Hungary	-76,101
24	Chile	-70,277
25	Slovakia	-69,328
26	Panama	-65,998
27	Kazakhstan	-61,573
28	Tunisia	-57,117
29	Mozambique	-52,581
30	Serbia	-45,608
31	Bangladesh	-39,401
32	Mongolia	-35,677
33	Philippines	-34,841
34	Italy	-33,388
35	Belarus	-32,488
36	Czech Republic	-31,593
37	Croatia	-30,775
38	Cyprus	-28,598
39	Zambia	-27,893
40	Cambodia	-22,254
41	Bulgaria	-21,561
42	Ukraine	-20,737
43	Democratic Republic of the Congo	-19,779
44	Malaysia	-18,761
45	Uganda	-18,640
46	Nicaragua	-16,921
47	Latvia	-14,861

48	Lithuania	-12,895
49	Armenia	-9,822
50	Thailand	-9,738
51	Montenegro	-9,044
52	Slovenia	-8,855
53	Albania	-8,028
54	Estonia	-6,992
55	Bhutan	-2,644
56	Finland	4,655
57	Iceland	4,882
58	Malta	9,003
59	South Africa	28,593
60	Luxembourg	31,284
61	Austria	66,321
62	Sweden	112,903
63	Kuwait	115,211
64	Argentina	117,113
65	Israel	153,015
66	Belgium	199,098
67	Denmark	271,675
68	Russia	356,501
69	South Korea	565,066
70	Canada	658,507
71	Saudi Arabia	682,800
72	Netherlands	813,400
73	Switzerland	838,700
74	Singapore	896,000
75	Norway	996,600
76	Taiwan	1,342,780
77	Hong Kong	1,563,100
78	People's Republic of China	2,123,995
79	Germany	2,747,700
80	Japan	3,453,557

Exercise 33: *Compare Switzerland, Taiwan and Canada, which one has the highest* $\frac{NIIP}{GDP}$ *ratio?*

Switzerland: $\frac{NIIP}{GDP} = \frac{838700}{70989}$ *=11.81*

Taiwan: $\frac{NIIP}{GDP} = \frac{1342780}{574905}$ *=2.33*

Canada: $\frac{NIIP}{GDP} = \frac{658507}{1736426}$ *=0.379*

Exercise 34: *Compare Ireland, Mexico and Australia, which one has the riskiest* $\frac{NIIP}{GDP}$ *ratio?*

Ireland: $\frac{NIIP}{GDP} = \frac{-671100}{388699}$ *= -1,73 (Ireland)*

Mexico: $\frac{NIIP}{GDP} = \frac{-655500}{1258287}$ *= -0.52*

Australia: $\frac{NIIP}{GDP} = \frac{-624571}{1392681}$ *= -0.45*

Exercise 35*: As an economist I would never take Luxembourg, Singapore, Hong Kong, Malta and Kuwait as a "role model" of economic growth and development. Why do you think is that?*

Because these countries have unique structure. All of them are small city states with small population size.

Exercise 36: *As you can see from NIIP numbers US is the biggest debtor nation out of all. But still, global world trust is US and nations continue investing into that country, otherwise how would US attract so much money? Choose 10 countries from above table as an investor and shortly explain why you chose those countries.*

Japan, Germany, People's Republic of China, Hong Kong, Taiwan, Norway, Singapore, Switzerland, Netherlands, Saudi Arabia.

By looking at NIIP, these countries show their creditworthiness. Trust is the biggest asset for any investment. No trust, no investment!

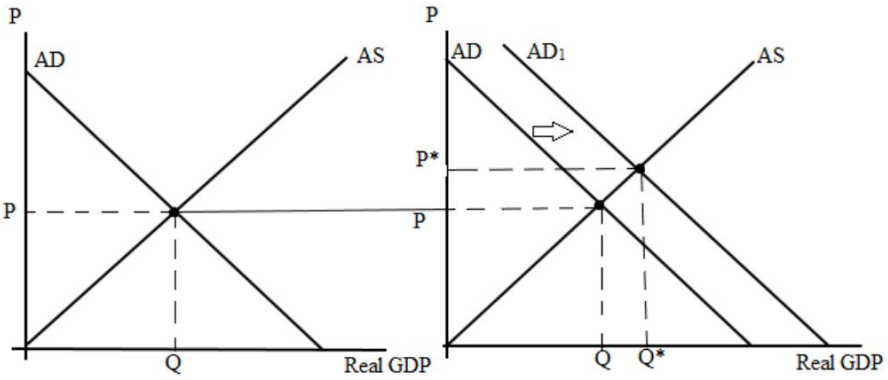
Budget

Budget is the most important instrument of governments in regulating and intervening into the market. Budget means government revenue and government spending. Revenue comes from taxes, licenses, tariffs, fees, penalties, state enterprises, and investments. Spending includes social spending (stipends, grants, subsidies, support money, schools, universities, retiree houses,

handicapped support, utilities, research, awards, etc.), state employee salaries, government projects, internal and external debt obligations, defense and security spending, compensations, emergency spending (forest fire, earthquake, flooding, etc.) and others.

GDP=C+G+I+ (Export-Import)

G in GDP formula is a budget spending (government spending). When budget spending equals to revenue, it is called **balanced budget.** When budget spending exceeds revenue it creates **budget deficit.** If budget revenue is more than expenses (spending) we will have a **budget surplus.** Budget deficit prolonged for a long time causes financial problems for government causing governments to take measures for "budget restructuring" like cutting spending and borrowing funds from internal or external markets, which negatively impacts population welfare. Inversely, budget surplus lead governments to expand their social spending programs, projects and investment areas increasing population welfare. Economists use term "**deficit spending"** to explain why states increase budget spending (even if it leads to budget deficit) in recession and crisis. Cutting spending in recession will lead to more contraction of economy, which will lead into deeper recession. Instead, economists understood that increasing budget spending "smooth" down the recession and actually leads to quick recovery. Deficit spending in crisis stimulates the economy by increasing aggregate demand. Budget deficit created in crisis and recession times will quickly be closed during expansion of economy later on.



State budget size shows the role of the state in an economy. The bigger the budget, the bigger the role of the government. At the same time, budget means "power to intervene" in to the market. The bigger the budget, bigger the market power of the states. The smaller the budget, smaller the market power of the states. **We can conclude that budget is the "airbag" of an economy, but at the same time it is an "instrument of market intervention" for governments. Big budget will show big influence of states to economy.**

Table of top 50 countries according to budget size
(In millions of USD)

№	Countries	Revenue	Expenditure
1	United States	6,028,001	6,807,161
2	China	3,312,308	3,787,245
3	Japan	1,678,000	1,888,000
4	Germany	1,598,000	1,573,000
5	France	1,334,000	1,412,000
6	United Kingdom	1,077,300	1,120,000
7	Italy	884,500	927,800
8	Canada	623,700	657,400
9	Brazil	618,853	779,532
10	India	544,422	725,052
11	Spain	492,400	535,900
12	Australia	461,000	484,900
13	Korea, South	351,600	338,000
14	Netherlands	344,800	340,200
15	Saudi Arabia	260,000	294,933
16	Russia	253,900	287,500
17	Sweden	274,800	269,900
18	Belgium	249,700	260,000
19	Mexico	292,800	314,900
20	Switzerland	223,500	222,100
21	Norway	214,300	198,000
22	Austria	197,800	201,900
23	Turkey	173,900	190,400
24	Denmark	173,500	175,500
25	Finland	136,800	140,700
26	Indonesia	130,600	154,800
27	Argentina	123,300	161,100
28	Greece	95,360	98,080
29	Taiwan	93,000	91,670
30	Portugal	92,990	96,830
31	Israel	92,820	102,100
32	South Africa	92,380	103,300
33	Poland	90,800	102,200
34	Ireland	85,410	87,220
35	Colombia	85,140	95,280
36	Czech Republic	83,620	83,930
37	United Arab Emirates	83,440	112,400
38	Thailand	82,000	107,000
39	Venezuela	77,890	160,000
40	Iran	77,220	86,260
41	New Zealand	73,200	71,900
42	Hong Kong	66,190	62,860
43	Iraq	63,970	76,350
44	Hungary	63,630	66,960

45	Peru	59,660	65,480
46	Romania	58,500	65,540
47	Chile	56,730	64,890
48	Singapore	53,400	56,490
49	Kuwait	52,870	61,390
50	Cuba	52,360	60,570

Exercise 37: *Top 5 countries with biggest budget deficit*

USA, China, Japan, Brazil, India.

Exercise 38*: Top 5 countries with biggest budget surplus*

South Korea, Netherlands, Sweden, Norway, Hong Kong

Exercise 39: *Top 5 countries with biggest* $\frac{Budget\ spending}{GDP}$ *ratio*

United States ratio: $\frac{6807161}{21427700} = 0.32$

China: $\frac{3787245}{14342903} = 0.26$

Japan: $\frac{1888000}{5081770} = 0.37$

Germany: $\frac{1573000}{3845630} = 0.41$

France: $\frac{1412000}{2715518} = 0.52$

Exercise 40: *Top 5 countries with smallest* $\frac{Budget\ spending}{GDP}$ *ratio*

Indonesia 0.14, Singapore 0.15, Taiwan 0.17, Russia 0.17, Hong Kong 0.17

Exercise 41*: Top 5 countries with balanced budget (minimum difference)*

Switzerland 1.400, Greece -2.720, Taiwan 1.330, Czech Republic -310, New Zealand 1,300.

Exercise 42: *Make analysis of numbers above and tell me, what is average* $\frac{Budget\ spending}{GDP}$ *ratio?*

Average budget spending ratio of top 5 countries is: (USA, Japan, China, Germany, and France:

$\frac{0.32+0.26+0.37+0.41+0.52}{5} = 0.376$

Social and political indicators

Above tables were macro numbers (related to macroeconomics) now, let's continue with micro numbers (numbers related to microeconomics). Economic growth and development occurs in those countries where opportunities are given and environment is eligible for that. How would you develop and grow in the country where civil unrest is present, bureaucracy is over the roof, corruption is high, monopoly is allowed, competition is hampered, discrimination exists, transparency is non-existent, middle class is vanished or entrepreneurship is not allowed at all? States are obliged to provide all required environment for growth and development if they are really planning to grow and develop. In order to grow and develop these things are "MUST BE":

i. **Peace**
ii. **Laws are respected and implemented**
iii. **Human rights are respected and protected**
iv. **Less bureaucracy (zero bureaucracy)**
v. **Corruption is illegal (fully implemented)**
vi. **Monopoly is illegal (fully implemented)**
vii. **Competition is encouraged and supported**
viii. **Discrimination is illegal (fully implemented)**
ix. **Government is transparent**
x. **Majority of population belongs to middle class**
xi. **Entrepreneurship is encouraged and supported**
xii. **Investment is encouraged, supported and protected**
xiii. **Financial freedom is protected**
xiv. **Property rights are protected**
xv. **Economic freedom is protected**
xvi. **Trade freedom is protected**
xvii. **Efficient fiscal policy**
xviii. **Efficient monetary policy**

Without having these principles at work and trying to grow is as silly as trying to fly without wings, trying to breath underwater with lungs of mammals, trying to run with tied up legs, trying to speak without tongue, or trying to do a heart surgery without having any knowledge about anatomy. It is not only silly; it is actually impossible! Before digging seeds to soil one must make sure soil is good (required quality), otherwise no matter how good you look after seeds, water them, chemicals and other stuff, your harvest is going to be poor.

There are many indicators showing how country is developed socially and politically. I want to mention here the most important ones for economists.

Rule of Law Index

I will start from the indicator that shows the political development level of a country: **Rule of Law Index**. Simply, **Rule of Law Index represents how law is respected and complied among all citizens regardless of their status and evaluate equality of law for all.** In society where "**nobody is above the law**" opportunity for growth and development is high. Just having law is not enough; adherence to the rule of law must be of highest level. People feel confident when they know that their rights will not be chewed, discriminated and even when happens something they can easily go and fight for their rights in courts. Confidence from future leads to investment, and investment leads to growth and development. Higher the index shows the strong compliance of the rule of law, the smaller the index represents weak compliance. World Justice Project, international organization based in USA is initiator (creator) of this index. Index calculation is done through using surveys of experts and companies from all around the world.

Table of top 50 countries by Rule of law Index

1	Denmark	0.9
2	Norway	0.89
3	Finland	0.87
4	Sweden	0.86
5	Netherlands	0.84
6	Germany	0.84
7	New Zealand	0.83
8	Austria	0.82
9	Canada	0.81
10	Estonia	0.81
11	Australia	0.8
12	Singapore	0.79
13	United Kingdom	0.79
14	Belgium	0.79
15	Japan	0.78

16	Hong Kong SAR, China	0.76
17	Republic of Korea	0.73
18	Czech Republic	0.73
19	Spain	0.73
20	France	0.73
21	United States	0.72
22	Uruguay	0.71
23	Portugal	0.7
24	Slovenia	0.69
25	Costa Rica	0.68
26	Chile	0.67
27	Italy	0.66
28	Poland	0.66
29	Barbados	0.65
30	United Arab Emirates	0.65
31	St. Vincent and the Grenadines	0.64
32	Romania	0.63
33	St. Kitts and Nevis	0.63
34	Antigua and Barbuda	0.63
35	Namibia	0.63
36	St. Lucia	0.62
37	Rwanda	0.62
38	Mauritius	0.61
39	Croatia	0.61
40	Greece	0.61
41	The Bahamas	0.61
42	Georgia	0.6
43	Botswana	0.6
44	Grenada	0.59
45	South Africa	0.59
46	Dominica	0.58
47	Malaysia	0.58
48	Argentina	0.58
49	Jamaica	0.57
50	Jordan	0.57

***Exercise 43**: How many nations from Asia do you see on above list?*

Republic of Korea, Japan, China, Malaysia, Singapore

***Exercise 44:** How many nations from Middle East do you see on above list?*

United Arab Emirates and Jordan.

***Exercise 45**: How many nations from Africa do you see on above list?*

Botswana, Rwanda, Mauritius, Namibia, South Africa.

***Exercise 46**: How many nations from Americas do you see on above list?*

Canada, USA, Argentina, Chile, Dominica, Uruguay, Costa Rica, Jamaica, St, Lucia, St. Vincent and the Grenadines, St. Kitts and Nevis.

***Exercise 47**: How many nations from Eastern Europe do you see on above list?*

Romania, Slovenia, Czech Republic

***Exercise 48:** How many nations from Baltic States do you see on above list?*

Estonia.

Human Development Index (HDI)

Second indicator is important social indicator: **Human Development Index.** Index was first developed by Indian economist Amartya Sen in second half of twentieth century. The index measures how developed society is in a given country (as name mentions it). It contains three components: **Life Expectancy Index (LEI), Education Index (EI)** and **Income Index (II)**. Let's learn each of them separately.

Life expectancy index is calculated by simply taking life expectancy of the population of that country (in years) and subtracting 20 (years), which represents minimum life of population in our times, and dividing it to maximum life expectancy 85 (years), subtracted to minimum life time 20 (years). Long life always represented happy and healthy life in every culture. Only healthy, wealthy and happy people can live long life. Stressed, hungry and sick people have less chances to live long life. This is the main reason behind taking life expectancy as a variable in measuring human development. In a country where on average people live longer, signals that people there are healthy, wealthy and happy.

$$Life\ expectancy\ index\ (LEI) = \frac{Life\ expectancy\ (LE)}{85 - 20}$$

$$\boldsymbol{LEI = \frac{LE - 20}{85 - 20}}$$

Exercise 49: *Assume average life expectancy of population of Norway is 80, then calculate please life expectancy index of Norway.*

$$LEI = \frac{LE - 20}{85 - 20}$$

$$LEI = \frac{80 - 20}{85 - 20} = 0.92$$

Life expectancy index of Norway is 0.92

Education Index (EI) is found by taking average of two indexes: **Mean Years of Schooling Index (MYSI)** and **Expected Years of Schooling Index (EYSI).** Education is a luxury good! Not everybody has a chance to get education. Especially secondary and tertiary education is time and money consuming. That is why in developing and underdeveloped countries you will witness very low levels of secondary and tertiary education levels. Education level shows how person is skilled, so how would you expect from low education level country to produce high-tech goods? Only highly educated person can produce pharmaceutical goods, technological goods, chemical goods, planes, rockets, software programs, etc. Without investing into education and increasing productivity of your labor force countries will never be able to achieve high levels of development.

$$\boldsymbol{Education\ Index\ (EI) = \frac{MYSI + EYSI}{2}}$$

Mean Years of Schooling Index is found by dividing Mean Years of Schooling of given country to **15** (years that a person aged 25 or older has spent in formal education).

$$\boldsymbol{Mean\ Years\ of\ Schooling\ Index(MYSI) = \frac{Mean\ Years\ of\ Schooling\ (MYS)}{15}}$$

$$\boldsymbol{MYSI = \frac{MYS}{15}}$$

Exercise 50*: Mean Years of Schooling in Norway is 14 years. Find Mean Years of Schooling Index of Norway.*

$$\boldsymbol{Mean\ Years\ of\ Schooling\ Index(MYSI) = \frac{Mean\ Years\ of\ Schooling\ (MYS)}{15}}$$

$$\boldsymbol{Mean\ Years\ of\ Schooling\ Index(MYSI) = \frac{14}{15} = 0.93}$$

Mean Years of Schooling Index of Norway is 0.93

Expected Years of Schooling Index (EYSI) is found simply dividing Expected Years of Schooling of a given country to **18** (total expected years of schooling for children under 18 years of age).

$$\boldsymbol{Expected\ Years\ of\ Schooling\ Index(EYSI) = \frac{Expected\ Years\ of\ Schooling}{18}}$$

$$\boldsymbol{EYSI = \frac{EYS}{18}}$$

Exercise 51: *Expected Years of Schooling Index of Norway.*

$$\boldsymbol{Expected\ Years\ of\ Schooling\ Index(EYSI) = \frac{Expected\ Years\ of\ Schooling}{18}}$$

$$\boldsymbol{EYSI = \frac{15}{18} = 0.83}$$

Expected Years of Schooling Index of Norway is 0.83

Exercise 52: *What is the Education Index of Norway?*

$$\boldsymbol{Education\ Index(EI) = \frac{MYSI + EYSI}{2}}$$

$$\boldsymbol{EI = \frac{0.93 + 0.83}{2} = 0.88}$$

Education Index of Norway is 0.88

Income Index (II) is found simply taking national logarithm of Gross National Income per capita (same as GDP per capita) subtracting from it natural logarithm of 100 (minimum income is considered 100 USD per capita) and dividing all this to natural logarithm of 75000 (75000 USD is considered maximum income per capita) subtracted from it natural logarithm of 100 (minimum income is considered 100 USD per capita). Can hungry person work productive enough? I do not think so. Personally, I work really well till I start feeling hunger. It is one of the most vital conditions for any human being to be productive member of society: you must have food, you must have shelter and you must have safety! If these things are non-existent you can forget about productivity. Without meeting my basic needs, you can never make me work. To meet all those basic needs, I must have an income. No money, no honey! The

higher income I have, the more chances that my basic needs are met. This means now I can focus on other things, like work, art, education, research and development. Human beings start to develop, create and grow only after its basic needs are met. Otherwise the only thing in his mind will be: food, home, and safety. Only people with enough income go to museums, academies, conferences, etc. where they get their inspiration for further development and growth.

$$\boldsymbol{Income\ Index\ (II) = \frac{\ln(GNI\ per\ capita) - \ln(100)}{\ln(75000) - \ln(100)}}$$

Exercise 53: *Find Income Index of Norway if Gross National Income per capita of Norway is 65000 USD.*

$$\boldsymbol{Income\ Index\ (II) = \frac{\ln(GNI\ per\ capita) - \ln(100)}{\ln(75000) - \ln(100)}}$$

$$\boldsymbol{Income\ Index\ (II) = \frac{\ln(65000) - \ln(100)}{\ln(75000) - \ln(100)} = 0.98}$$

Income Index of Norway is 0.98

Human Development Index (HDI) of any country is found by:

$$\boldsymbol{HDI = \sqrt[3]{(LEI) * (EI) * (II)}}$$

Exercise 54: *What is HDI of Norway?*

$$\boldsymbol{HDI = \sqrt[3]{(LEI) * (EI) * (II)}}$$

$$\boldsymbol{HDI = \sqrt[3]{(0.92) * (0.88) * (0.98)} = 0.92}$$

HDI of Norway is 0.92

Human development index top 50 countries

1	Norway	0.954
2	Switzerland	0.946
3	Ireland	0.942
4	Germany	0.939
4	Hong Kong	0.939
6	Australia	0.938
6	Iceland	0.938
8	Sweden	0.937
9	Singapore	0.935
10	Netherlands	0.933

11	Denmark	0.93
12	Finland	0.925
13	Canada	0.922
14	New Zealand	0.921
15	United Kingdom	0.92
15	United States	0.92
17	Belgium	0.919
18	Liechtenstein	0.917
19	Japan	0.915
20	Austria	0.914
21	Luxembourg	0.909
22	Israel	0.906
22	South Korea	0.906
24	Slovenia	0.902
25	Spain	0.893
26	Czech Republic	0.891
26	France	0.891
28	Malta	0.885
29	Italy	0.883
30	Estonia	0.882
31	Cyprus	0.873
32	Greece	0.872
32	Poland	0.872
34	Lithuania	0.869
35	United Arab Emirates	0.866
36	Andorra	0.857
36	Saudi Arabia	0.857
36	Slovakia	0.857
39	Latvia	0.854
40	Portugal	0.85
41	Qatar	0.848
42	Chile	0.847
43	Brunei	0.845
43	Hungary	0.845
45	Bahrain	0.838
46	Croatia	0.837
47	Oman	0.834
48	Argentina	0.83
49	Russia	0.824
50	Belarus	0.817

Top 50 countries according to life expectancy (in years)

№	Countries	Overall
1	Hong Kong	84.7
2	Japan	84.5
3	Singapore	83.8

4	Italy	83.6
5	Spain	83.4
6	Switzerland	83.4
7	Australia	83.3
8	Iceland	82.9
9	Israel	82.8
10	South Korea	82.8
11	Sweden	82.7
12	France	82.5
13	Malta	82.4
14	Canada	82.3
14*	Norway	82.3
16	Greece	82.1
16	Ireland	82.1
16	Luxembourg	82.1
16	Netherlands	82.1
16	New Zealand	82.1
21	Portugal	81.9
22	Andorra	81.8
23	Finland	81.7
25	Belgium	81.5
26	Austria	81.4
27	Germany	81.2
28	Slovenia	81.2
29	United Kingdom	81.2
30	Cyprus	80.8
31	Denmark	80.8
32	Liechtenstein	80.5
33	Costa Rica	80.1
34	Chile	80
35	Czech Republic	79.2
36	Barbados	79.1
37	Lebanon	78.9
38	United States	78.9
39	Cuba	78.6
40	Estonia	78.6
41	Poland	78.5
42	Croatia	78.3
43	Panama	78.3
44	Turkey	78.3
45	United Arab Emirates	77.8
46	Uruguay	77.8
47	Oman	77.6
48	Slovakia	77.4
49	Bosnia and Herzegovina	77.3
50	Bahrain	77.2

Exercise 55: *Calculate HDI of New Zealand if you know that Education Index of New Zealand is 0.8 (use tables given in this chapter for finding indexes, use GDP per capita for Gross National Income (GNI) per capita, it is the same thing)*

EI=0.8

Income Index (II) $= \frac{\ln(43953)-\ln(100)}{\ln(75000)-\ln(100)} = 0.92$

Life Expectance Index (LEI) = $\frac{82.1-20}{85-20} = 0.95$

HDI= $\sqrt[3]{0.95 * 0.8 * 0.92} = 0.89$

Exercise 56*: Find Education Index of United Kingdom using above tables*

Income Index $= \frac{ln(48710)-ln(100)}{ln(75000)-ln(100)} = 0.93$

Life Expectancy Index (LEI) $= \frac{81.2-20}{85-20} = 0.94$

HDI=0.92

0.92= $\sqrt[3]{0.94 * 0.93 * EI}$

$(0.92)^3 = (\sqrt[3]{0.94 * 0.93 * EI})^3$

0.779=0.94*0.93*EI

$\frac{0.779}{0.8742}$ = EI

EI=0.89

Exercise 57: *Using above given tables find out which component of HDI (LEI, EI, or II) of Israel is the lowest.*

Income Index (II) = $\frac{\ln(42194)-\ln(100)}{\ln(75000)-\ln(100)}$ = 0.91

Life Expectance Index (LEI) = $\frac{82.8-20}{85-20}$ = 0.97

HDI=0.906

0.906 = $\sqrt[3]{0.97 * 0.91 * EI}$

$0.906^3 = (\sqrt[3]{0.97 * 0.91 * EI})^3$

$0.7437=0.97*0.91*EI$

$\frac{0.743677}{0.8827}$ = EI

EI=0.8425 II=0.91 LEI=0.97

***Exercise 58**: Using above given tables find out which component of HDI (LEI, EI, or II) of Canada is the lowest*

Income Index (II) = $\frac{\ln(51342)-\ln(100)}{\ln(75000)-\ln(100)}$ = 0.94

Life Expectancy Index (LEI) = $\frac{82.3-20}{85-20}$ = 0.96

HDI=0.922

$0.922 = \sqrt[3]{0.94 * 0.96 * EI}$

$0.922^3 = (\sqrt[3]{0.94 * 0.96 * EI})^3$

$0.7838=0.94*0.96*EI$

$\frac{0.7838}{0.9024}$ = EI

EI=0.87 LEI=0.96 II=0.94

Index of Economic Freedom (IEF)

Next very important index for any economist is **Index of Economic Freedom** (**IEF**), which measures how freely citizens of the given nation can pursue their own economic interests. The degree of trade, economic, financial and competition freedom will impact economy overall. The more stringent and restrictive the policies are, the lesser the growth and development. Higher degree of freedom of action gives people more opportunity to expand and grow. The index was initiated by The Heritage Foundation, American think-tank group.

Table of top 50 countries by Index of economic freedom

1	Singapore	89.4
2	Hong Kong	89.1
3	New Zealand	84.1
4	Australia	82.6

5	Switzerland	82
6	Ireland	80.9
7	United Kingdom	79.3
8	Denmark	78.3
9	Canada	78.2
10	Estonia	77.7
11	Taiwan	77.1
12	Georgia	77.1
13	Iceland	77.1
14	Netherlands	77
15	Chile	76.8
16	Lithuania	76.7
17	United States	76.6
18	United Arab Emirates	76.2
19	Luxembourg	75.8
20	Finland	75.7
21	Mauritius	74.9
22	Sweden	74.9
23	Czech Republic	74.8
24	Malaysia	74.7
25	South Korea	74
26	Israel	74
27	Germany	73.5
28	Norway	73.4
29	Austria	73.3
30	Japan	73.3
31	Qatar	72.3
32	Latvia	71.9
33	Rwanda	70.9
34	Armenia	70.6
35	Macau	70.3
36	Bulgaria	70.2
37	Cyprus	70.1
38	Romania	69.7
39	Kazakhstan	69.6
40	Botswana	69.6
41	North Macedonia	69.5
42	Malta	69.5
43	Thailand	69.4
44	Azerbaijan	69.3
45	Colombia	69.2
46	Poland	69.1
47	Uruguay	69.1
48	Belgium	68.9
49	Jamaica	68.5
50	Saint Lucia	68.2

Exercise 59: *If you were to do a business in African continent which countries would you have chosen? (Use above list)*

Rwanda and Mauritius.

Exercise 60*: Would you choose Baltic States for a business? Do all Baltic States are on the list? List all Baltic States.*

Yes, because of ease of doing business there. There are only 3 Baltic States: Latvia, Estonia, Lithuania, and all of them are on the list.

Exercise 61: *Would you choose Caucasus region countries for a business? Do all Caucasus region countries are on the list? List all Caucasus region countries.*

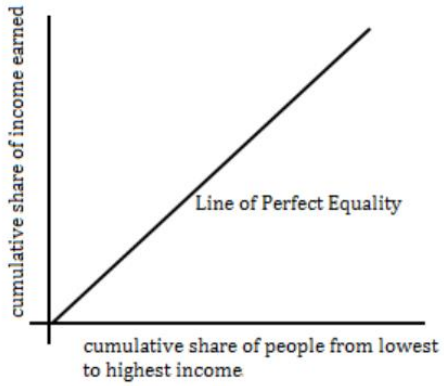
Yes, I would choose all 3 regions, because in all of them it is easy to build up a business. Caucasus regions are Armenia, Azerbaijan, Georgia and all of them are on above list.

Gini Index

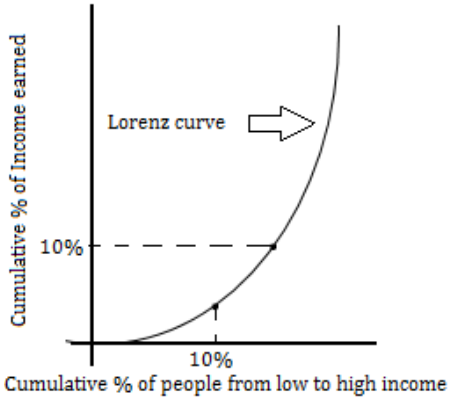
Middle class is the main force driving an economy! The most of the experts, specialists, academics, engineers, generals, artists, etc. come from middle-class. We have learned previously that we need three things to produce an output: labor, technology and capital. If we agree that capital comes from freedom, then we must also agree that labor and technology comes from middle class. Roughly, we can divide population into three categories according to their income: high income class, middle income class and low income class. We know that low income class has low chances of being productive because they have bigger problem; they must meet their basic needs. High income class is the main capital provider of an economy. Middle class is the main working machine, productive machine, power machine of an economy. Development and growth will be smooth and sustainable only if middle class is strong and in majority compared to high income and low income classes. **Weak middle class means weak economy! Gini Index** or **Gini Coefficient** (shortly **Gini**) measures how income is distributed among the population. The index takes its name from Italian sociologist who initiated this index at the beginning of twentieth century, Corrado Gini. Assume a country is a big family and this family earns income, they all work. The distribution of this income will impact wellbeing of every member of this family. The more fairly distributed income will bring peace, happiness and further growth, while unfairly distributed income will lead to all kind of social and economic problems in future. Now, a country is a family,

and family income is a GDP. How GDP is distributed in an economy? Who earns the most profits and who are left with nothing? Do high income class is earning more and more each year and middle income class is shrinking? Low income class is increasing instead of decreasing? If such thing is true, then this country has a problem with income distribution. Pay attention, I did not say "problem with income", I said "problem with income distribution". The cause of the problem might be inefficient fiscal policy (tax system), monetary policy (inflation, discriminatory credit policy, etc.), labor policy (wage controls, wage limits, wage inefficiency, etc.), institutional (bureaucracy, corruption, etc.) or even cultural (Caste system of India, low women's rights, cultural discrimination, aristocracy, etc.) and many other. **Income distribution is faire when the one who worked the hardest earned the most, and the one who worked the less earned the lowest.** Since middle income class is the hardest working class in any economy, the biggest portion of income must go to them!

How Gini Index is calculated? First, how would an economy with perfect income distribution looks like? It will look like this:

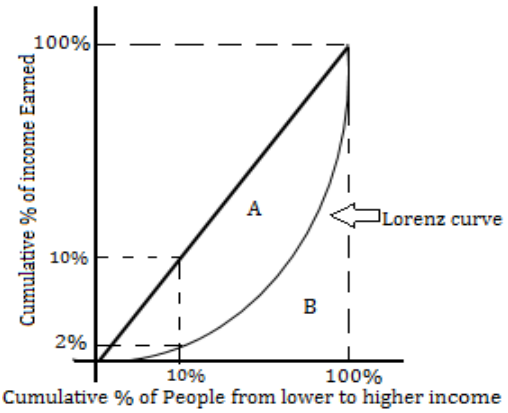


Real world distribution unfortunately does not look like that. Real distribution of wealth calculation is done using **Lorenz curve,** developed by American economist at the beginning of twentieth century, which simply divides total population into 10% groups according to their earnings. Curve is formed by lines drown from the top points of those ten groups.



Gini Index is found by dividing area in between Lorenz curve and Line of Perfect Equality to whole area under Line of Perfect Equality (half of square!).

$$Gini\ Index = \frac{A}{A+B}$$



High index shows high inequality of income distribution in economy. Lower the index, the lower the inequality in income distribution in an economy.

Many specialists make mistake when dealing with income distribution: Wrongly comparing income distribution with wealth distribution. **Income** distribution is distribution of earnings. **Wealth** distribution is distribution of wealth among the population. Thus, income might be distributed fairly, but this does not mean that wealth is distributed fairly. Income distribution and wealth distribution is absolutely different indicators.

Wealth:

A. Land
B. Resources: Gold, oil, natural gas, silver, etc.
C. Capital: Money, bonds, shares, estates, etc.

Income:

A. Salary: Compensation for labor work
B. Interest earnings: From deposits to the bank
C. Dividends: From investments
D. Profits: From doing business

E. Rent: From giving capital and land for usage

Top 130 countries by income inequality Gini coefficient (income distribution in %, Gini Index*100%)

1	South Africa	63
2	Namibia	59.1
3	Suriname	57.6
4	Zambia	57.1
5	Sao Tome and Principe	56.3
6	Central African Republic	56.2
7	Eswatini	54.6
8	Mozambique	54
9	Belize	53.3
10	Botswana	53.3
11	Honduras	52.1
12	Angola	51.3
13	St. Lucia	51.2
14	Guinea-Bissau	50.7
15	Colombia	50.4
16	Panama	49.2
17	Republic of Congo	48.9
18	Guatemala	48.3
19	Costa Rica	48
20	Benin	47.8
21	Cabo Verde	47.2
22	Venezuela	46.9
23	Seychelles	46.8
24	Brazil	46.6
25	Cameroon	46.6
26	South Sudan	46.3
27	Nicaragua	46.2
28	Paraguay	46.2
29	Jamaica	45.5
30	Ecuador	45.4
31	Mexico	45.4
32	Comoros	45.3
33	Lesotho	44.9
34	Malawi	44.7
35	Djibouti	44.6
36	Guyana	44.6
37	Chile	44.4
38	Philippines	44.4
39	Zimbabwe	44.3
40	Dominican Republic	43.7
41	Rwanda	43.7
42	Ghana	43.5

43	Chad	43.3
44	Togo	43.1
45	Nigeria	43
46	Peru	42.8
47	Uganda	42.8
48	Madagascar	42.6
49	Bolivia	42.2
50	DR Congo	42.1
51	Papua New Guinea	41.9
52	Turkey	41.9
53	ôt 'Ivo r	41.5
54	Argentina	41.4
55	United States	41.4
56	Haiti	41.1
57	Malaysia	41
58	Iran	40.8
59	Kenya	40.8
60	Tanzania	40.5
61	Bulgaria	40.4
62	Senegal	40.3
63	Trinidad and Tobago	40.3
64	Micronesia	40.1
65	Sri Lanka	39.8
66	Uruguay	39.7
67	Morocco	39.5
68	Indonesia	39
69	Israel	39
70	Montenegro	39
71	Burundi	38.6
72	El Salvador	38.6
73	China	38.5
74	Gabon	38
75	India	37.8
76	Russia	37.5
77	Bhutan	37.4
78	Lithuania	37.3
79	Mauritius	36.8
80	Fiji	36.7
81	Yemen	36.7
82	Georgia	36.4
83	Laos	36.4
84	Thailand	36.4
85	Serbia	36.2
86	Romania	36
87	The Gambia	35.9
88	Italy	35.9
89	Syria	35.8
90	Sierra Leone	35.7
91	Vietnam	35.7

92	Latvia	35.6
93	Sudan	35.4
94	Burkina Faso	35.3
95	Liberia	35.3
96	Uzbekistan	35.3
97	Ethiopia	35
98	Luxembourg	34.9
99	United Kingdom	34.8
100	Spain	34.7
101	Armenia	34.4
102	Australia	34.4
103	Greece	34.4
104	Niger	34.3
105	North Macedonia	34.2
106	Tajikistan	34
107	Canada	33.8
108	Portugal	33.8
109	Guinea	33.7
110	Jordan	33.7
111	Palestine	33.7
112	Pakistan	33.5
113	Albania	33.2
114	Bosnia and Herzegovina	33
115	Mali	33
116	Japan	32.9
117	Ireland	32.8
118	Nepal	32.8
119	Tunisia	32.8
120	Mongolia	32.7
121	Switzerland	32.7
122	Mauritania	32.6
123	United Arab Emirates	32.5
124	Bangladesh	32.4
125	Germany	31.9
126	Lebanon	31.8
127	France	31.6
128	South Korea	31.6
129	Egypt	31.5
130	Cyprus	31.4

Top 130 countries by inequality of wealth distribution (wealth distribution in %, Gini Index*100%)

1	Ukraine	0.955
2	Kazakhstan	0.952
3	Egypt	0.909
4	Thailand	0.902
5	Nigeria	0.894

6	Lebanon	0.889
7	Russia	0.875
8	Turkey	0.871
9	Sweden	0.865
10	India	0.854
11	United States	0.852
12	Grenada	0.842
13	Dominica	0.84
14	Indonesia	0.84
15	Antigua and Barbuda	0.838
16	Denmark	0.835
17	Ireland	0.83
18	Philippines	0.826
19	Brazil	0.823
20	Haiti	0.82
21	Malaysia	0.82
22	Germany	0.816
23	Belize	0.815
24	Saudi Arabia	0.81
25	Colombia	0.807
26	South Africa	0.806
27	Honduras	0.804
28	Morocco	0.802
29	Yemen	0.801
30	Mexico	0.8
31	Lesotho	0.795
32	Panama	0.795
33	Peru	0.795
34	Argentina	0.792
35	Norway	0.791
36	Barbados	0.788
37	Jamaica	0.788
38	Latvia	0.788
39	Zambia	0.787
40	Paraguay	0.785
41	Botswana	0.783
42	Equatorial Guinea	0.781
43	Bolivia	0.779
44	Nicaragua	0.778
45	Ecuador	0.776
46	Namibia	0.776
47	Chile	0.773
48	Costa Rica	0.769
49	Central African Republic	0.768
50	Finland	0.767
51	Comoros	0.766
52	Israel	0.766
53	Austria	0.764
54	Papua New Guinea	0.76

55	El Salvador	0.759
56	Algeria	0.758
57	Czech Republic	0.758
58	Singapore	0.758
59	Gambia	0.755
60	Guyana	0.75
61	Trinidad and Tobago	0.748
62	United Kingdom	0.747
63	Switzerland	0.741
64	Uruguay	0.741
65	Netherlands	0.736
66	Portugal	0.736
67	Malawi	0.733
68	Kenya	0.732
69	Iceland	0.731
70	Romania	0.728
71	Rwanda	0.728
72	Canada	0.726
73	Cameroon	0.725
74	Poland	0.722
75	Togo	0.719
76	Guinea	0.716
77	Chad	0.715
78	Estonia	0.715
79	China	0.714
80	Uganda	0.714
81	New Zealand	0.708
82	Vietnam	0.708
83	Zimbabwe	0.707
84	Iran	0.705
85	Senegal	0.705
86	Cambodia	0.704
87	Madagascar	0.702
88	Gabon	0.7
89	Mozambique	0.7
90	Guinea-Bissau	0.697
91	Spain	0.697
92	Fiji	0.694
93	Benin	0.689
94	Italy	0.689
95	France	0.687
96	Sri Lanka	0.687
97	Tunisia	0.683
98	Ghana	0.682
99	Greece	0.682
100	Mali	0.682
101	Seychelles	0.679
102	Georgia	0.678
103	Jordan	0.677

104	Burkina Faso	0.674
105	Kyrgyzstan	0.673
106	Bangladesh	0.671
107	Sierra Leone	0.671
108	South Korea	0.67
109	Mauritania	0.667
110	Luxembourg	0.663
111	Hungary	0.662
112	Niger	0.66
113	Belgium	0.659
114	Australia	0.658
115	Lithuania	0.655
116	North Macedonia	0.655
117	Burundi	0.654
118	Pakistan	0.65
119	Tanzania	0.65
120	Bulgaria	0.647
121	Slovenia	0.646
122	Armenia	0.645
123	Azerbaijan	0.643
124	Mauritius	0.64
125	Croatia	0.631
126	Japan	0.631
127	Malta	0.631
128	Albania	0.629
129	Belarus	0.614
130	Ethiopia	0.612

***Exercise 62**: Scandinavian countries are known with their low Gini Indexes (income distribution). How come all of them are on the top of the worst "wealth distributed" countries list?*

Because Wealth and income are absolutely different things, which means that income is evenly distributed in Scandinavian countries, but majority of wealth belong to only small amount of people.

***Exercise 63:** How come that none of the European and North American countries are on the top of the worst "income distributed" countries list (Gini Index Income distribution) while on the top of the worst "wealth distributed" countries list there are many of them?*

Because wealth and income are absolutely different things (as it was said before), which means that income is evenly distributed in European and North American countries, but majority of wealth belong to only small amount of people.

***Example 64**: How many countries are on both top 50 list (income distribution and wealth distribution)?*

Mexico, Panama, Jamaica, Nigeria.

Let's for the sake of learning and analysis create a new index from two Gini Indexes (income distribution and wealth distribution) which will measure **absolute inequality in economy**. We will take an average of two indexes and call it a **Gini Y Index (**Y is coming from Yhlas, my name**):**

$$\boldsymbol{Gini\ Y\ Index\ (GYI)} = \frac{\boldsymbol{GI(income\ distribution)+GI(wealth\ distribution)}}{\mathbf{2}}$$

The higher the **Gini Y Index**, the bigger the **absolute inequality** in economy!

***Exercise 65**: Find top 10 countries with the worst **Gini Y Indexes** (use above Gini Index tables)*

1. *South Africa 71.8*
2. *Namibia 68.35*
3. *Zambia 67.9*
4. *Belize 67.4*
5. *Central African Republic 66.5*
6. *Honduras 66.25*
7. *Nigeria 66.2*
8. *Botswana 65.8*
9. *Colombia 65.55*
10. *Brazil 64.45*

***Exercise 66:** Find top 10 European countries with worst **Gini Y Indexes**.*

1. *Russia 62. 5*
2. *Ireland 57.9*
3. *Latvia 57.2*
4. *United Kingdom 54,7*
5. *Romania 54.4*
6. *Portugal 53,7*
7. *Switzerland 53.4*
8. *Bulgaria 52.55*
9. *Italy 52.4*
10. *Spain 52.2*

Exercise 67: *Find top 10 Asian countries with worst* ***Gini Y Indexes****.*

1. *Turkey 64.5*
2. *Philippines 63.5*
3. *Thailand 63.3*
4. *India 61.6*
5. *Malaysia 61.5*
6. *Indonesia 61*
7. *Lebanon 60.35*
8. *New Guinea 58.95*
9. *Yemen 58.4*
10. *Israel 57.8*

Exercise 68*: Find top 10 North and South American countries with worst* ***Gini Y Indexes****.*

1. *Belize 67.4*
2. *Honduras 66.25*
3. *Colombia 65.55*
4. *Brazil 64.45*
5. *Panama 64.35*
6. *Dominica 63.85*
7. *United States of America 63.3*
8. *Mexico 62.7*
9. *Costa Rica 62.45*
10. *Paraguay 62.35*

Exercise 69*: Find top 10 African countries with worst* ***Gini Y Indexes***

1. *South Africa 71.8*
2. *Namibia 68.35*
3. *Zambia 67.9*
4. *Central African Republic 66.5*
5. *Nigeria 66.2*
6. *Botswana 65.8*
7. *Lesotho 62.2*
8. *Mozambique 62*
9. *Comoros 60.95*
10. *Guinea Bissau 60.2*

Corruption Perception Index (CPI)

Every society has its own moral and ethical value which is derived from their historical, religious, cultural and natural background. The same behavior which is considered "bad" in one country can be very "normal" in another one. While certain actions are considered a "crime" in one nation, the same action might be in the list of "normal" actions in another one. **We are different, nations are different, and this diversity makes our world beautiful**! How boring our world would have been if there were only one religion, one code of ethics, one law, one philosophy, one politics, one history, one color, etc. this would have been unlivable world! But, there are certain "sins" that is considered as a "sin" everywhere, like stealing, killing, lying, forcing, and bribing. Yes, bribery and corruption is considered as a "bad behavior" in all cultures and criminally punished in all countries. What about perception of corruption? We all know that for example lying is a "bad behavior" but how we perceive a lie in terms of giving criteria to this "bad behavior"? Some people will consider lying as a "bad behavior", some consider it to be "very bad" some radicals will say "extremely bad". There will be those who would say lying is actually "not that bad behavior". **What I want to say is that we, the people, perceive actions according to our moral and ethical values.** A child grown up in academicals family will have different ethical values than child who grow up in favelas. How we perceive any action depends on our inner moral codes, ethics and discipline which is derived from environment we are surrounded. High injustice, crime rate, unemployment, inflation, conflicts, drug abuse, alcoholism, etc. increases our distrust to environment and lowers down expectations from future and this in return shatters our inner moral and code of ethics. Our code of ethics fades away. We start perceiving even worst crimes as a "normal behavior". As our environment changes, we change.

Corruption as a crime existed, exists and will exist in all nations as long as we have hierarchy! Hierarchy persists everywhere in human beings‟ life: Family, Work, Business, Government, Army, etc. because human beings love order. Actually, all animals also love order that is why hierarchy persists in their lives too. Hierarchy requires bosses and subjects. The boss holds on authority for executive power. Using that executive power for your self-interest rather than public (common) interest is a corruption. **Corruption Perception Index (CPI)** measures how people perceive corruption: Good, Bad or Normal? When people perceive it as a "bad" behavior, it shows the high ethical standard and strong moral code of those people. We can say that, nations that scored high in CPI have high ethical standards and those who scored low has low ethical standards overall.

Top 50 countries by their Corruption Perception Index scores

1	New Zealand	87

1	Denmark	87
3	Finland	86
4	Sweden	85
4	Switzerland	85
4	Singapore	85
7	Norway	84
8	Netherlands	82
9	Luxembourg	80
9	Germany	80
11	Iceland	78
12	Canada	77
12	United Kingdom	77
12	Australia	77
12	Austria	77
16	Hong Kong	76
17	Belgium	75
18	Ireland	74
18	Estonia	74
20	Japan	73
21	Uruguay	71
21	United Arab Emirates	71
23	France	69
23	United States	69
25	Bhutan	68
26	Chile	67
28	Taiwan	65
29	Bahamas	64
30	Spain	62
30	Barbados	62
30	Portugal	62
30	Qatar	62
34	Botswana	61
35	Brunei	60
35	Israel	60
35	Slovenia	60
35	Lithuania	60
39	Saint Vincent and the Grenadines	59
39	South Korea	59
41	Poland	58
41	Cyprus	58
41	Cape Verde	58
44	Czech Republic	56
44	Georgia	56
44	Latvia	56
44	Costa Rica	56
48	Dominica	55
48	Saint Lucia	55

50	Malta	54

***Exercise 70**: How many countries from African continent are on Top 50 list above?*

Botswana.

***Exercise 71**: How many countries from Americas are on Top 50 list above?*
United States, Canada, Chile, Costa Rica, Dominica, Saint Vincent and the Grenadines, Uruguay, Saint Lucia.

***Exercise 72**: How many countries from Asia are on Top 50 list above?*

Japan, South Korea, Singapore, Brunei, Bhutan, Qatar.

***Example 73**: Scandinavian countries are on top 10 of Top 50 list above, what do you say about that?*

It is a proof that Scandinavian countries have high ethical standards.

Ease of Doing Business (EDB) rating

Bureaucracy is the biggest enemy of progress, growth and development! Related and unrelated documents, hours of standing in endless lines among tired and rightly angered people, meticulous pretentions by supervisor, days of waiting for final answer which can easily turn to weeks and months, all these are just for one tiny approval! **People lose time, energy, health, employment opportunity and trust! Economy loses tax revenues** (Businessman would have started paying taxes earlier if business permit was given early! Employees would have started paying income taxes!), **loses decreasing unemployment numbers** (Without permit nobody is allowed to work, thus unemployment is not decreasing!), and **loses entrepreneurship soul** (Population loses interest in doing anything!). Even after getting a permit, entrepreneur cannot get the grip of bureaucrat out of his neck, because endless audits, revisions, controls, inefficient rules, conditions, penalties, fines, heavy tax burdens, and fees just does not let him focus on entrepreneurship. The lesser the bureaucracy, the faster the growth and development! **Ease of Doing Business** rating exactly measures that, the level of

bureaucracy in economy. High rating means less bureaucracy. Low rating means high bureaucracy.

Top 50 countries by Ease of Doing Business rating

1	New Zealand
2	Singapore
3	Hong Kong
4	Denmark
5	South Korea
6	United States
7	Georgia
8	United Kingdom
9	Norway
10	Sweden
11	Lithuania
12	Malaysia
13	Mauritius
14	Australia
15	Taiwan
16	United Arab Emirates
17	North Macedonia
18	Estonia
19	Latvia
20	Finland
21	Thailand
22	Germany
23	Canada
24	Ireland
25	Kazakhstan
26	Iceland
27	Austria
28	Russia
29	Japan
30	Spain
31	China
32	France
33	Turkey
34	Azerbaijan
35	Israel
36	Switzerland
37	Slovenia
38	Rwanda
39	Portugal
40	Poland
41	Czech Republic
42	Netherlands

43	Bahrain
44	Serbia
45	Slovakia
46	Belgium
47	Armenia
48	Moldova
49	Belarus
50	Montenegro

Exercise 74: *Caucasus region nations (Georgia, Armenia and Azerbaijan) are on Top 50 list, especially Georgia is among top 10, and did this surprise you? Make a research about those nations, especially Georgia, from open sources and find out how they achieved such a marvelous result.*

Georgia started economic and political reforms in mid 2000s which boosted their government transparency and economic efficiency. Same is applicable to Armenia and Azerbaijan.

Exercise 75: *Japan scored lower than expected, find out why is that?*

Japan has very high standards of social safety, such as ecological standards and recycling regulations. This creates a bit problem for new businesses to adapt.

Homework:

I. Economy of Turtle Island has only three sectors: fishing, coconut harvesting and tourism. Last year 90 tons of fish was caught and sold (3 TMT/KG), 200 tons of coconut was harvested and sold (9 TMT/KG), and total of 50000 tourists visited the island (each tourist spent on average 5000 TMT on island). What is Turtle Island's last year's GDP?

II. This year, Turtle Island produced everything at the same amount (even quantity of tourists is same) as last year. Prices

changed a bit: fish 4 TMT/KG, coconut 9.5 TMT and tourists spent 6000 TMT. What is Turtle Island's this year's GDP?

III. Third year in a row production in Turtle Island remains unchanged. Same quantity of fish, coconuts and tourists. The only thing changing is prices: fish 2 TMT, coconut 8 TMT and tourists spent 4000 TMT. What is third year's GDP of Turtle Island?

IV. Entrepreneur moved to Turtle Island from Mary, Turkmenistan built giant seafood restaurant and buys all fish that is caught in island and sells them to costumers for 1 KG fish=5 TMT. What will be the GDP of Turtle Island? (Assume that all production, prices and tourist quantities as in question I).

V. One more entrepreneur moved from Ashgabat, Turkmenistan to Turtle Island opened up a coconut oil factory and buys all coconuts and makes oil from it and sells for (1 LITRE=40 TMT, to make 1 LITRE coconut oil 2 KG of coconuts needed). What will be the GDP of Turtle Island? (Assume that all production, prices and tourist quantities as in question I, seafood restaurant also working for the same price $\rightarrow$ 5 TMT).

VI. Calculate GDP per capita of Turtle Island if on 10000 people live there.

VII. Technological upgrade increased production of fishermen by 50% and coconut growers harvest by 80%. If we assume prices remains the same as in question V, how GDP of the island will change?

VIII. What will happen to new GDP per capita after economic boom?

IX. Tsunami destroys fishing industry by 70% and coconut trees by 80%. If we assume prices and quantities of production remains the same as in question V, what will be the impact of Tsunami to GDP of island?

X. What will happen to GDP per capita after catastrophe?

XI. Calculated HDI of Turtle Island assuming all numbers be the same as in question 5 and Education Index=0.7. On average people die at age around 72 on the island.

XII. Calculate unemployment rate of Turtle Island if total labor force is 6000 and 230 people are looking for job.

XIII. How will unemployment rate change if coconut oil factory shuts down (1000 people worked there)?

XIV. There are two towns in Turtle Island: Town1 and Town2. Calculate inflation rate of the island by using table below consumer basket:

	Town1 Prices 01.2018	**Town2 Prices 01.2018**	**Town1 Prices 01.2019**	**Town2 Prices 01.2019**
Fish	3	3.1	3.1	3.2
Milk	5	4.8	5	5
Bread	2	2.2	2.2	2.3
Coconut	10	11	11	11
Chocolate	20	19	19	19
Chicken	15	15.5	16	16.5
Shirt	22	23	24	25
Slippers	8	7.5	8.3	8.4
Beer	4.4	4.5	4.7	4.8
Squirt	33	35	36	37
Book	16	14	15	17
Education	200	210	210	220
Haircut	7	7	7.4	7.5
Rice	10	11	11	13

XV. Calculate GDP per capita of the island if 1000 migrants moved in to island.

XVI. What will happen to unemployment rate?

XVII. Calculate Gini Index of Turtle Island if bottom 10% of population earn 10% lesser than next 10% of population. Second lowest earning 10% earns 10% lesser than next 10% of population, and so on.

XVIII. Below is given table of businesses in one economy. Calculate GDP of this economy using all three methods.

Annual production	**Input**	**Final sales price in TMT**	**Annual salary per employee in TMT**	**Interest expense in BLN TMT**	**Profit in BLN TMT**
Diamond mining: 80000 carat	Employees 2000 and bank loan 1BLN TMT	5000/carat	100000	**?**	**?**
Iron mining: tons **?**	Employees 4000 and bank loan 1BLN TMT	4000 /ton	100000	**?**	1.1
Jewelry 7000 pieces	Employees 1000, diamond 50000 carat and bank loan **?** BLN TMT	100000 /piece	70000	0.08 BLN	**?**
Ship making 2 ships	Employees 1000, iron 100000 tons, and bank loan 2BLN TMT	0.6 BLN TMT/ship	**?**	**?**	0.5
Construction 100 building	Employees 10000, cement 1000000 tons, iron 50000 tons and bank loan **?** BLN TMT	**?** /building	70000	0.3 BLN	0.8
Cement **?** tons	Employees 3000 and bank loan 5BLN TMT	1000 TMT/ton	80000	**?**	1.26
Health service 20000 patients	Employees 300 and bank loan **?** BLN TMT	2000 TMT/patient	90000	0.02	**?**
Banks Total loan amount **?**	Employees 200	10% annual rate	100000		1.8

XIX. Below given a table of businesses in economy. Calculate GDP of this economy using all three methods.

Annual production	Input	Final sales price in TMT	Annual salary per employee in TMT	Income tax and profit taxes	Profit in BLN TMT
Cotton production 12000000 tons	Employees 20000 and 3 BLN loan	1000	500000		
Textile industry 5000000 tons	Employees 10000, cotton 2000000 ton and 5 BLN loan	2500	600000		
Clothing sector 5000000 tons	Employees 5000 and textile 1000000 tons and 5 BLN loan	3000	700000		
Agriculture sector: grapes 2000000 tons, wheat 3000000 tons, vegetables 7000000 tons, other fruits 5000000 tons, sugar cane 4000000 tons, maize 10000000 tons, cereal products 30000000 tons	Employees 100000, chemicals 1000000 tons, trucks 30000 units and 50 BLN bank loan	Grapes 5000/ton Wheat 2000/ton Vegetables 2000/ton Fruits 4000/ton Sugar cane 3000/ton, maize 5000/ton, cereal products 1000/ton			
Sugar factory Sugar 2000000 tons	Employees 2000, sugar cane 4000000 tons and 5 BLN bank loan	7000/ton	700000		
Vineyard 1000000 LITRE vines	Employees 1000, grapes 2000000 tons and 3 BLN loan	13000/Liter	700000		
Bakeries, pastries, macaroni factories, etc. 4000000 tons	Employees 20000, wheat 3000000 tons, sugar 1000000 tons, electricity 0.1 BLN megawatts and 10 BLN bank loan	10000/ton	700000		
Livestock, meat production (chicken, cattle, etc.) 2000000 tons	Employees 8000 and 5 BLN bank loan	20000/ton	800000		

Canned food factory, processed food 15000000 tons	Employees 10000, meat 1000000 tons, vegetable 5000000 tons, fruits 3000000 tons, chemicals 1000000 tons, 10000000 tons maize, 2000000 tons macaroni, electricity 0.1 BLN megawatts and 10 BLN bank loan	25000/ton	800000		
Supermarkets, shopping malls	Employees 5000, canned foods 15000000, bakery products 1000000 tons, vines 500000 LITRE, vegetables 1000000 tons, fruits 1000000, clothing 4000000 tons and electricity 0.1 BLN megawatts	20% of initial purchased price	600000		
Restaurants, hotels, casinos 20000000 clients	Employees 10000, meat 1000000 tons, bakery products 1000000, sugar 1000000 tons, fruits 1000000 tons, vines 500000 LITRE, vegetables 1000000 tons, clothing 1000000, electricity 0.1 BLN megawatts tons and 10 BLN bank loans	5000/client	800000		
Chemical industry chemicals 5000000 tons	Employees 8000 electricity 0.1 BLN megawatts and 20 BLN bank loan	25000/ton	1000000		
Pharmaceutical industry medicals 1000000 units	Employees 5000, electricity 0.1 BLN megawatts chemicals	110000/ton	1500000		

	3000000 tons				
Iron mining 1000000 tons	Employees 10000 electricity 0.1 BLN megawatts and 4 BLN bank loan	25000/ton	1000000		
Aluminum mining 1000000 tons	Employees 9000 electricity 0.1 BLN megawatts and 6 BLN bank loan	25000/ton	1100000		
Titanium mining 500000 tons	Employees 5000 electricity 0.1 BLN megawatts and 3 BLN bank loan	80000/ton	1200000		
Copper mining 1000000 tons	Employees 8000 electricity 0.1 BLN megawatts and 6 BLN bank loan	35000/ton	1000000		
Gold mining 100000 tons	Employees 10000 and electricity 0.1 BLN megawatts	1000000/ton	1400000		
Metallurgy company steel 3000000 tons	Employees 9000, iron 1000000 tons, aluminum 700000 tons, titanium 400000 tons, copper 900000 tons electricity 0.3 BLN megawatts and 10 BLN bank loan	50000/ton	1300000		
Petroleum Gasoline 10 BLN ton	Employees 50000 Cars and equipment 170000, Military aircrafts and helicopters 3000 units, electricity 0.2 BLN megawatts and 200 BLN bank Loans	50/ton	1200000		
Natural gas 20 BLN cub. Meters	Employees 70000, electricity 0.1 BLN megawatts, 1 BLN cub. Meters natural gas, Cars	20/cub. Meter	1300000		

	and equipment 100000, and 100 BLN bank Loans				
Electricity megawatts 3.3 BLN	Employees 60000 and 5 BLN cub. Meters natural gas	100/megawatts	1100000		
Military industry tanks, helicopters, aircrafts 10000 units	Employees 3000, electricity 0.1 BLN megawatts, steel 1000000 tons	7000000/unit	1200000		
Car industry Cars, trucks, buses, equipment 300000 units	Employees 5000, steel 2000000 tons electricity 0.1 BLN megawatts, and 20 BLN bank loan	800000/unit	1100000		
Transportation services, trains, plains, buses, taxis, et. 10 BLN unit of services	Employees 90000, 10 BLN cub. Meter of natural gas, 8 BLN ton gasoline, 1 BLN megawatts electricity and 80 BLN bank loan	90/service	1000000		
Education sector (kindergartens, schools, universities) Serving 200000 students	Employees 10000 and electricity 0.1 BLN megawatts	100000/student	1000000		
Health sector (hospitals and clinics) 3000000 patients	Employees 10000 electricity 0.1 BLN megawatts and medicals 1000000	50000/patient	1400000		
Banks	Employees 10000 electricity 0.1 BLN megawatts and gold 100000 tons	10% annual interest on loan amount	900000		
State sector (ministries, agencies, police, soldiers, etc.)	Employees 100000 Military tanks, aircrafts 7000 units, electricity 0.1 BLN megawatts, 1 BLN ton of gasoline		1000000		

Income tax (All employees pay it) =10%
Profit tax (only companies pay it):
Profit tax for Agricultural and Textile sector (from top to bottom until supermarkets and shopping malls, 9 cells) =20%
Leisure sector (supermarkets and casinos, 2 cells) =25%
Mining and chemical (6 cells) =30%
Metallurgy, Pharmaceutical and health services (2 cells) =35%
Energy and transportation sector (petrol, gas and electricity, transportation, 4 cells) =50%
Military, car and banks (3 cells) =30%
Education (2 cells) pays no profit taxes only income taxes
State employees only pay income tax.

A. Find total budget revenue and budget expenses. Is this government is having budget deficit or budget surplus?
B. Find total Factor incomes
C. Find total Consumer spending on final goods in economy
D. Are Factor incomes and Consumer spending equal?
E. Which sector has biggest profitability?
F. Which sector has lowest profitability?
G. Can you calculate unemployment rate in this economy if you were told that only 50000 people are looking for job?
H. Calculate GDP per capita if total population is 20000000 people.
I. Calculate total inflation rate if transportation cost increases by 10%
J. Who is the biggest employer?
K. Can you tell me how this economy is structurally, market, mixed or command?
L. Calculate total loan amount of banks.
M. Which goods are exported and which are imported?
N. Add three different business to this economy and calculate new GDP
O. Calculate HDI of this economy if life expectancy is 80 years and EI=0.85
P. If we assume that all businesses are privately owned (even education sector) and all of them are family owned businesses, then calculate the Gini Index
Q. If we assume that all businesses are public companies and that no investor owns more than 10% shares of the company, how would Gini Index will look like?

XX. Take Netherlands and Switzerland and using those indicators which were used in this chapter, compare these two countries.

XXI. As you have paid attention, economies that locate in one certain regions tend to have very similar characteristics, (Brazil, Argentina and Chile), Baltic states, (Estonia, Latvia and Lithuania), Eastern European economies (Bulgaria, Romania, Albania, Macedonia) , Western European economies, Middle east (Saudi Arabia, United Arab Emirates, Qatar, Kuwait, and Oman), North African region (Maghreb region: Algeria, Morocco, Tunis, Libya, and Egypt), Scandinavian region (Denmark, Sweden, Norway and Finland), Caribbean region, Pacific region, Southeast Asian region, Central Asian region. Take one of those regions and compare economic indicators and try to answer why those economies have close characteristics.

XXII. There are hypothetical only two countries exist, Country A and Country B. Country A has current account balance in an amount of 400 BLN Coins. This information about countries are given: GDP of Country A is 1 TRILLION Coins and GDP of Country B is 1.1 TRILLION Coins. Consumption in Country A is 30% lesser than Country B. Government spending in Country B exceeds government spending in Country A by 20%. Investment spending is the same in both countries, around 100 BLN Coins. Find consumption and budged spending in both countries.

XXIII. Explore economy of Mexico through economic and other indicators. What kind of problems does Mexico have?

XXIV. Inflation rate in Venezuela is exploded recently. Why is that?

XXV. 3C+1.2G+1.3I+0.2 Trillion=2 Trillion
1.5C+G+I-0.1 Trillion=1.4 Trillion
C+G+I=1 Trillion
GDP's of three hypothetical countries are given above. Find C, G and I's.

XXVI. $CPI_{01.2019}$=3004508.3 Pesos
$CPI_{01.2020}$=3012908 Pesos

Calculate inflation rate.

XXVII. Initial price of the paining was 0.1 BLN FRANKS. After a century painting is sold for 0.2 BLN FRANKS. What added a value for this good? Why value of painting increased? Is it applicable to all goods?

XXVIII. Make analysis of Indian economy. What problems do India have?

XXIX. Where you would prefer to live and do a business, in Brazil or Nigeria? Why?

XXX. How tourism sector impacts GDP, through trade or consumption?

Solutions:

I. Fish consumption: 3*90000=270.000 TMT
Coconut consumption: 9*200.000=1.800.000 TMT
Income from tourism: 5000*50.000=250.000.000 TMT
GDP=270.000+1.800.000+250.000.000= 252.070.000 (We calculate only using Aggregate spending method since we don't have any other information for further calculations).

II. Fish consumption: 4*90.000=360.000 TMT

Coconut consumption: 9.5*200.000=1.900.000 TMT
Income from tourism: 6000*50.000= 300.000.000 TMT
GDP=300.000.000+1.900.000+360.000=302.260.000 TMT

III. Fish consumption:2*90.000=180.000 TMT
Coconut consumption: 8*200.000=1.600.000 TMT
Income from tourism: 4.000*50.000=200.000.000 TMT
GDP= 200.000.000+1.600.000+180.000=201.780.000 TMT

IV. Fish consumption: 5*90.000=450.000 TMT
Coconut consumption: 9*200.000=1.800.000 TMT
Income from tourism: 5.000*50.000=250.000.000 TMT
GDP= 250.000.000+450.000+1.800.000 = 252.250.000 TMT

V. $\frac{200.000}{2} = 100.000$ Liter of coconut
Coconut oil consumption: 40*100.000=4.000.000 TMT
Fish consumption: 5*90.000=450.000 TMT
Income from tourism 5.000*50.000=250.000.000 TMT
GDP=4.000.000+450.000+250.000.000=254.450.000 TMT

VI. $GDP\ per\ capita = \frac{254.450.000}{10000} = 25445\ TMT$

VII. Fish consumption : 90000+90000*50%=135000 TMT
135000*5=675000 TMT
Coconut growers: 200.000+200.000*80%=360.000
$\frac{360000}{2}$=180.000
180.000*40=7.200.000 TMT
Income from tourism 5.000*50.000=250.000.000 TMT
GDP=250.000.000+7.200.000+675.000=257.875.000 TMT

VIII. $GDP\ per\ capita = \frac{257.875.000}{10.000} = 25787{,}5\ TMT$

IX. Fishing industry: 135.000-135.000*70%=40.500
40.500*5=202.500
Coconut trees: 360.000-360.000*80%=72.000
$\frac{72000}{2}$=36.000
36.000*40=1.440.000 TMT
Income from tourism: 5.000*50.000=250.000.000 TMT
GDP=1.440.000+202.500+250.000.000=251.642.500 TMT

X. $GDP\ per\ capita = \frac{251.642.500}{10.000} = 25164{,}25\ TMT$

XI. Education Index=0.7

Life expectancy Index (LEI)$= \frac{72-20}{85-20} = 0.8$

Income Index (II)$= \frac{\ln(25445)-\ln(100)}{\ln(75000)-\ln(100)} = 0.84$

HDI=$\sqrt[3]{(0.84) * (0.8) * (0.7)} = 0.777\ or\ 77.7\%$

XII. Labor Force=6000

Looking for a job=230

Unemployment rate = $\frac{230}{6000}$ =0.038 or 3.8%

XIII. 1000+230=1230 (number of unemployed)

Unemployment rate = $\frac{1230}{6000} = 0.205$ or 20.5%

XIV. $Inflation\ rate = \frac{CPI_{current\ year} - CPI_{last\ year}}{CPI_{last\ year}} * 100$

Primarily, we need to find CPI to find inflation rate, so to find Consumer price index we add up all prices and take weighted average:
Total Price in **Town 1** in 01.2019=
3.1+5+2.2+11+19+16+24+8.3+4.7+36+15+210+7.4+
11= 372.7 TMT
Total price in **Town 2** in 01.2019=
3.2+5+2.3+11+19+16.5+25+8.4+4.8+37+17+220+7.5+13=
389.7 TMT

$$CPI_{current\ year} = \frac{372.7 + 389.7}{2} = 381.2$$

Total Price in **Town 1** in 01.2018=
3+5+2+10+20+15+22+8+4.4+33+16+200+7+10=355.4 TMT

Total price in **Town 2** in 01.2018=
3.1+4.8+2.2+11+19+15.5+23+7.5+4.5+35+14+210+7+11=
367.6 TMT

$$CPI_{last\ year} = \frac{355.4 + 367.6}{2} = 361.5\ TMT$$

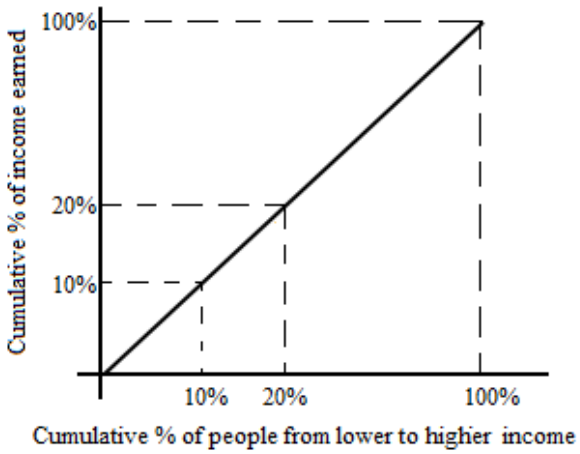
$$Inflation\ rate = \frac{381.2 - 361.5}{361.5} * 100\% = 5.45\%$$

XV. 10000+1000=11000

GDP per capita $= \frac{251.642.500}{11000} = 22876.6$ TMT

XVI. Unemployment rate$= \frac{1230}{7000} = 0.176\ or\ 17.6\%$

XVII.



XVIII.

Annual production	**Input**	**Final sales price in TMT**	**Annual salary per employee in TMT**	**Interest expense in BLN TMT**	**Profit in BLN TMT**
Diamond mining: 80000 carat	Employees 2000 and bank loan 1BLN TMT	5000/carat	100000	**0.1**	**0.1**
Iron mining: tons **400000**	Employees 4000 and bank loan 1BLN TMT	4000 /ton	100000	**0.1**	1.1
Jewelry 7000 pieces	Employees 1000, diamond 50000 carat and bank loan **0.8** BLN TMT	100000 /piece	70000	0.08	**0.3**

Ship making 2 ships	Employees 1000, iron 100000 tons, and bank loan 2 BLN TMT	0.6 BLN TMT/ship	**100000**	**0.2**	0.5
Construction 100 building	Employees 10000, cement 1000000 tons, iron 50000 tons and bank loan **3** BLN TMT	**30000000** /building	70000	0.3	0.8
Cement **2000000** tons	Employees 3000 and bank loan 5BLN TMT	1000 TMT/ton	80000	**0.5**	1.26
Health service 20000 patients	Employees 300 and bank loan **0.2** BLN TMT	2000 TMT/patient	90000	0.02	**-0.007**
Banks Total loan amount **13 BLN**	Employees 2000	10% annual rate	100000		**1.1**

a) Diamond mining:

Π=TR-TC (pay attention that we include to the total cost fixed interest rate 10% of the loan borrowed from the bank!)

TR=P*Q=5.000*80.000=400.000.000

TC=w*L+ r*K=2.000*10.000+100.000.000=300.000.000

Π=TR-T→400.000.000-300.000.000=100.000.000 TMT

b) Iron mining:

Π=1.100.000.000 TMT

TR=4.000Q

TC=100.000*4.000+100.000.000=500.000.000

Π=TR-TC→ 1.100.000.000=4.000Q-500.000.000

1.100.000.000+500.000.000=4000Q

Q=400.000

c) Jewelry

TR=P*Q=7000*100.000=700.000.000
TC=1000*70.000+50.000*5000+80.000.000=400.000.000 TMT
Π=TR-TC→ 700.000.000-400.000.000=300.000.000 TMT

d) Ship making:

Π=500.000.000 TMT
TR=2*600.000.000=1.200.000.000
TC=1000*w+200.000.000+4000*100.000=600.000.000+1000*w
Π=TR-TC→ 500.000.000=1.200.000.000-600.000.000-1000*w
-100.000.000=-1000*w
w=100.000TMT

e) Construction:

Π=800.000.000TMT
TR=P*Q=100P
TC=10.000*70.000+1.000.000*1000+50.000*4000+300.000.000 TMT=2.200.000.000TMT
Π=TR-TC→ 800.000.000=100P-2.200.000.000
P=30.000.000TMT

f) Cement:

Π=1.260.000.000TMT
TR=P*Q=1000Q
TC=3000*80.000+500.000.000=740.000.000TMT
Π=TR-TC→ 1.260.000.000=1000Q-740.000.000
Q=2.000.000

g) Health service:

TR=P*Q=2000*20.000=40.000.000 TMT
TC=300*90.000+20.000.000=47.000.000 TMT
Π=TR-TC→ 40.000.000-47.000.000 = -7000 TMT

h) Banks

TC=2000*100.000=200.000.000
TR= (sum of total interest earned from loan) = 100.000.000+100.000.000+80.000.000+200.000.000+300.000.000+500.000.000+20.000.000=1.300.000.000 TMT

Π=TR-TC→1.300.000.000-200.000.000=1.100.000.000 TMT

Now let's calculate GDP by using *aggregate spending method* (don't forget that we include to spending only final goods):

GDP= 30.000*5000+250.000*4000+7000*100.000+ 2*600.000.000+100*30.000.000+1.000.000*1000+20.000* 2000=7090 BLN TMT (bank is excluded in both cases)

Now let's calculate GDP by using *factor income method* (w*L+Π+r):

Banking sector is already calculated in factor income method through rental rate. That's why we don't independently banking sector in calculation of GDP.

w*L=2000*100.000+4000*100.000+1000*70.000+100.000* 1000+10.000*70.000+3000*80.000+300*90.000=1,737 BLN TMT
Π=0.1+ 1.1+0.3+ 0.5+ 0.8+ 1.26-0.007=4,053 BLN TMT
r=0.1+0.1+0.08+0.2+0.3+0.5+0.02=1.3 BLN TMT
GDP=1,737 BLN +4,053 BLN +1.3 BLN= 7090 BLN TMT

XIX. We can't calculate GDP by using value added method due to lack of information.

Annual production	**input**	**Final sales price in TMT**	**Annual salary per employee in TMT**	**Rental rates In BLN TMT**	**Salaries in BLN TMT**	**Profit in BLN TMT**	**Income and profit tax**
Cotton production 12000000 tons	Employees 20000 and 3 BLN loan	1000	500000	**0.3**	**10**	**1.7**	**1.34**
Textile industry 5000000 tons	Employees 10000, cotton 2000000 ton and 5 BLN loan	2500	600000	**0.5**	**6**	**4**	**1.4**
Clothing sector 5000000 tons	Employees 5000 and textile 1000000 tons and 5 BLN loan	3000	700000	**0.5**	**3.5**	**8.5**	**2.05**

Agriculture sector: grapes 2000000 tons, wheat 3000000 tons, vegetables 7000000 tons, other fruits 5000000 tons, sugar cane 4000000 tons, maize 10000000 tons, cereal products 30000000 tons	Employees 100000, chemicals 1000000 tons, trucks 30000 units and 50 BLN bank loan	Grapes 5000/ton Wheat 2000/ton Vegetables 2000/ton Fruits 4000/ton Sugar cane 3000/ton, maize 5000/ton, cereal products 1000/ton	500000	**5**	**50**	**38**	**12.6**
Sugar factory Sugar 2000000 tons	Employees 2000, sugar cane 4000000 tons and 5 BLN bank loan	7000/ton	700000	**0.5**	**1.4**	**0.1**	**0.16**
Vineyard 1000000 LITRE vines	Employees 1000, grapes 2000000 tons and 3 BLN loan	13000/Liter	700000	**0.3**	**0.7**	**2**	**0.47**
Bakeries, pastries, macaroni factories, etc. 4000000 tons	Employees 20000, wheat 3000000 tons, sugar 1000000 tons, electricity 0.1 BLN megawatts and 10 BLN bank loan	10000/ton	700000	**1**	**14**	**2**	**1.8**
Livestock, meat production (chicken, cattle, etc.) 2000000 tons	Employees 8000 and 5 BLN bank loan	20000/ton	800000	**0.5**	**6.4**	**33.1**	**7.26**
Canned food factory, processed food 15000000 tons	Employees 10000, meat 1000000 tons, vegetable 5000000 tons, fruits 3000000 tons, chemicals 1000000 tons, 10000000 tons maize, 2000000 tons macaroni, electricity 0.1 BLN megawatts and 10 BLN bank loan	25000/ton	800000	**1**	**8**	**219**	**44.6**
Supermarkets, shopping malls	Employees 5000, canned foods 15000000, bakery products 1000000 tons, vines 500000 LITRE, vegetables	20% of initial purchased price	600000	**-**	**3**	**68.9**	**17.525**

	1000000 tons, fruits 1000000, clothing 4000000 tons and electricity 0.1 BLN megawatts						
Restaurants, hotels, casinos 20000000 clients	Employees 10000, meat 1000000 tons, bakery products 1000000, sugar 1000000 tons, fruits 1000000 tons, vines 500000 LITRE, vegetables 1000000 tons, clothing 1000000, electricity 0.1 BLN megawatts tons and 10 BLN bank loans	5000/client	800000	**1**	**8**	**28.5**	**7.925**
Chemical industry chemicals 5000000 tons	Employees 8000 electricity 0.1 BLN megawatts and 20 BLN bank loan	25000/ton	1000000	**2**	**8**	**105**	**32.3**
Pharmaceutical industry medicals 1000000 units	Employees 5000, electricity 0.1 BLN megawatts chemicals 3000000 tons	110000/ton	1500000	**-**	**7.5**	**17.5**	**6.875**
Iron mining 1000000 tons	Employees 10000 electricity 0.1 BLN megawatts and 4 BLN bank loan	25000/ton	1000000	**0.4**	**10**	**4.6**	**2.38**
Aluminum mining 1000000 tons	Employees 9000 electricity 0.1 BLN megawatts and 6 BLN bank loan	25000/ton	1100000	**0.6**	**9.9**	**4.5**	**2.34**
Titanium mining 500000 tons	Employees 5000 electricity 0.1 BLN megawatts and 3 BLN bank loan	80000/ton	1200000	**0.3**	**6**	**23.7**	**7.71**
Copper mining 1000000 tons	Employees 8000 electricity 0.1 BLN megawatts and 6 BLN bank loan	35000/ton	1000000	**0.6**	**8**	**16.4**	**5.72**
Gold mining 100000 tons	Employees 10000 and electricity 0.1 BLN megawatts	1000000/ ton	1400000	**-**	**14**	**76**	**24.2**
Metallurgy company steel 3000000 tons	Employees 9000, iron 1000000 tons, aluminum 700000 tons, titanium 400000 tons, copper 900000 tons electricity 0.3 BLN megawatts and 10	50000/ton	1300000	**1**	**11.7**	**1.3**	**1.625**

	BLN bank loan						
Petroleum Gasoline 10 BLN ton	Employees 50000 Cars and equipment 170000, Military aircrafts and helicopters 3000 units, electricity 0.2 BLN megawatts and 200 BLN bank Loans	50/ton	1200000	**20**	**60**	**243**	**127.5**
Natural gas 20 BLN cub. Meters	Employees 70000, electricity 0.1 BLN megawatts, 1 BLN cub. Meters natural gas, Cars and equipment 100000, and 100 BLN bank Loans	20/cub. Meter	1300000	**10**	**91**	**189**	**103.6**
Electricity megawatts 3.3 BLN	Employees 60000 and 5 BLN cub. Meters natural gas	100/ megawatts	1100000	**-**	**66**	**164**	**88.6**
Military industry tanks, helicopters, aircrafts 10000 units	Employees 3000, electricity 0.1 BLN megawatts, steel 1000000 tons	7000000/ unit	1200000	**-**	**3.6**	**6.4**	**2.28**
Car industry Cars, trucks, buses, equipment 300000 units	Employees 5000, steel 2000000 tons electricity 0.1 BLN megawatts, and 20 BLN bank loan	800000/ unit	1100000	**2**	**5.5**	**122.5**	**37.3**
Transportation services, trains, plains, buses, taxis, et. 10 BLN unit of services	Employees 90000, 10 BLN cub. Meter of natural gas, 8 BLN ton gasoline, 1 BLN megawatts electricity and 80 BLN bank loan	90/service	1000000	**8**	**90**	**102**	**60**
Education sector (kindergartens, schools, universities) Serving 200000 students	Employees 10000 and electricity 0.1 BLN megawatts	100000/ student	1000000	**-**	**10**	**0**	**1**
Health sector (hospitals and clinics)	Employees 10000 electricity 0.1 BLN megawatts and	50000/ patient	1400000	**-**	**14**	**16**	**1.4**

3000000 patients	medicals 1000000						
Banks	Employees 10000 electricity 0.1 BLN megawatts and gold 100000 tons	10% annual interest on loan amount	900000	-	-	-	**36.5**
State sector (ministries, agencies, police, soldiers, etc.)	Employees 100000 Military tanks, aircrafts 7000 units, electricity 0.1 BLN megawatts, 1 BLN ton of gasoline		1000000	-	-	-	**10**

Cotton Production:
TR=P*Q=1000*12.000.000=12 BLN TMT
TC=20.000*500.000(wage) +3.000.000.000*10%=10.3 BLN TMT
П=TR-TC→ 12-10.3=1.7 BLN TMT

Textile industry:
TR=P*Q=2500*5.000.000=12.5 BLN TMT
TC=600.000(wage)*10.000+2.000.000*1000 (Cotton)+5.000.000.000*10%=8.5 BLN TMT
П=TR-TC→ 12.5-8.5=4 BLN TMT

Clothing sector:
TR=P*Q=3000*5.000.000=15 BLN TMT
TC=700.000*5000(wage) +2500*1.000.000 (Textile)+5.000.000.000*10%=6.5 BLN TMT
П=TR-TC→ 15-6.5=8.5 BLN TMT

Agriculture sector:
TR=P*Q=2.000.000(grape)*5000+3.000.000(wheat)*2000+ 7.000.000(vegetable)*2000+5.000.000(fruit)*4000+ 4.000.000(sugar cane) *3000+10.000.000(maize)*5000 +30.000.000(cereal)*1000=142 BLN
TC=500.000(wage)*100.000+1.000.000 (Chemicals)*25000+30.000(trucks)*800.000+ 50.000.000.000*10%=104 BLN TMT
П=TR-TC→ 142-104=38 BLN TMT

Sugar factory:
TR=P*Q=2.000.000*7000=14 BLN TMT
TC=2000*700.000(wage) +4.000.000 (Sugar cane) *3000+5.000.000.000*10%=13.9 BLN TMT

Π=TR-TC→ 14-13.9=0.1 BLN TMT

Vineyard:
TR=P*Q= 13000*1.000.000=13 BLN TMT
TC=1000*700.000(wage) +2.000.000
(Grape)*5000+3.000.000.000*10%=11 BLN TMT
Π=TR-TC→ 13-11=2 BLN TMT

Bakery:
TR=P*Q=10.000*4.000.000=40 BLN TMT
TC=20.000(wage)*700.000+2000*3.000.000(wheat)+
7000*1.000.000(sugar)+10.000.000.000*10%+10.000.000.0
00
(Electricity)=38 BLN TMT
Π=TR-TC→ 40-38=2 BLN TMT

Livestock:
TR=P*Q=2.000.000*20.000=40 BLN TMT
TC=8000(wage)*800.000+5.000.000.000*10%=6.9 BLN
TMT
Π=TR-TC→ 40-6.9=33.1 BLN TMT

Canned food factory:
TR=P*Q=25000*15.000.000=375 BLN TMT
TC=10.000*800.000(wage)+1.000.000(meat)
*20.000+5.000.000(vegetable)*2000+3.000.000(fruits)*
4000+1.000.000(chemicals)*25000+10.000.000(maize)*
5000+2.000.000(macaroni)*10.000+10.000.000.000
(electricity)+ 10.000.000.000*10%=156 BLN TMT
Π=TR-TC→ 375-156=219 BLN TMT

Supermarket:
TC=5000*600.000(wage)+25000*15.000.000(canned food)
+1.000.000(bakery)*10.000+500.000*13000(vine)+
1.000.000(vegetables)*2000+1.000.000(fruits)*4000+
4.000.000(clothing)*3000+10.000.000.000(electricity)=
422.5 BLN TMT
TR=P*Q= (we calculate only for sale products: from canned
food up to clothing) 409.5+409.5*20%=491.4 BLN TMT
Π=TR-TC→ 491.4-422.5=68.9 BLN TMT

Restaurants, casinos and hotels:

TR=P*Q=5000*20.000.000=100 BLN TMT
TC=10.000*800.000(wage)+1.000.000(meat)*20000+
1.000.000(bakery)*10.000+1.000.000(sugar)*7000+
1.000.000(fruit)*4000+500.000(vine)*13000+
1.000.000(vegetable)*2000+3000*1.000.000(clothing)+
10.000.000.000(electricity)+ 10.000.000.000*10%=71.5
BLN TMT
Π=TR-TC→ 100-71.5=28.5 BLN TMT

Chemical industry:
TR=25000*5.000.000=125 BLN TMT
TC=8000*1.000.000(wage)+10.000.000.000
(Electricity)+ 20.000.000.000*10%=20 BLN TMT
Π=TR-TC→ 125-20=105 BLN TMT

Pharmaceutical industry:
TR=P*Q=1.000.000*110.000=110 BLN TMT
TC=5000*1.500.000(wage) +10.000.000.000
(Electricity)+3.000.000(chemicals)*25000=92.5 BLN TMT
Π=TR-TC→ 110-92.5=17.5 BLN TMT

Iron mining:
TR=P*Q=25000*1.000.000=25 BLN TMT
TC=10.000*1.000.000(wage)+ 10.000.000.000
(Electricity)+ 4.000.000.000*10%=20.4 BLN TMT
Π=TR-TC→ 25-20.4=4.6 BLN TMT

Aluminum mining:
TR=P*Q=1.000.000*25000= 25 BLN TMT
TC=9000*1.100.000(wage) + 10.000.000.000
(Electricity)+ 6.000.000.000*10%=20.5 BLN TMT
Π=TR-TC→ 25-20.5=4.5 BLN TMT

Titanium mining:
TR=P*Q=80.000*500.000=40 BLN TMT
TC=5000*1.200.000(wage)+ 10.000.000.000
(Electricity)+ 3.000.000.000*10%=16.3 BLN TMT
Π=TR-TC→ 40-16.3=23.7 BLN TMT

Copper mining:
TR=P*Q=35000*1.000.000=35 BLN TMT
TC=8000*1.000.000(wage)+ 10.000.000.000

(Electricity)+ 6.000.000.000*10%=18.6 BLN TMT
Π=TR-TC→ 35-18.6=16.4 BLN TMT

Gold mining:
TR=P*Q=100.000*1.000.000=100 BLN TMT
TC =10.000*1.400.000(wage)+ 10.000.000.000
(Electricity)=24 BLN TMT
Π=TR-TC→ 100-24=76 BLN TMT

Metallurgy Company:
TR=50.000*3.000.000= 150 BLN TMT
TC=9000*1.300.000(wage)+1.000.000
(Iron)*25000+700.000(aluminum)*25000+
400.000(titanium)*80.000+900.000(copper)*35000
+30.000.000.000(electricity) + 10.000.000.000*10%=
148. 7 BLN TMT
Π=TR-TC→ 150-148.7=1.3 BLN TMT

Petroleum gasoline:
TR=P*Q=10.000.000.000*50=500 BLN TMT
TC=50.000*1.200.000(wage) +170.000 (cars and
equipment)*800.000+3000 (military
aircrafts)*7.000.000+20.000.000.000(electricity)+
200.000.000.000*10%=257 BLN TMT
Π=TR-TC→ 500-257= 243 BLN

Natural gas:
TR=P*Q=20.000.000.000*20=400 BLN TMT
TC=70.000*1.300.000(wage) +
10.000.000.000(electricity) +1.000.000.000*20(natural gas)
+100.000 (cars and equipment)
*800.000+100.000.000.000*10%=211 BLN TMT
Π=TR-TC→ 400-211=189 BLN TMT

Electricity:
TR=P*Q=3.300.000.000*100=330 BLN TMT
TC=60.000*1.100.000(wage) +5.000.000.000(natural gas)
*20=166 BLN TMT
Π=TR-TC→ 330-166=164 BLN TMT

Military industry:
TR=P*Q=10.000*7.000.000=70 BLN TMT

TC=3000*1.200.000(wage) +10.000.000.000(electricity)+
1.000.000(steel)*50.000=63.6 BLN TMT
Π=TR-TC→ 70-63.6=6.4 BLN TMT

Car industry:
TR=P*Q=300.000*800.000=240 BLN TMT
TC=5000*1.100.000(wage) +2.000.000(steel)*50.000+
10.000.000.000(electricity) + 20.000.000.000*10%=
117.5 BLN TMT
Π=TR-TC→ 240-117.5=122.5 BLN TMT

Transportation services:
TR=P*Q=90*10.000.000.000=900 BLN TMT
TC=90.000*1.000.000(wage) +10.000.000.000(natural gas)
*20+8.000.000.000(gasoline)*50+100.000.000.000
(Electricity) + 80.000.000.000*10%= 798 BLN TMT
Π=TR-TC→ 900-798=102 BLN TMT

Education sector:
TR=P*Q=200.000*100.000 =20 BLN TMT
TC=10.000*1.000.000(wage) +10.000.000.000(electricity)
=20 BLN TMT
Π=TR-TC→ 20-20=0

Health sector:
TR=P*Q=3.000.000*50.000= 150 BLN TMT
TC=10.000*1.400.000(wage) + 10.000.000.000(electricity)
+1.000.000(medicals)*110.000= 134 BLN TMT
Π=TR-TC→ 150-134=16 BLN TMT

Banks:
TC=10.000*900.000+10.000.000.000(electricity) =19 BLN
TMT
TR= (sum of all interest rates from loan of above industries)
+ 100.000(gold)*1.000.000=155.5 BLN TMT
Π=TR-TC→ 155.5-19= 136.5 BLN TMT (one asset turned
into another).

State sector:
TC=100.000*1.000.000(wage) +7.000.000*7000(military
tanks) +10.000.000.000(electricity)
+1.000.000.000*50(gasoline)=209 BLN TMT

Let's calculate using *factor income method:*

Sum up all salaries=516.2 BLN TMT
Sum up all interest rates (r) =55.5 BLN TMT
Sum up all profits=1507.7 BLN TMT
GDP=2079.4 BLN

Now let's calculate GDP by using *aggregate spending method:*

Final goods	**Quantity**	**Price**
Cotton	10.000.000	1000
Textile	4.000.000	2500
Aluminum	300	25000
Titanium	100	80
Copper	100	35
Petroleum	2 000.000.000	50
Natural Gas	4 000.000.000	20
Transportation	10 000.000.000	90
Education	200 000	100 00
Health sector	3 000.000	50 000
Gold mining	100 000	1 000.000.000
Electricity	100	200 000 000
Supermarket	-	491.4
Miltary industry	7000	7 000.000.000

GDP=10.000.000(cotton)*1000+4.000.000(textile)*2500+
30.000.000(cereal)*1000+20.000.000(casino)*5000+
300.000(aluminum)*25000+100.000(titanium)*80.000+
100.000(copper)*35000+2.000.000.000(gasoline)*50+
4.000.000.000 (natural gas)*20+90*10.000.000.000
(transportation) + 200.000(education)*100.000+
3.000.000 (health sector) *50.000+100.000*1.000.000(gold
mining) +20.000.000.000 (electricity) +7000*7.000.000.000
(military industry) +
491.4 BLN (supermarket) = 2079.4 BLN TMT

A. Total budget revenue=648.46 BLN TMT
   Total budget expenditure=209 BLN TMT
   Budget revenue – budget expenditure=648.46-209=439.46 BLN TMT (budget surplus)
B. Total factor income =2079.4 BLN TMT

C. Total consumer spending on final goods=2079.4 BLN TMT
D. Yes, total factor income is equal to total consumer spending on final goods.
E. Petroleum Gasoline sector is the most profitable sector.
F. Sugar factory is the least profitable sector.
G. Unemployment rate = $\frac{50000}{663000+50000}$*100=7%
H. GDP per capita= $\frac{2\ 079\ 400\ 000\ 000}{20\ 000\ 000}$ = 103 970 TMT
I. If transportation cost increases by 10%, increase in price of transported goods will also be 10%.
Inflation rate= $\frac{2287.34-2079.4}{2079.4}$*100=10%
J. State and agriculture sectors.
K. We can know the structure of the economy by calculating its budget spending ratio. Bigger ratio shows that its structure is command economy and smaller the ratio is that its structure is market economy.
$\frac{Budget\ spending}{GDP} = \frac{209}{2079.4} = 0.1$ or 10% (market economy)
L. Total loan amount of banks 55.5 BLN TMT.
M. There are no imported and exported goods.
N. 3 different businesses:

Annual production	**Input**	**Final sales price in TMT**	**Annual salary per employee in TMT**	**Rental rates in BLN TMT**	**Salaries in BLN TMT**	**Profit in BLN TMT**
Diamond mining 10 000 000 carat	Employees 10000 , electricity 0.01 BLN megawatts, bank loan 20 BLN	6000 per carat	3 000 000	2	30	27
High-Tech industry 10 MLN	Employees 40000, electricity	10000	200 000	1	8	73.5

(smartphones, computers, TV, gadgets.)	0.1,300000 aluminum, bank loan 10 BLN TMT					
Chocolate industry 10000000 ton	Employees 30000, electricity 0.09, bank loan 20 BLN	2000 per ton	10000	2	0.3	8.7

In input of above new sectors we add from previous final goods 20 BLN electricity and 300000 aluminums, so from our previous GDP we deduct them because they are now inputs of other goods:
300000*25000+20 BLN= 27.5 BLN TMT
2079.4-27.5=2051.9 BLN TMT
And add our new produced final goods:
GDP (using aggregate spending method)
=180+2051.9-2231.9 BLN TMT
GDP (using factor income method)
=152.5+2079.4=2231.9 BLN TMT.

O. GDP per capita $= \frac{2.231.900.000.000}{20.000.000} = 111595$ TMT

Life expectancy Index (LEI) $= \frac{80-20}{85-20} = 0.92$

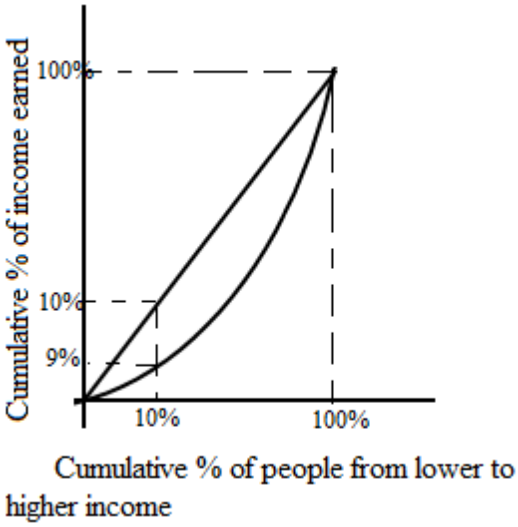
Income Index (II) $= \frac{\ln(111595)-\ln(100)}{\ln(75000)-\ln(100)} = 1.06$

EI=0.8

HDI= $\sqrt[3]{1.06 * 0.92 * 0.8} = 0.92$

P. According to factor income method, we can see that almost 25% of annual income goes to salary, and the rest 75% goes to owners of the company as profit. Since only poor and middle class live for salary by looking at GDP we can say that high income inequality exist in this economy.

Q. Out of 2079.4 BLN TMT, if investors earn no more than 10%, then:
1507.7*90%+ 516.2=1873.13
Will be earnings of middle and poor class people, which means that income is fairly distributed if almost 90% of GDP is earnings of middle and poor class people.



XX. Looking at the indicators below, we can say that citizens of Switzerland are richer than Netherlands; according to HDI we can say that human development is higher in Switzerland than in Netherland. According to Economic freedom indicator citizens of Switzerland are more liberal in perusing in economic activity. Economic equality in both countries is in acceptable level.

Indicators	**Switzerland**	**Netherlands**
GDP per capita	70989	59687
HDI	94.6%	93.3%
Economic freedom index	82%	77%
NIIP	813400 MLN	839700
Inflation rate	0.36%	2.67%
Net export	50100	91000

XXI. Close geographical location increases historic, cultural and economic relations between states and close historic ties make neighbor nations look alike. By taking advantage of geographical location, it brings good income for its population. As a result, good income increases social standards of living, life expectancy and enlarges education opportunities.

XXII. Let's for the sake of ease write GDP equations in terms of only country B.

$C_B=1.3C_A$

$G_B=1.2G_A$

$I_B=I_A$

If there are only two countries, higher export of country A will lead to higher import of country B.

$$\begin{cases} 1000\ BLN = C_A + C_A + 100\ BLN + 400\ BLN \\ 1100\ BLN = 1.3C_A + 1.2C_A + 100\ BLN - 400\ BLN \end{cases}$$

Multiply 1st equation to 1.3 and get:

$$\begin{cases} 13000\ BLN = 1.3C_A + 1.3C_A + 130\ BLN + 520\ BLN \\ 1100\ BLN = 1.3C_A + 1.2C_A + 100\ BLN - 400\ BLN \end{cases}$$

From 1st equation deduct 2nd and get:

200 BLN=0.1G_A +950 BLN

-750 BLN=0.1G_A

G_A= -7500 BLN

1000 BLN = C_A - 7500 BLN + 100 BLN + 400 BLN

C_A =8000 BLN

If C_B=1.3C_A then

C_B=1.3*(8000) =10400 BLN

G_B=1.2G_A=1.2*(-7500) = -9000 BLN

XXIII. According to the indicators below, we can say that Mexico has low inflation rate, but high international homicide rate, also vast incarceration victims. According to NIIP we can see that this country is a debtor, also has a budget deficit problem. According to Gini index wealth as well as income is unevenly distributed among its population.

Indicators	**Mexico**
Inflation rate	6,04
International homicide rate	29,07
Incaceration victims	203364
NIIP	-355,500
Budget	-22100
Gini (income)	45.4
Gini wealth	80
Gini Y	62.7

XXIV. Economic sanctions imposed by USA and Europe caused exports and imports to halt. This caused scarcity which reflected in prices.

XXV. $\begin{cases} 3C + 1.2G + 1.3I + 0.2 = 2 \\ 1.5C + G + I - 0.1 = 1.4 \\ C + G + I = 1 \end{cases}$

From 2nd we deduct 3rd equation and get:

0.5C-0.1=0.4

0.5C=0.5

C=1

C+G+I=1

1+G+I=1

G+I=0

G=-I

And put it to 1st equation:

3*1+1.2*(-I) +1.3I+0.2=2

3+0.1*I+0.2=2

0.1*I=-1.2

I=-12

If G=-I then G=-(-12)

G=12

XXVI. Inflation rate = $\frac{3012908-3004508.3}{3004508.3} * 100\% = 0.28\%$

XXVII. Goods loose value due to depreciation, but certain goods actually gain value as they get older. Goods that are considered as an art is one of those goods which gain value over the years.

XXVIII. According to the indicators below on the table we can say that India is a debtor country, also India has a budget deficit problem. According to the Gini indexes we can see that income and wealth are unevenly distributed among its population.

Indicators	**India**
NIIP	-426,525
Budget size	180630
Gini (income)	37,8%
Gini (wealth)	85,4%
Gini Y	61,6%
Net export	128000
GDP	3845630 MLN

XXIX. According to the indicators below, I would choose Brazil. According to standards of living, Brazil is higher than Nigeria, also according CPI; Nigeria has lower ethical

standards which might cause some problems in doing business in Nigeria.

Indicators	Brazil	Nigeria
GDP per capita	8,920	2,028
HDI	76.1%	53.4%
Gini (income)	46.6%	43%
Corruption perception index	35%	26%

XXX. Trade in economics mean import and export of goods. Only local purchase of goods is included to consumption. Since tourists do both, buy and consume goods locally, tourism impacts GDP in both ways.

Chapter 5: Basic Macroeconomics

Progress is in the nature of human beings. From the day we born, we start our journey of advancement. We crawl, then walk, after we mumble words, and later on speak unstoppably. We ask millions of questions; child curiosity seems never be fully satisfied. Some of us are even got reprimanded for being "too curious" and asking "silly questions". At that childhood age we learn about competition. Who is taller, who is faster, smarter, and also, who eats more chocolates a day. This continues our whole life: we want to be the best in class at school, the most charming guy in class, the fastest and strongest athlete and of course get into the best university. At university we want to be again best in class, date the best girl in class and write the best course works. When we grow up, we also learn that world is not a perfect. We learn how to coup with that too! After university years, we get into the adult world where you are practically on your own. This feeling of competition deeply rooted from childhood, pushes us all our lives. We want the best house, the most beautiful wife, the best jacket and the best place "under the sun". As we grow and develop, our goals grow too. You want to live in the best city, in the best province, in the best country after all. What does "best country" means? In economics science, "best country" means "wealthy and developed country". How to be wealthy and developed country? You must grow sustainably and continuously! How to grow? Using your inputs: labor, capital and technology (knowledge, method, process, know-how, etc.). As we have learned before, you cannot produce any output without these three things: labor, capital and technology. How will doctors evaluate your condition? By looking at your health. If your health condition is good, they will say that you are doing well. They do not care about how much you earn or grades in math courses. They have their own criteria of evaluation: health. If you go to your boxing coach ask him to evaluate you, first thing he will say is about your skills in ring because that is his criteria of evaluation. He is not interested in anything else. Well, then how economists evaluate how economy is doing: good or bad? They evaluate economy by how productive it is. More output means more income. Growth is a growth of output. Growth of economy means growth of output in economy. Growth of quality of output, quantity of output, all is included in growth. So "economic growth" means, "growth of production of goods and services in economy". So, if you produced 10 chairs last year and produced the same quantity, economically, you are not growing. Even if chair prices doubled this year. That is not a "real" growth that is "inflationary or nominal" growth. But if you produced 10 chairs but these chairs are of better quality and they sell for better price that is growth. The real one. You produced 11 chairs, this is a growth. 13 chairs, you are doing well, 20 chairs, it is called an economic boom. In order to measure how well we are

doing we have economic indicators which we learned in previous chapter: GDP and GDP per capita. We use inflation adjusted GDP (real GDP) to eliminate price factor in calculating economic growth. Otherwise inflation will distort our numbers. Let's see how we calculate real GDP:

Assume we produce only two things in this hypothetical economy, bananas and apples. Price and quantity of production is given for last year:

$P_{b,\ 2019}$=4 TMT

Q_b=3000 Tons

$P_{a,\ 2019}$=3 TMT

Q_a=4000 Tons

Total production last year (GDP_{2019}) = $P_{b,\ 2019} * Q_b + P_{a,\ 2019} * Q_a$= 24000 TMT

Production for this year:

$P_{b,\ 2020}$=5 TMT

Q_b=2800 Tons

$P_{a,\ 2020}$=4 TMT

Q_a=3500 Tons

Total production this year (GDP_{2020}) = $P_{b,\ 2020} * Q_b + P_{a,\ 2020} * Q_a$= 28000 TMT

How much did economy grow? If your answer will be:

$$\frac{28000 - 24000}{24000} * 100\% = 16.7\%$$

Then you are absolutely wrong! Compare quantity of outputs and you will find out that current year's production is lesser than last year's! How come this year's GDP is higher than last year's? The reason is the inflation rates. When we calculated this year's GDP_{2020}, we have calculated it using current year's prices, P_{2020}, which got us wrong results. If we want to find out real growth of GDP, eliminating negative impact of inflation, we must calculate current year's GDP using previous year's prices:

$P_{b,\ 2019}$=4 TMT

$Q_{b,\ 2020}$=2800 Tons

$P_{a,\ 2019}$=3 TMT

$Q_{a,\ 2020}$=3500 Tons

Total production this year (GDP_{2020}) = $P_{b,\ 2020} * Q_b + P_{a,\ 2020} * Q_a$= 21700 TMT

We will find out that economy is actually shrunk!

$$\frac{21700 - 24000}{24000} * 100\% = -9.6\%$$

GDP of current year found using prices of previous years is called a real GDP or inflation adjusted GDP. GDP that was calculated using current year's prices is called nominal GDP.

GDP per capita is used to measure how wealthy the population is, but at the same time GDP per capita measures productivity of labor force. Do you remember the saying about "quality over quantity"? That is exactly true when it comes to productivity of labor. Economy can grow simply by increasing number of labor, but at the same time it can grow by increasing productivity of labor. Take agriculture for example. How can you increase your harvest? Two ways: either by increasing your land, trees or by implementing better techniques and technology (biochemistry, chemicals, or equipment). In economics science, growth of economy due to increase in inputs (labor, land, etc.) is called extensive growth. Economic growth due to increased productivity by using more efficient methods, technology, etc. is called intensive growth. In our modern world borders of countries are well known and well protected. Resources are also known. There might be countries which still need to expand extensively, but majority of countries are focusing on intensive growth. Economists understood it long time ago that simply increasing inputs will bring to short run growth, which will end as soon as inputs are exhausted or simply because of diminishing marginal return of inputs in production function. Assume we have a production function in the form of:

$$Q=AK^{\alpha}L^{\beta}$$

where A is a technology, K is a capital input, L is a labor input, and production function does not have an increasing returns to scale characteristics (thus $\alpha, \beta < 1$). Then, as we increase labor:

$$\frac{\partial Q}{\partial L} = \frac{\beta AK^{\alpha}L^{\beta}}{L} = \frac{\beta}{L}(AK^{\alpha}L^{\beta})$$

$$\frac{\beta}{L}(AK^{\alpha}L^{\beta}) < (AK^{\alpha}L^{\beta})$$

$$\text{Because } \frac{\beta}{L} < 1, \text{ since } \beta < 1$$

As it is seen from above derivation, both inputs (K, L) have diminishing marginal return characteristics (you can try to repeat it with K yourself). **What does diminishing marginal return of labor means? This means, as we increase labor (keeping everything**

unchanged) our output per labor falls. Let's see this in numerical example, assume we have a production function of this form:

$$Q=K^{0.6}L^{0.5}$$

We want to find out how many goods I can produce with L=100 and K=100. Then:

$$Q=K^{0.6}L^{0.5}$$
$$Q = (100)^{0.6}(100)^{0.5}$$
$$Q=158$$

Now, let's calculate how many goods were produced per labor:

$$\text{Output per labor} = \frac{158}{101} = 1.58$$

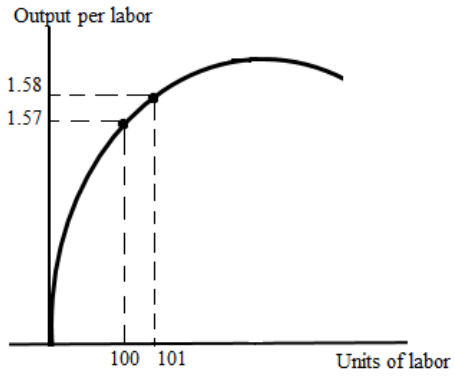
Let's increase our labor input by 1 unit keeping everything constant. Then:

$$Q=K^{0.6}L^{0.5}$$
$$Q = (100)^{0.6}(101)^{0.5}$$
$$Q=159$$

Now, let's calculate how many goods were produced per labor:

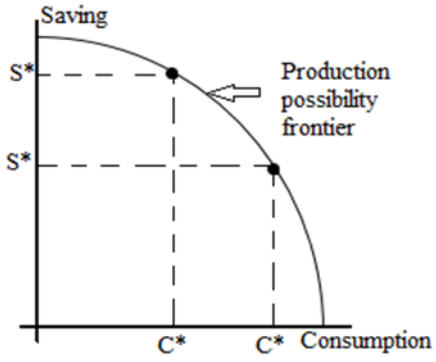
$$\text{Output per labor} = \frac{159}{101} = 1.57$$

As you can see, labor has decreasing marginal return characteristics. Output per labor fell as we increased labor only! Same is also true for capital. Try it yourself!



If nations target to grow in a long run, intensive growth methods must be implemented. So, how can we increase GDP per capita, what methods are there? There are many different ideas and theories about growth, but almost all of them have these in common:

A. Saving (Investment): Saving is a left over portion of income after consumption (remember the first chapter?). Consumption spending is to meet today's needs and savings are for tomorrow. That is true for both households and businesses. Households save for education, new car, new house, travelling, etc. long word short households save for investing for better future. Same is true for businesses. Businesses use profits for expansion, otherwise they must borrow the needed capital. Big savings for households means better life, big savings for businesses means expansion. Savings leads to growth! Saving here, does not mean saving money under the pillow. Saving here (and in economics science) means investment, thus making saved money work for you. Not just sit there (out of money circulation) and lose value due to inflation, but earn some return for owner. **The secret of fast growth and development of Asian economies recently, is the saving habits of its population.** It is in the culture of the most Asian nations to "be satisfied with minimum" (minimalism). While Western culture value today more than tomorrow, they are a **"consumer nation"**. Internal and external debt rates of Western and Eastern economies will prove my words. In poor economies people spend most of their income for consumption goods such as food and energy, while in developed economies basic needs spending (consumption) tends to take smaller portion of total income.



B. Human capital: Simply, human capital is a quality of labor force. Education level, health status, social status, economic status, communication level, and other personal attributes all included in human capital. How can you produce high- tech and extremely high quality goods with lowly educated labor force? If you are struggling with epidemic diseases and infections? No way! **Productivity of labor will grow only if it is given a "right environment" to grow!** You cannot expect a fully tied up person to swim like a fish right? How would he do that? Economic growth depends on growth of human capital. The "right environment" for growth was mentioned in previous chapter when we discussed about HDI. Let's explore it a little bit more:

xix. Peace: First and foremost, condition for any kind of development.

xx. **Laws are respected and implemented:** Having a written law is one thing, how it is implemented is another.

xxi. **Human rights are respected and protected**: Human being must be the most valuable input in any economy. Nothing is more valuable than human being. Labor force is the most valuable asset.

xxii. **Less bureaucracy (zero bureaucracy):** Bureaucracy is the enemy of progress because bureaucracy seeks wealth without a progress! Bureaucrat wants wealth, he does not want changes. Progress demands hard work and it is hectic, hard and risky; that is all what bureaucrat hates. Zero bureaucracy for maximum growth.

xxiii. **Corruption is illegal (fully implemented):** Corruption is children of bureaucracy. The more bureaucracy lives, the more children she will have.

xxiv. **Monopoly is illegal (fully implemented):** Monopoly is a business dictatorship. All said.

xxv. **Education is accessible:** Kindergartens, schools and universities are cradles of civilization. Everything and everyone starts there!

xxvi. **Health services are accessible:** The longer I live, the longer I am beneficial to society. The longer I live, the more I can produce. The longer I live, the more experienced I will be, and I can transfer my experience to the next generation.

xxvii. **Competition in all sectors are encouraged and supported:** Competition is a driving force for everything. We must not be afraid of competition, we must love competition, a healthy competition. Losing and winning is a part of game. A sector without a competition is like a game where everyone wins. It is not possible. In a sector where competition does not exist, everyone is in a loss, except the owner of the sector.

xxviii. **Discrimination is illegal (fully implemented):** There must not be any kind of discrimination, full stop.

xxix. **Government is transparent:** Government is a big teacher. Students learn and implement what they were taught. Transparency is a symbol of honesty! Honesty is the first and the most important lesson for every student because honesty is fundamental principle of any kind of relationship: economic, diplomatic, civil, family, etc.

xxx. **Majority of population belongs to middle class:** Middle class is the trampoline of growth. The size of middle class in economy will show the level of political and economic stability. Society starts to rot from within when middle class vanishes.

xxxi. **Entrepreneurship is encouraged and supported:** It is the brave heart of entrepreneurs broke the borders of “dark minds”. Entrepreneurship soul is the biggest asset for any economic development.

xxxii. **Investment is encouraged, supported and protected:** Investment is the main source of funding of growth. No funding, no growth.

xxxiii. **Financial freedom is protected:** Financial limitations, barriers and all kinds of prohibitions are going to limit transactions. Transactions are logistics of money. Money will not go to the areas where "roads" are missing.

xxxiv. **Property rights are protected**: Property is an extension of its owner. Property rights mean human rights.

xxxv. **Economic freedom is protected:** Economic limitations, barriers, and all kinds of prohibitions are going to limit economic operations. How would economy grow then?

xxxvi. **Trade freedom is protected:** Trade is"energy" of economic growth. Nothing will move without energy, ask Isaac Newton about it.

xxxvii. **Efficient fiscal policy:** Taxes must be progressive: rich must pay more, poor must pay less. Otherwise economic injustice will grow. Injustice means unrest.

xxxviii. **Efficient monetary policy:** Inflation is the biggest enemy of investment and inflation is the only head ache of Central bankers. Controlling inflation is the 99% success of efficient monetary policy.

Top 100 countries by Human Capital Index (%)
Measures maximum of human potential reached

1	Singapore	0.88
2	South Korea	0.84
3	Japan	0.84
4	Hong Kong	0.82
5	Finland	0.81
6	Ireland	0.81
7	Australia	0.8
8	Sweden	0.8
9	Netherlands	0.8
10	Canada	0.8
11	Germany	0.79
12	Austria	0.79
13	Slovenia	0.79
14	Czech Republic	0.78
15	United Kingdom	0.78
16	Portugal	0.78
17	Denmark	0.77
18	Norway	0.77
19	Italy	0.77
20	Switzerland	0.77
21	New Zealand	0.77
22	France	0.76
23	Israel	0.76

24	United States	0.76
25	Macao	0.76
26	Belgium	0.76
27	Serbia	0.76
28	Cyprus	0.75
29	Estonia	0.75
30	Poland	0.75
31	Kazakhstan	0.75
32	Spain	0.74
33	Iceland	0.74
34	Russia	0.73
35	Latvia	0.72
36	Croatia	0.72
37	Lithuania	0.71
38	Hungary	0.7
39	Malta	0.7
40	Slovakia	0.69
41	Luxembourg	0.69
42	Greece	0.69
43	Seychelles	0.68
44	Bulgaria	0.68
45	Chile	0.67
46	China	0.67
47	Bahrain	0.67
48	Vietnam	0.67
49	United Arab Emirates	0.66
50	Ukraine	0.65
51	Mongolia	0.63
52	Mauritius	0.63
53	Turkey	0.63
54	Oman	0.62
55	Malaysia	0.62
56	Albania	0.62
57	Costa Rica	0.62
58	Bosnia and Herzegovina	0.62
59	Montenegro	0.62
60	Qatar	0.61
61	Georgia	0.61
62	Trinidad and Tobago	0.61
63	Argentina	0.61
64	Mexico	0.61
65	Thailand	0.6
66	Ecuador	0.6
67	Romania	0.6
68	Uruguay	0.6
69	Azerbaijan	0.59
70	Colombia	0.59
71	Iran	0.59
72	Peru	0.59

73	Saudi Arabia	0.58
74	Sri Lanka	0.58
75	Moldova	0.58
76	Kyrgyzstan	0.58
77	Kuwait	0.58
78	Armenia	0.57
79	Jordan	0.56
80	Kosovo	0.56
81	Brazil	0.56
82	Palestine	0.55
83	Tuvalu	0.55
84	Philippines	0.55
85	Jamaica	0.54
86	Lebanon	0.54
87	Indonesia	0.53
88	North Macedonia	0.53
89	Tajikistan	0.53
90	Paraguay	0.53
91	Panama	0.53
92	Nicaragua	0.53
93	Algeria	0.52
94	Kenya	0.52
95	Tonga	0.51
96	Tunisia	0.51
97	El Salvador	0.5
98	Morocco	0.5
99	Guyana	0.49
100	Cambodia	0.49

C. **Technology and physical capital:** You cannot produce new things with old tools, can you? Robots, machinery, equipment, software, artificial intellect, etc. these things increase output by many folds. No doubt about that! Physical capital (or just capital) also increases output but as you will see below, it has diminishing marginal return characteristics. Simply increasing physical capital will not increase output per capital continuously. Technological advancement is possible only if human capital is high! Only well educated person can come up with new ideas to solve issues. Come up with new methods, new ways, and new technology of accomplishing things. Even though many economics literatures include technology and physical capital as a different factor of growth, I strongly believe that there is only one single factor: **Human capital!** All other factors are derived from human capital. Technology is derived (born, created) from human capital!

Growth is one of the oldest topics of economics science and boggled minds of enlightened people ever since. Macroeconomics, Developmental economics, Growth economics are one of major branches of economics which almost solely researches and tries to understand in tiny detail what is growth and how to achieve it. Economists construct tiny version of our economy which is called economic models and try to grasp the dynamics of growth in there. Economic models cannot answer to all questions of economists but it gives a picture, at least some picture about what is going on in real economy. There are many models, here: we will study the fundamental one, Solow's growth model. I can say that this model is forefather of all recent models. As a Shakespeare, Tolstoy, Magtymguly Pyragy, Solow's growth model is also one of "classics" of economics science.

Solow's growth model

Solow's growth model was developed by Robert Solow, American economist in the midst of twentieth century. This is a classic exogenous growth model, fundamental for beginner macroeconomics. "Exogenous" because many variables such as savings rate, population growth rate and technological advancement rates in models taken as "given" variables. Thus, model does not explain them; it just takes them as it is. Even though this model has many weaknesses, it was the first macro model that could mathematically explain intuitive economic believes such as "savings/investment is clue for growth" and others. This is fundamental macroeconomic model which is still not losing its actuality and taught in top economic schools.

Macroeconomics has evolved ever since; there are many types of macro models which helps economists understanding economic growth. "Infinite horizons model", "Overlapping generations model" and "Endogenous growth models" are the main types of macroeconomic growth models which are widely taught and learned today. These topics are for advanced level of macroeconomics and demand huge analytical and quantitative skills, in addition to knowledge of data analysis and econometrics.

Assumptions of the model:

1) **We use Cobb-Douglas; output function is used in the model and it has constant returns to scale characteristics.**

$$Y_t = F(K_t, AL_t)$$

Y is output, **K** is capital, **A** represents technology and knowledge, and **L** represents labor force in an economy at a time ***t***. Then:

$$F(K_t, AL_t) = K^{\propto} AL^{1-\propto}$$

Due to the constant return to scale characteristics of output function, we can:

$$\frac{Y}{AL} = F\left(\frac{K}{AL}, \frac{AL}{AL}\right) = \left(\frac{K}{AL}\right)^{\propto} \left(\frac{AL}{AL}\right)^{1-\propto}$$

$$y = F(k,1) = (k)^{\propto}(1)^{1-\propto}$$

$$y = F(k) = k^{\propto}$$

$\frac{\boldsymbol{Y}}{\boldsymbol{AL}}$ is output per labor; we will define $\frac{\boldsymbol{Y}}{\boldsymbol{AL}}$ = y,
$\frac{\boldsymbol{K}}{\boldsymbol{AL}}$ is capital per unit of labor; we will define $\frac{\boldsymbol{K}}{\boldsymbol{AL}}$ = k

2) **Second assumption is labor and technology grow at constant rate;** for example, population growth is 2%, then *n=0.02*, technology grows by 3%, then *g=0.03*.

$$\frac{\partial L_t}{\partial t} = n(L_t) \qquad \frac{\partial A_t}{\partial t} = g(A_t)$$

3) **Third assumption is that capital depreciates by constant rate of (δ) and constant investment rate (saving rate) is (θ):**

$$\frac{\partial K_t}{\partial t} = \theta Y_t - \delta K_t$$

Capital growth depends on savings rate and depreciation rate. One unit of output spared for investment creates one unit of capital. You can understand above formula this way: output is income (Y), so the portion of income you invest and other portion you consume. Capital depreciates, loses value. Example: equipment is capital and it gets wears and tears due to constant usage. Building is also a capital, and it gets old if not maintained time to time. Money is also a capital and it also loses value due to inflation. If depreciation rate is high, capital will lose value faster. Example: high inflation rate will lose value of your money faster; often usage will olden the equipment faster; building which was used by a lot of people will lose its new appearance faster. Assume δ = 0.2, this means that capital depreciation rate is 20%. So if initial value of capital was 1000 TMT, a year later capital will lose its 20% value and will be 800 TMT this year. θ represents the portion of income saved. $\theta = 0.3$ means that 30% of income was saved and invested to create new capital in future. $\frac{\partial K_t}{\partial t}$ represents change in capital stock. Capital stock will depend on two things, the portion of income you save and invest

and also to depreciation rate of capital. Intuitively we can say that our capital stock will decrease ($\frac{\partial K_t}{\partial t} < 0$) if our investment is lesser than depreciated capital.

Exercise 1: *Assume this, we have a 1000 TMT and inflation rate is 10%. What will happen to my capital stock (to my wealth) if I invest this money for 5% return?*

My capital stock (change in capital or $\frac{\partial K_t}{\partial t}$) = 0.05*1000-0.1*1000= -50

My capital stock will decrease by 50 TMT. Now my total capital stock is not a 1000 TMT anymore, but it is 950 TMT.

Exercise 2: *Company bought new equipment for 1000000 TMT and annual depreciation rate of this equipment is 10%. What is the value of this equipment after 8 years?*

$\frac{\partial K_t}{\partial t}$ =1000000-(0.1*1000000) *8=200 000 *(linear depreciation)*

Or 1000000*(1-0.1)^8= 430 467.21 *(non-linear depreciation)*

Exercise 3: *Company increased its capital stock by 1000 TMT this year. Company's revenue is 20000 TMT and its savings rate is 20%. What is the value of its total capital if annual depreciation rate of its capital is 15%?*

$\frac{\partial K_t}{\partial t}$=1000

1000=20%*20000-k*15%

1000-4000=-k*15%

k=20000

Exercise 4: *Turtleland has a population of 1000000 people. If annual population growth rate of the country is constant 3%, how many years it will take for Turtleland to reach 2000000 population size?*

2000000=1000000*(1+0.03) ^{n}

$\frac{2000000}{1000000} = (1 + 0.03)^n$

ln (2) = n ln (1+0.03)

$$\frac{ln2}{ln1.03} = n$$

n=23 years

4) **All exogenous constant variables δ, θ, n, g > 0**
5) **There is no government**
6) **Single good is produced in economy**

Let's focus on $\frac{K}{AL}$= k, capital stock per unit of labor. How will capital stock per labor change over time?

$$\frac{\partial \mathrm{k}}{\partial \mathrm{t}} = \frac{\left(\frac{\partial \mathrm{K}}{\partial \mathrm{t}}\right)(\mathrm{AL}) - \left(\frac{\partial \mathrm{AL}}{\partial \mathrm{t}}\right)\mathrm{K}}{(\mathrm{AL})^2} = \frac{\left(\frac{\partial \mathrm{K}}{\partial \mathrm{t}}\right)(\mathrm{AL}) - \left(\frac{\partial \mathrm{A}}{\partial \mathrm{t}}(\mathrm{L}) + \frac{\partial \mathrm{L}}{\partial \mathrm{t}}(\mathrm{A})\right)\mathrm{K}}{(\mathrm{AL})^2}$$

We know that ($\frac{\partial K_t}{\partial \mathrm{t}} = \theta Y_t - \delta K_t$) from third assumption!

We also know that ($\frac{\partial L_t}{\partial t} = n(L_t)$) and ($\frac{\partial A_t}{\partial t} = g(A_t)$) from second assumption!

Then, just put this into the original function:

$$\frac{\partial \mathrm{k}}{\partial \mathrm{t}} = \frac{(\theta \mathrm{Y} - \delta \mathrm{K})(\mathrm{AL}) - (\mathrm{g}(\mathrm{A})(\mathrm{L}) + \mathrm{n}(\mathrm{L})(\mathrm{A}))\mathrm{K}}{(\mathrm{AL})^2}$$

Now, it is turn for ($\frac{K}{AL} = k$) we can rewrite this as k(AL)=K.
Put this into our original equation:

$$\frac{\partial \mathrm{k}}{\partial \mathrm{t}} = \frac{(\theta \mathrm{Y} - \delta \mathrm{k}(\mathrm{AL}))(\mathrm{AL}) - (\mathrm{g}(\mathrm{A})(\mathrm{L}) + \mathrm{n}(\mathrm{L})(\mathrm{A}))\mathrm{k}(\mathrm{AL})}{(\mathrm{AL})^2}$$

We can simplify this into:

$$\frac{\partial \mathrm{k}}{\partial \mathrm{t}} = \frac{(\theta \mathrm{Y})(\mathrm{AL})}{(\mathrm{AL})^2} - \frac{(\delta \mathrm{k})(\mathrm{AL})(\mathrm{AL})}{(\mathrm{AL})^2} - \frac{\mathrm{gk}(\mathrm{AL})(\mathrm{AL})}{(\mathrm{AL})^2} - \frac{\mathrm{nk}(\mathrm{AL})(\mathrm{AL})}{(\mathrm{AL})^2}$$

After cancellations:

$$\frac{\partial \mathrm{k}}{\partial \mathrm{t}} = \frac{(\theta \mathrm{Y})}{(\mathrm{AL})} - \delta \mathrm{k} - \mathrm{gk} - \mathrm{nk}$$

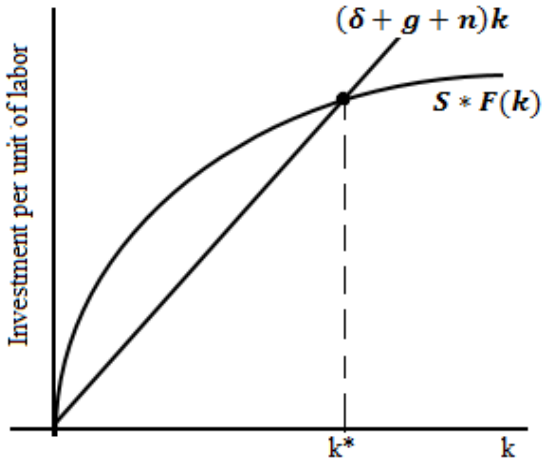
$$\frac{\partial k}{\partial t} = \frac{(\theta Y)}{(AL)} - (\delta + g + n)k$$

Finally, from first assumption we know that:

$$\frac{Y}{AL} = y = F(k)$$

Replace it in original equation:

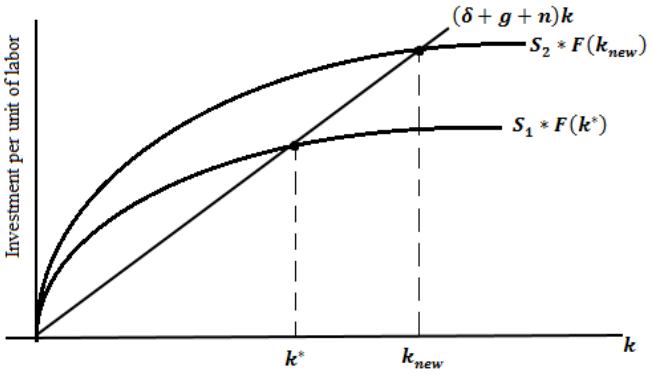
$$\frac{\partial k}{\partial t} = \theta F(k) - (\delta + g + n)k$$



This is the main findings of Solow's growth theorem! Let's explore it now and see what this equation tells us:

$$\theta F(k) > (\delta + g + n)k \quad \text{Then } \frac{\partial k}{\partial t} > 0$$

Which means: In order to increase capital stock per capita in economy, investment must be bigger than total population growth (g+n) and depreciation (δ). When investment is lower than population growth and depreciation, economy will witness meltdown of capital stock per capita.



Balanced growth occurs (steady state) when:

$$\boldsymbol{\theta F(k) = (\delta + g + n)k} \qquad \textbf{Then } \boldsymbol{\frac{\partial k}{\partial t} = 0}$$

When investment is equal to total population growth and depreciation steady state occurs: Investment covers all expenses and capital stock per capita does not grow. What is a maximum of $\boldsymbol{\frac{\partial k}{\partial t}}$? Using well known *first order condition for maxima and minima:*

$$\frac{(\frac{\partial k}{\partial t})}{\partial t} = 0$$

We know that from first assumption:

$$y=F(k)= k^{\propto}$$

Then, replace it to original function:

$$\frac{\partial k}{\partial t} = \theta F(k) -(\delta + g + n)k$$

$$\frac{\partial k}{\partial t} = \theta k^{\propto} -(\delta + g + n)k$$

Then, *first order condition for finding maximum and minimum:*

$$\boldsymbol{\frac{(\frac{\partial k}{\partial t})}{\partial t} = \theta \propto k^{\propto -1} - (\delta + g + n)k = 0}$$

Solve it:

$$\theta \propto k^{\propto -1} - (\delta + g + n)k = 0$$
$$\theta \propto k^{\propto -1} = (\delta + g + n)k$$

Prove that it is a maximum; *second order condition must be:*

$$\boldsymbol{\left(\frac{\frac{(\frac{\partial k}{\partial t})}{\partial k}}{\frac{\partial k}{\partial k}}\right) < 0}$$

Then:

$$\frac{(\frac{\partial k}{\partial t})}{\partial t} = \theta \propto k^{\propto -1} - (\delta + g + n)k = 0$$

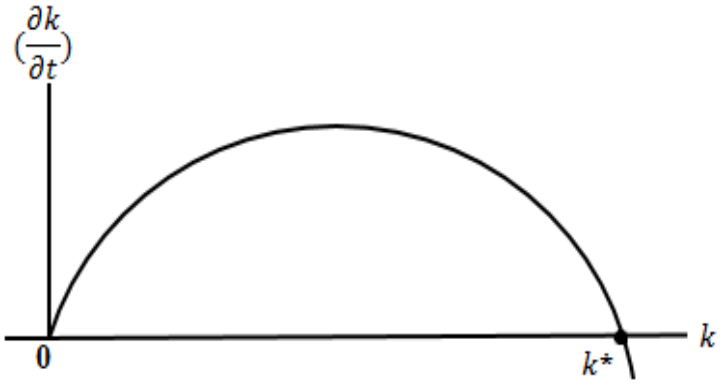
Second derivative:

$$\theta \propto (\propto -1)k^{\propto -2}$$

We know that $0<\propto<1$ from first, constant returns to scale assumption. Then $(\propto -1) < 0$ which makes all function less than zero:

$$\theta \propto (\propto -1)k^{\propto -2} < 0$$

This satisfies our second order condition!



At steady state, we have maximum capital per capita! In this kind of situation, steady state, only technological advancement can push economy for further growth. At least that is what Dr. Solow thinks and it is very hard to disagree with him. Thank you Dr. Solow!

Exercise 5: *Assume you have constant returns to scale production function in the form of:*

$$Y = F(K, L) = K^{\propto} L^{1-\propto}$$

$$Y = K^{0.7} L^{0.3}$$

Also, after calculations it was found that optimal combination of labor and capital is L=2K, (L=1000 and K=500). Then total output is:

$$Y = K^{0.7} L^{0.3}$$

$$Y = (500)^{0.7}(1000)^{0.3} = 615$$

Total output is 615. Now, let's find capital per labor $\frac{K}{L} = k$ *and output per labor* $\frac{Y}{L} = y$*:*

$$\frac{\mathrm{K}}{\mathrm{L}} = \mathrm{k} = \frac{500}{1000} = 0.5$$

$$\frac{\mathrm{Y}}{\mathrm{L}} = \mathrm{y} = \frac{615}{1000} = 0.615$$

Assume also that depreciation rate is δ *= 5% and labor grows with constant rate of* n*= 2% annually. What must be saving rate* θ *in this economy to keep steady state condition* $\theta F(k) = (\delta + g + n)k$*? Let's solve this using what we have learned above:*

Steady state in our case: $\theta F(k) = (\delta + n)k$

Put all findings into the function:

$$\theta F(k) = (\delta + g + n)k$$

$$\theta(0.615) = (0.05 + 0.02)(0.615)$$

$$\theta(0.615) = (0.035)$$

$$\theta = 0.057$$

To keep economy at steady state, savings rate (investment rate) must be approximately 5.7%.

Below is table of countries by their GDP growth rates (%)

1	South Sudan	11.3
2	Rwanda	10.1
3	Libya	9.9
4	Dominica	9.2
5	Ethiopia	9.0
6	Bangladesh	7.9
7	Armenia	7.6
8	Tajikistan	7.5
9	Djibouti	7.5
10	Nepal	7.1
11	Cambodia	7
12	Vietnam	7
13	Ivory Coast	6.9
14	Myanmar	6.5

15	Benin	6.4
16	Turkmenistan	6.3
17	Tanzania	6.3
18	Ghana	6.1
19	China	6.1
20	Tuvalu	6.0
21	The Gambia	6.0
22	Philippines	5.9
23	Mauritania	5.9
24	Niger	5.8
25	Burkina Faso	5.7
26	Maldives	5.7
27	Guinea	5.6
28	Kenya	5.6
29	Uzbekistan	5.6
30	Egypt	5.6
31	Ireland	5.5
32	Cape Verde	5.5
33	Togo	5.3
34	Senegal	5.3
35	Antigua and Barbuda	5.3
36	Georgia	5.1
37	Sierra Leone	5.1
38	Mali	5.1
39	Mongolia	5.1
40	Dominican Republic	5.1
41	Papua New Guinea	5.0
42	Indonesia	5.0
43	Hungary	4.9
44	Uganda	4.9
45	Madagascar	4.8
46	Laos	4.7
47	Guyana	4.7
48	Guinea-Bissau	4.6
49	Kazakhstan	4.5
50	Malawi	4.5
51	Kyrgyzstan	4.5
52	Malta	4.4
53	Democratic Republic of the Congo	4.4
54	Estonia	4.3
55	Malaysia	4.3
56	India	4.2
57	Serbia	4.2
58	Poland	4.1
59	Romania	4.1
60	Kosovo	4.0
61	Iraq	3.9

62	Lithuania	3.9
63	Seychelles	3.9
64	Brunei	3.9
65	Eritrea	3.8
66	Cameroon	3.7
67	Montenegro	3.6
68	Moldova	3.6
69	Guatemala	3.6
70	North Macedonia	3.6
71	Israel	3.5
72	Mauritius	3.5
73	American Samoa	3.5
74	Gabon	3.4
75	Bulgaria	3.4
76	Colombia	3.3
77	Pakistan	3.3
78	Cyprus	3.2
79	Ukraine	3.2
80	Grenada	3.1
81	Timor Leste	3.1
82	Afghanistan	3.0
83	Panama	3.0
84	Central African Republic	3.0
85	Chad	3.0
86	Botswana	3.0
87	Croatia	2.9
88	Vanuatu	2.9
89	Saint Kitts and Nevis	2.9
90	Somalia	2.9
91	Bolivia	2.8
92	Taiwan	2.7
93	Bosnia and Herzegovina	2.7
94	Honduras	2.7
95	Czech Republic	2.6
96	Slovenia	2.4
97	Marshall Islands	2.4
98	El Salvador	2.4
99	Thailand	2.4
100	Denmark	2.4
101	United States	2.3
102	Suriname	2.3

Example 6: *India's GDP is approximately 2.8 Trillion USD while GDP of Vietnam is 0.26 Trillion USD. If we assume that both countries grow with the same rate, how long will it take Vietnam to reach the same economic size of India?*

India's economy grows with rate of 4.2% (from above table) and Vietnam's economy grows with 7% and rates are constant. A question asks us to find:

$$GDP_{India}\,(1+0.042)^{t} = GDP_{Vietnam}\,(1+0.07)^{t}$$

$$2.8\,(1.042)^{t} = 0.26\,(1.07)^{t}$$

$$\frac{(1.07)^{t}}{(1.042)^{t}} = \frac{2.8}{0.26}$$

$$(1.0269)^{t} = 10.77$$

Take ln of both sides:

$$\ln\,(1.0269)^{t} = \ln\,(10.77)$$

$$t\,(\ln\,(1.0269)) = 2.377$$

$$t\,(0.0265) = 2.377$$

$$t = 89.7$$

It will take approximately 89.7 years for Vietnam to reach the same economic size of India! What will be their size of economy at that time? Their size of economy will be:

$$GDP_{India}\,(1+0.042)^{t} = 2.8\,(1.042)^{89.7} = 112 \text{ Trillion USD}$$

$$GDP_{Vietnam}\,(1+0.07)^{t} = 0.26\,(1.07)^{89.7} = 112 \text{ Trillion USD}$$

Homework:

I. Cobb-Douglass production function economy with constant returns to scale characteristics is producing 1000 units of goods. $\frac{K}{L}$ =0.25 and population + depreciation rate is 10%. What must be investment rate in this economy to keep capital per capita at steady state? $Q=K^{0.2}L^{0.8}$

II. When India's GDP will reach 10 Trillion USD?

III. If both economies grow at constant current rate, can Nigeria catch up with China one day?

IV. Developing economies grow faster than developed economies. Why do you think is that?

V. $2000= (3000)^{\alpha} (1000)^{1-\alpha}$ find α?

VI. In above Exercise V, assume L=3000, K=1000, labor and depreciation rate is the same 4%. What must be investment rate to keep capital per capita at steady state?

VII. Calculate GDP_{2019}; calculate nominal GDP_{2020} and real GDP_{2020} using table of prices and quantities below.

P_{2019}	Q_{2019}	P_{2020}	Q_{2020}
7000	150000	7050	160000
96	2000000	100	1900000
6	400000	5	500000
30	2500000	27	3000000
1000	560040	1100	500000
560	350000	550	380000
10	70000000	11	70000000
850	700700	820	800000
200	505050	220	500000
24	9000000	25	8000000
45000	50600	45500	40000
77	440000	80	500000
300	200000	280	150000
30000	5200000	31000	5000000
4500	340	4400	400
8800	70000	9000	70000
95	840000	100	900000
110	9200000	100	9000000

VIII. Which country will reach economic size faster: Pakistan or Kazakhstan to 1 Trillion GDP?

IX. $3000 = (5000)^{\alpha}(2000)^{1-\alpha}$, L=5000 and K=2000, find α and show that labor and capital have diminishing returns characteristics.

X. Use function above and if depreciation rate of capital is 3% and labor grows with constant rate of 6%, what must be the investment rate so that economy can increase capital stock by 200?

XI. Can Suriname catch up China one day if they continue growing with the same rate? Show it.

XII. Switzerland's GDP per capita is way above Nigeria's. If we assume that Nigeria's population will grow with constant rate of 1% and economy will grow by 3%, how long will it take for Nigeria to reach the wealthy level of Switzerland? (GDP per capita)?

XIII. Growth rate of economy slows down as economy grows up. Why is that?

XIV. According to Human Capital Index, Ireland is well above the Mexico. What might be reasons?

XV. Luxemburg is very wealthy city state (tiny country). But still, 41st on Human Development Index. What might be the reasons?

XVI. China is the second largest economy in the world. First is USA. If they both grow with the same rate, how long will it take China to catch up with USA?

XVII. How education impacts the growth, positive or negative? Why? Take for example top 100 universities in the world and see where they are located. In poor or wealthy countries?

XVIII. Competition is vital for any growth. Give example of your first competition. Some people cannot compete, they cannot lose, they afraid to lose. How this behavior could be won?

XIX. $\frac{Y}{L} = 0.7$ and $\frac{K}{L} = 0.4$ and labor and depreciation grows with constant rates. At investment rate of 9% stock of capital per capita remains in steady state. What is combined growth rate of labor and depreciation?

XX. Total output (Q) is 1000 and capital (K) is 2000. What must be the investment rate, labor growth rate and depreciation rate so that stock of capital per capita remains at steady state?

XXI. In above question $1000= (2000)^{\alpha}(L)^{1-\alpha}$ has constant returns to scale characteristics. If $\alpha=0.2$, then find Labor.

XXII. Inputs of an economy are L=3000, K=2000, and production function has a form of $Q= (3000)^{0.2} (2000)^{0.8}$. Population grows with 2% and depreciation rate is 10%. Find current investment rate if you know that economy is in steady state.

XXIII. What will happen to stock of capital per capita if depreciation rate jumps to 15% while all other things remain constant? Show impact on graph

XXIV. What will happen to stock of capital per capita if investment rate doubles from above question?

XXV. Production function has increasing marginal return to both capital and labor. Prove it. $Q=(K)^{1.3}(L)^{1.1}$

XXVI. Production function has constant returns to scale characteristics and L=K=Q=1000. If depreciation and labor growth rates are equal. What kind of mathematical relationship must investment rate, depreciation and labor growth rates have to keep stock of capital per capita at steady state?

XXVII. Imagine a country where labor growth is controlled and is not allowed, thus labor growth rate is 0. What must be investment rate and depreciation rate to keep the wealth levels at steady state?

XXVIII. What is the difference between growth and sustainable growth?

XXIX. Capital has a constant depreciation rate of 10% of initial purchased price. If capital was bought for 50000TMT when the value of the capital will be zero?

XXX. Does cultural, historical and religious background of country play role in economic growth? Give examples.

Solutions:

I. $Q=K^{0.2}L^{0.8}$

$\frac{K}{L}=0.25$

$Q=1000$

$\delta + g = 10\%$

$K = 0.25L$

$1000 = (0.25L)^{0.2}L^{0.8}$

$1000 = (0.25)^{0.2}L$

$\frac{1000}{0.758} = L$

$L=1319$

$K=1319*0.25$

$K=330$

$k = \left(\frac{K}{L}\right) = \frac{330}{1319} = 0.25$

$y=\left(\frac{Y}{L}\right) = \frac{1000}{1319} = 0.758$

$\boldsymbol{\theta F(k) = (\delta + g + n)k}$

$\theta(0.758) = (10\%)*0.25$

$\theta = \frac{0.025}{0.758} = 0.033$ or 3.3%

To keep economy at steady state, investment rate (saving rate) must be approximately 3.3%.

II. Current GDP of India is about 3 Trillion USD, and growth rate is 4.2% (given in above table):

$10 \text{ Trillion}=3 \text{ Trillion}*(1+0.042)^{n}$

$\frac{10}{3} = (1+0.042)^{n}$

$\ln \frac{10}{3} = n \ln (1+0.042)$

$n = \frac{\ln \frac{10}{3}}{\ln (1+0.042)} = 29.3$

n = 29.3 years

III. GDP_{China}=13 Trillion USD

Growth rate of China=6.6 %

$GDP_{Nigeria}$=397 BLN USD

Growth rate of Nigeria=2%

$1300*(1+0.066)^{n} = 397*(1+0.02)^{n}$

$\ln \left(\frac{1300}{397}\right) = n \ln \left(\frac{1.02}{1.066}\right)$

$n = \frac{\ln \left(\frac{1300}{397}\right)}{\ln \left(\frac{1.02}{1.066}\right)} = -27$

n = -27 (there is no negative years in real world).

This shows that it will never happen! Because China will continue growing. Nigeria can catch up China only in one case: if China stops growing.

IV. Speed of growth is high until economies reach efficiency level. Developed economies have more efficient economy than developing nations. Economic growth is like growth of living organism. After certain size, organism slows down the growth. Organism stops growing when it reaches maximum of genetic potential. Economies are similar; growth slows down when potential efficiency is reached.

V. $2000= (3000)^{\alpha} (1000)^{1-\alpha}$
$\ln 2000=\boldsymbol{\alpha} \ln 3000+(1-\boldsymbol{\alpha}) \ln 1000$
$\ln 2000=\boldsymbol{\alpha} \ln 3000+\ln 1000-\boldsymbol{\alpha}\ln 1000$
$\ln 2000-\ln 1000=\boldsymbol{\alpha}\ln 3000-\boldsymbol{\alpha}\ln 1000$
$0.693=\boldsymbol{\alpha}1.099$
$\boldsymbol{\alpha}=0.631$

VI. K=1000
L=3000
$k=(\frac{K}{L})= \frac{1000}{3000} = 0.33$
$y=(\frac{Y}{L})= \frac{2000}{3000} = 0.667$
$\boldsymbol{\theta F(k) = (\delta+g+n)k}$
$\theta(0.667) = (4\%)*0.33$
$\theta = \frac{0.0132}{0.0667} = 0.0198$

To keep economy at steady state, investment rate (saving rate) must be approximately 1.98%

VII. $GDP_{2019}=P_{2019}*Q_{2019}=$
7000*150000+96*2000000+6*400000+30*2500000+
1000*560040+560*350000+10*70000000+
850*700700+200*505050+24*900000+45000*50600+
77*440000+300*200000+30000*5200000+4500*340+
8800*70000+95*840000+110*9200000=
163 768 255 000 TMT

Nominal $GDP_{2020}=P_{2020}*Q_{2020}=$
7050*160000+100*190000+5*500000+27*3000000+
1100*500000+550*380000+11*70000000+820*800000+
220*500000+25*8000000+45500*40000+
80*500000+280*150000+31000*5000000+4400*400+
9000*70000+100*900000+100*
9000000= 162 420 260 000 TMT

Real $GDP_{2020}=P_{2019}*Q_{2020}$=7000*160000+
96*1900000+6*500000+30*3000000+1000*500000+
560*380000+10*70000000+850*800000+200*500000+
24*8000000+45000*40000+77*500000+300*150000+
30000*5000000+4500*400+8800*70000+95*900000+
110*9000000= 156 237 000 000 TMT

VIII. $GDP_{Pakistan}$ (2019) =284 BLN USD
Growth rate of Pakistan = 3.3%

1000 BLN =284 BLN *$(1+0.033)^{n}$

$\ln\left(\frac{1000}{284}\right)$= n ln (1.033)

$n = \frac{\ln\left(\frac{1000}{284}\right)}{\ln(1.033)}$

n = 38.8

$GDP_{Kazakhstan}$ (2019) = 180.16 BLN
Growth rate of Kazakhstan =4.5%

1000 BLN=180.16 BLN *$(1+0.045)^{n}$

$\ln\left(\frac{1000}{180.16}\right)$= n ln (1.045)

$n = \frac{\ln\left(\frac{1000}{180.16}\right)}{\ln(1.045)}$

n = 39

Pakistan will reach 1 Trillion GDP a bit faster than Kazakhstan.

IX. $3000= (5000)^{\alpha} (2000)^{1-\alpha}$, L=5000 and K=2000

ln 3000= $\boldsymbol{\alpha}$ ln 5000+ (1-$\boldsymbol{\alpha}$) ln 2000

ln 3000=$\boldsymbol{\alpha}$ln 5000+ln 2000-$\boldsymbol{\alpha}$ln 2000

ln 3000-ln 2000=$\boldsymbol{\alpha}$ (ln 5000-ln 2000)

$\frac{\ln 3000-\ln 2000}{(\ln 5000-\ln 2000)} = \boldsymbol{\alpha} = 0.4425$

$\frac{Y}{L} = \frac{3000}{5000} = 0.6$ as we increase our labor for 10 units we get:

$Q= (5010)^{0.4425}(2000)^{1-0.4425}$=3003

Output per labor = $\frac{3003}{5010}$=0.599

$\frac{Y}{L} = \frac{3000}{2000} = 1.5$ as we increase our capital for 10 units we get:

$Q = (5000)^{0.4425}(2010)^{1-0.4425} = 3008$

Output per labor= $\frac{3008}{2010}$=1.496

X. δ =3% and n=6%

$k=(\frac{K}{L})=\frac{2000}{5000}=0.4$

if we want to increase capital stock by 200, our Q=3200

$y=(\frac{Y}{L})=\frac{3200}{3000}=0.64$

$\boldsymbol{\theta F(k) = (\delta + g + n)k}$

$\theta(0.64) = 0.03 + 0.06)*0.4$

$\theta = \frac{0.036}{0.64} = 0.056$

Investment rate must approximately 5.6% to increase capital stock by 200

XI. $GDP_{Suriname}$= 3.5 BLN USD
Growth rate of Suriname=2%
GDP_{China}= 13 Trillion USD (1300 BLN USD)
Growth rate of China =6.6%

$1300\ BLN*(1+0.066)^{n}=3.5BLN*(1+0.02)^{n}$

$\ln\ (\frac{1300}{3.5})= n \ln (\frac{1.02}{1.066})$

$n = \frac{\ln\ (\frac{1300}{3.5})}{\ln (\frac{1.02}{1.066})}$

$5.917= -004n$

$n = \frac{5.917}{-0.04} = -148$

In real world there is no negative year! This shows us that Suriname can never catch up China even if they grow with the same rate.

XII. Switzerland 's $GDP_{per\ capita}$ =82796 USD
Nigeria 's GDP=397.3 BLN USD
Population of Nigeria=195 900 000
If we want our GDP per capita to be equal to Switzerland 's then:

$82796 = \frac{397\ 300\ 000\ 000*(1+0.03)^n}{195\ 900\ 000*(1+0.01)^n}$

Simplify into: $82796 = \frac{397\ 300\ 000\ 000}{195\ 900\ 000} * \frac{(1+0.03)^n}{(1+0.01)^n}$

$82796 = 2028.075 * \frac{(1+0.03)^n}{(1+0.01)^n}$

$\frac{82796}{2028.075} = \frac{(1+0.03)^n}{(1+0.01)^n}$

$ln\ (\frac{82796}{2028.075}) = n\ ln\ (\frac{1+0.03}{1+0.01})$

$n = \frac{ln\ (\frac{82796}{2028.075})}{ln\ (\frac{1+0.03}{1+0.01})} = 189.17$ years

XIII. Speed of growth is high until economies reach efficiency level. Developed economies have more efficient economy

than developing nations. Economic growth is like growth of living organism. After certain size, organism slows down the growth. Organism stops growing when it reaches maximum of genetic potential. Economies are similar; growth slows down when potential efficiency is reached.

XIV. Human capital index includes many variables (as it was mentioned above in the chapter) according to which human capital is measured, such as economic freedom, bureaucracy, discrimination, peace and many others. If we measure according to these indexes separately Mexico's is much lower than Ireland 's which shows us indicators below:

Indexes	Mexico	Ireland
Economic freedom	66%	80.90%
Inflation (monetary policy)	3.64%	0.94%
Rule of law index	45%	90%
Corruption perception index	29%	74%
HDI	76%	94.20%

XV. Wealth does not guarantee high human development index. 3 indicators are used to calculate HDI, one of them is related with wealth while other 2 are related with social indicator, such as health and education.

XVI. GDP_{China}=13 Trillion USD
Growth rate of China 6.1%
GDP_{USA}=21 Trillion USD
Growth rate=1.7%

$$ln\left(\frac{21}{13}\right) = n\ ln\left(\frac{1.061}{1.017}\right)$$

$$n = \frac{ln\left(\frac{21}{13}\right)}{ln\left(\frac{1.061}{1.017}\right)} = 11.3$$

If 11 years China will grow at constant rate 6.1% and USA will grow at constant rate 1.7%, China will catch up USA.

XVII. Majority of top 100 universities locate in USA, Great Britain, Germany, and Australia where the human capital is one of the highest! Education affects growth only positively, otherwise how an uneducated person/people can produce high quality goods, more productive labor force. Furthermore, as was said above in the chapter, human capital is one of the main components of long run growth.

XVIII. Competition in between the siblings for parent's attention is first competition for any person.

XIX. $k=(\frac{K}{L}) = 0.4$

$y=(\frac{Y}{L}) = 0.7$

$\boldsymbol{\theta F(k) = (\delta + g + n)k}$

$0.09 * (0.7) = (\delta + n)*0.4$

$\frac{0.063}{0.4} = \delta + n$

$\delta + n = 0.1575$

$\delta + n = 15.75\%$

XX. $Q = 1000$

$K = 2000$

$y = (\frac{Y}{L}) = \frac{1000}{L} = x$

$k = (\frac{K}{L}) = \frac{2000}{L} = 2x$

$\frac{1000}{L} = \frac{2000}{L}$

$x=2x$

To keep economy at steady state, investment rate (saving rate) must be 2 times higher than population growth.

XXI. $1000= (2000)^{\alpha}(L)^{1-\alpha}$

$\alpha = 0.2$

$1000= (2000)^{0.2}(L)^{1-0.2}$

$L^{0.8} = \frac{1000}{2000^{0.2}} = 218.67$

$(L^{0.8})^{1.25}=(218.67)^{1.25}$

$L = 841$

XXII. $L=3000, K=2000$

$Q= (3000)^{0.2}(2000)^{0.8}$

$k=(\frac{K}{L})= \frac{2000}{3000} =0.67$

$y=(\frac{Y}{L})= \frac{2168.9}{3000} =0.72$

$\boldsymbol{\theta F(k) = (\delta + g + n)k}$

$\delta = 10\%$

$g = 2\%$

$\theta F(0.72) = (0.1 + 0.02)*0.67$

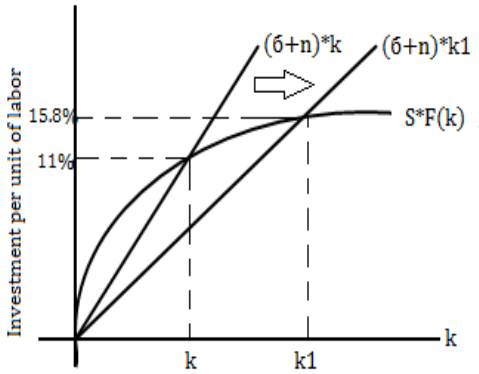
$\theta = \frac{0.0804}{0.72}$ =0.11 or 11%

XXIII. δ = 10%

θF(0.72) = (0.15 + 0.02)*0.67

θ = $\frac{0.1139}{0.72}$ =0.158 or 15.8%

Investment (saving) rate must increase to 15.8% in order to keep the wealth level at steady state.



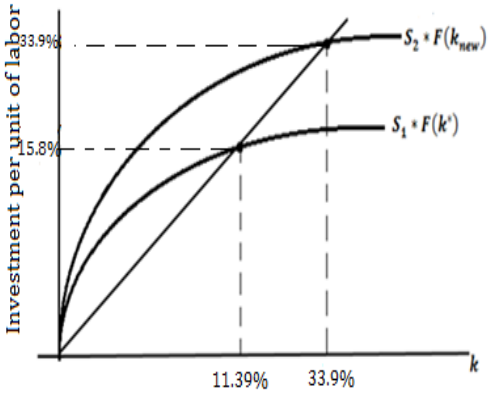
XXIV. If investment rate doubles:

θF(0.72) = (**δ + n**)*0.67

(2*0.158)*0.72 = (**δ + n**)*0.67

δ + n = $\frac{0.22752}{0.67}$ = 0.339 or 33.9%

If investment rate doubles, population growth must be 33.9% to keep wealth at steady state.



XXV. $Q=(K)^{1.3}(L)^{1.1}$

Let's see what will be quantity of output if K=100 and L=100

$Q= (100)^{1.3}(100)^{1.1}=63096$

My return on labor and capital is:

$\frac{Q}{L} = \frac{63096}{100}$ = 630.96

$\frac{Q}{K} = \frac{63096}{100}$ = 630.96

What if I double my labor input:
Q $(200)^{1.3}(100)^{1.1}$=155360
Output per labor: = $\frac{155360}{200}$ = 776.8
What if I double my capital input:
Q $(100)^{1.3}(200)^{1.1}$=135249
Output per capital: = $\frac{135249}{200}$ = 676.2
Even if I double my inputs, marginal return on labor and capital increases even more than twice! This is because powers of L and K, $\boldsymbol{\alpha}$; $\boldsymbol{\beta}$>1.

XXVI. L=K=Q=1000
$y=(\frac{Y}{L})=\frac{1000}{1000}=1$

$k=(\frac{K}{L})=\frac{1000}{1000}=1$
δ% = g%
θF(k) = (δ+g+n)k
θ(1) = (δ+n)*1
θ = (δ+n)
Investment rate must be equal to the sum of depreciation and labor growth rate in order to maintain stock of capital per capita at steady state.

XXVII. n = 0%
θF(k) = (δ+0%)k
θ = (δ+0%)
θ = δ
Investment rate and depreciation rate must be equal to keep the wealth level at steady state.

XXVIII. Growth can be due to some factors that increase our GDP, for example: increase in aggregate of output, increase in government purchase, investment, export.
GDP=C+I+G+ (E-IM)

But sustainable growth is a growth due to only 2 factors: 1st: do not harm environment (to keep clean ecosystem, do not overexploit the natural resources, land etc.)
2nd: growing without harming the next generation (do not fall in debt so that next generation after you will not pay for your fault, also clean environment for future generation.)

XXIX. P=50000 TMT
δ =10%
50000-50000*10% *n=0
50000=50000*10%*n

$\frac{50000}{50000*10\%}$ = n =10

After 10 years value of capital will be zero.

XXX. Cultural, Historical and religious background do play a role in economic development.

Let's explore historical: WWI, WWII or collapse of USSR affected immensely to economic development of participating countries. Say, after WWII a lot of people were dead and shortage of food lead to decrease of labor force.

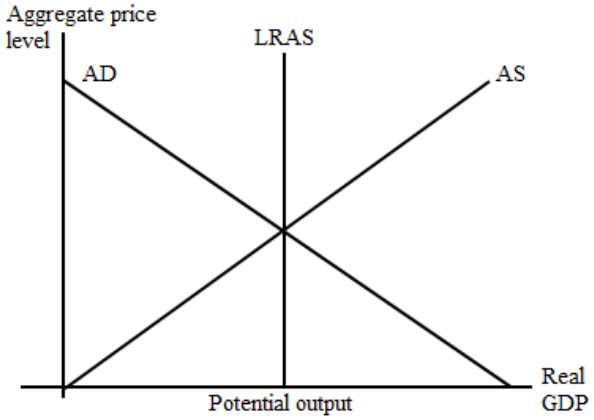
Role of culture in economic development: Each country has its own culture (each continent, to be more accurate), for example in Asia people have more "saving" habit (mostly called Asian Miracle) while in north and south countries people are more to" consume " So we know that saving affects to economy through investment while more consumption through increase in aggregate demand.

Chapter 6: Basic Macroeconomics 2

Monetary and fiscal policy is the main tools states use for regulation of economic activity. As we have learned before, there is no single country where economy is fully free, with no interference, with no regulations. There are plenty of reasons for that, but the most important one is that economy on its own cannot come to efficient results. Ecology, social classes, equality of opportunity and many other core issues just cannot be left to market for decision. They are too important to be left in hands of suppliers and consumers, we need regulator, and that regulator is government. Even though we believe that human beings are rational, we must accept that we have many natural fallacies like greed, envy and fear of future. These distorted emotions in an acceptable dose are good for people, because it pushes us to do something. Those emotions encourage us to risk, and as we know, risk brings rewards. Not always though. But overdose of those emotions makes our mind blurry, and decreases our chances of making right decision. Fishermen know that overfishing will end up tragically: all fish will be gone, but nobody will want to stop. This is oligopolistic game as we have seen in previous chapters, and fishermen know that. If I stop fishing, and my neighbor continues, I am going to be a loser side. So if we stop, all must stop, otherwise those who stop will lose. These kinds of situations will make people act against rationale; in our case people will continue fishing, unless somebody stops them. **Our natural fallacies will make us act in our short run interest, rather than in long run benefit.** We need regulator, a judge, any institution that will help to control our fallacies and turn/direct/ban/restrict/stop our hunt for short run interests and instead turn us to follow long run benefit.

The main function of government is to plan sustainable economic development programs which will benefit commons in a long run, and implement it for benefit of commons, even if it hurts our short run interests. Commons do not like to think in long run, we prefer short run interest. Preference of people of their short run interests to long run benefit creates inefficiencies which eventually leads markets to fail. Market failure means: huge rising debts, devastating unemployment rates, crisis, budget deficit, deficit of goods and services, hyperinflation, and civil unrest. Government uses two things to bring back failed market (or failing markets) to efficiency: Monetary policy which is generally main tool of central banks of countries which is also part of state apparatus, and fiscal policy, which is generally lead by one of ministries (Economics or Financial, or any other) of state authority. Government institutions have single target: long run stable growth of economy and well-being of its citizens. Their monetary and

fiscal policy plays a role of director in this "market orchestra". Since we have talked about long run benefits and short run interest, let's explore how the notions "long run" and "short run" is described in economics science.

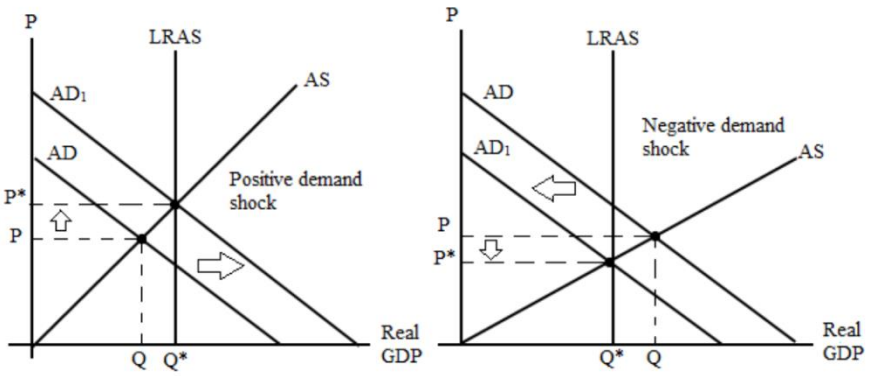


As it is seen from above graph, demand function represents aggregate demand for goods and services in economy, simply represents GDP. Short run supply curve represents suppliers in a short run. Long run supply curve represents long run optimal output, with given inputs: resource, labor, capital and technology. Intersections of all three functions represent equilibrium point or steady state of economy. Do you remember the steady state of economy? When income and investment portion of income is just enough to cover capital, labor and technological expenses? That is point when stock of capital per capita is equal to zero. First of all, let's learn the dynamics of these functions: Aggregate demand, Short run supply and Long run supply curves. What makes them move and in which directions? In economics, changes that impacted curves positively are called "**positive shock**" and changes that impacted curves negatively are called "**negative shock**". "**Positive shock**" move curve to the **right** no matter to which curve it is applied. "**Negative shock**" will move curve to the **left** no matter which curve the shock applied.

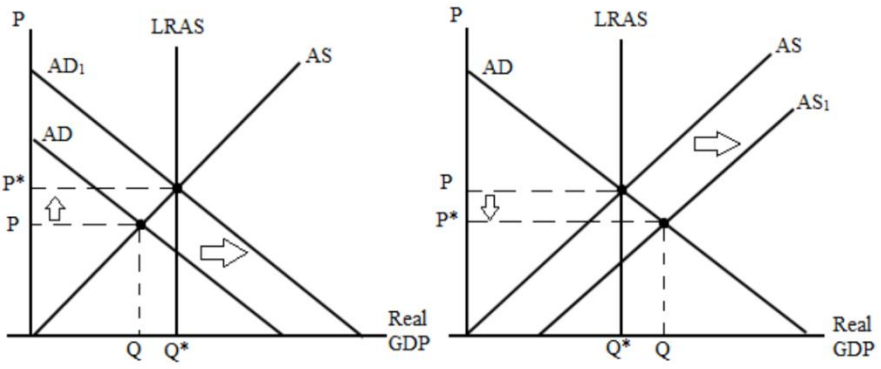
Aggregate demand curve:

i. **Income level or wealth:** As people get richer, they demand more goods and services. Income and aggregate demand is positively correlated. What increases population's wealth? Economic boom for example, opening up of new resources, huge investment, market reform, financial reform or even political and economic reform. These things done right will boost economy which in return will increase aggregate demand. Increase of wealth and income is positive shock for demand curve and this shock will move demand curve to the right. If crisis hits, or wealth is decreased this is a negative shock and move our aggregate demand curve to

the left. Positive shock to the demand curve will also move prices up, negative shock move prices down as it is seen from graph below:



ii. **Fiscal policy:** Tax cuts will increase income and this will stimulate spending. Increased spending means increased aggregate demand for goods and services. Tax cuts are positive shock and tax increase is negative shock. Here I must point out something: if tax cuts are to the consumer only, for example income tax cuts and so, this will be positive shock for demand curve. If tax cuts are to the suppliers, for example profit tax cut or corporate tax cut, this will be positive shock for short run supply curve. It is important because these cuts impact prices differently. Positive shock to demand curve drags prices up, but positive shock to short run supply curve (or simply suppliers) will drop prices instead. Fiscal reform might have different results and one might be very careful about that. When we talk about Fiscal policy, we mean budget. Budget spending also increases demand curve. Budget spending is purchasing of goods and services by government, social spending, subsidies, etc. all these impact aggregate demand positively shifting demand curve to the right. Cutting those spending impact demand curve negatively shifting demand curve to the left.

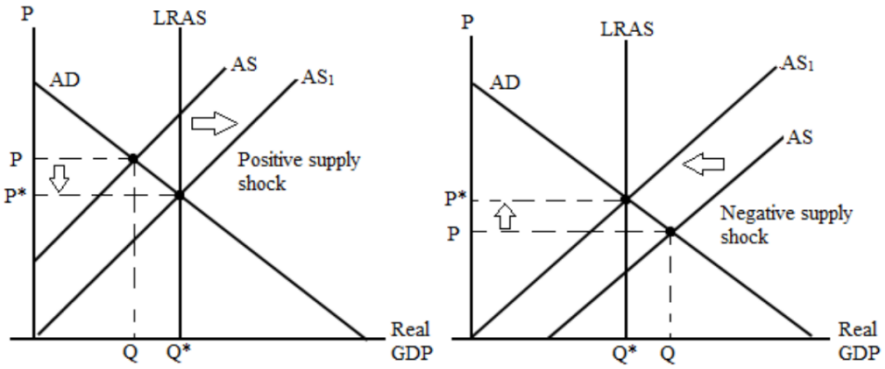


iii. **Monetary policy:** When governments increase money supply, aggregate demand is increased because simply there is a lot of money in market. Increasing of money supply is positive shock to aggregate demand and shifts demand curve to the right. Decreasing of money supply decreases demand, thus it is negative shock to demand curve, shifts demand curve to the left.

Short run supply curve:

i. **Input costs:** Input cost changes will impact suppliers deeply. Rising labor, energy, capital, rent and other costs will impact suppliers negatively, shifting supply curve to the left. Instead, lowering labor wages, cheap energy, capital and rent will impact suppliers positively, shifting supply curve to the right.

ii. **Technology, method, process, etc.:** New technology, method or process which makes production more efficient by decreasing costs, time or increasing product value will be a positive shock to the suppliers shifting supply curve to the right.

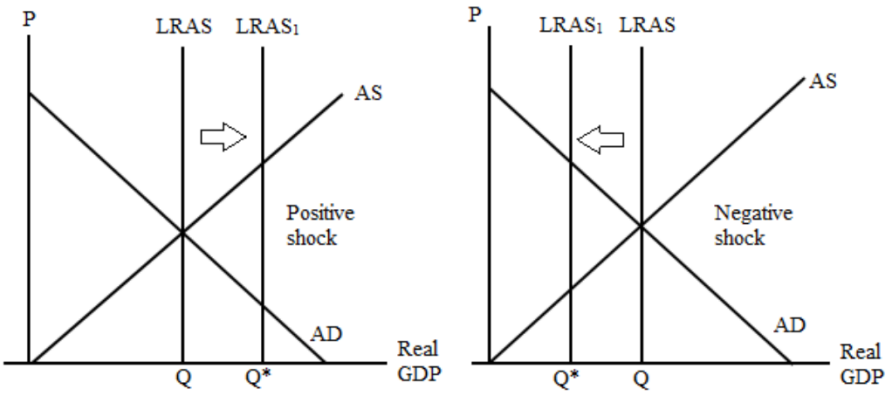


Long run supply curve:

i. **New resources:** When new resources (energy, metal, land, water, precious metals, etc.) are found in a country, this positively impacts whole economy by shifting long run supply curve to the right. New resources mean new potential for growth of economy. If instead resources are depleted, this will impact long run supply curve negatively, shifting long run supply curve to the left. Depleting resources mean shrinking economy.

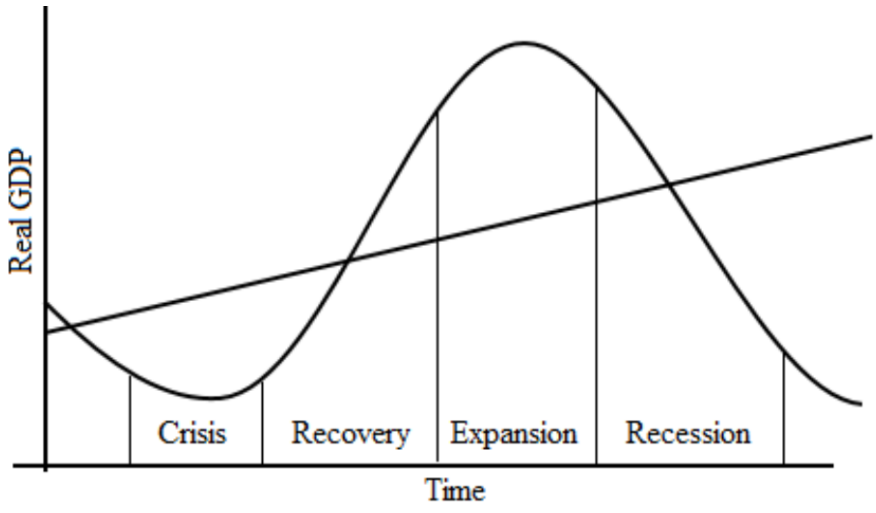
ii. **Increase of inputs:** Labor, capital, investment (or transfers), entrepreneurship, etc. these are all assets of economy; increasing assets have positive impact on economy, and on a long run supply curve, shifting curve to the right. Increase of inputs increases potential of economy.

iii. **Human capital:** Labor is central and the only derivative of real growth. Everything derives from labor: technology, capital, new resources, opportunities, new ideas, etc. that is why increasing human capital always impacts economy positively, shifting long run supply curve to the right.

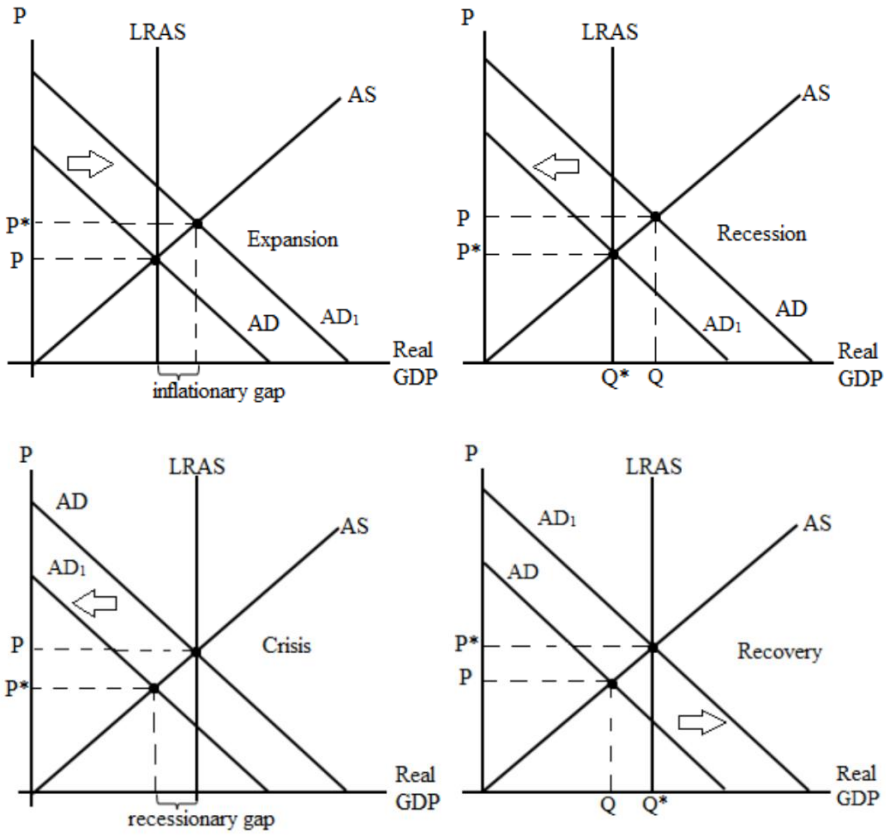


Shocks to economy and Business cycle:

Why we have shocks at all? Cannot economy simply grow with stable rates indefinitely? Answer is no, impossible. Economies grow with up and downs, and there are thousands of internal and external reasons. In economics science this ups and downs are called a Business cycle. **Expansion** is the time when output grows, unemployment decreases and interest rates are low. **Crisis** is the beginning of contraction of economy, unemployment rises, outputs fall and loans are limited (bankruptcies start). **Recession** is the deepening of crisis situation; unemployment hits the worst point, output decreases to minimum. **Recovery** is end of contraction of economy and start of expansion: unemployment starts to decrease, output grows.



Business cycle is natural; no economy can eliminate it! Economies grow though expansion and contraction. Question is not about "how not to have business cycle"; every economy will have it, no doubt. Question is how long each period of Business cycle will last? Of course, all economies wish to prolong Expansion period (recovery and expansion) as long as possible and cut fast the Contraction (crisis and recession) period. Economies want smooth business cycle, which mean volatility of fluctuation must not be critical! We do not want critical crisis and recession which will last decades. We know we cannot omit crisis and recession; we just want those periods last as short as possible. We understand that we cannot grow and expand forever, but we want those periods last as long as possible.



So, what economies do to shorten contraction period and prolong expansion period, to achieve smooth business cycle? They use monetary and fiscal policies. Can markets themselves deal with business cycle without any government intervention? Yes, they can deal. Business cycle will continue cyclical dynamics no matter what, but it might take very long time to get out of contraction period for markets if they want to do it themselves. Without intervention by government, recession might last for couple of decades, but intervention helps to recover in much shorter time. Intervention **"smoothen"** business cycles.

Fiscal policy

When we talk about fiscal policy we actually talk about government budget! As we have seen in previous chapter, some governments have huge $\frac{Budget\ spending}{GDP}$ ratios (France is around 50%). **Budget symbolizes economic and financial might of the government!** Bigger the budget, bigger the power of influence of government on market. Government budget is like an airbag for whole economy and effectively used to smooth the hard hits of business cycle. How this mechanism works? Budget has two components: **Debit side or Budget revenue. And Credit side or Budget spending.** Government collects revenue from different sources:

Fiscal policy: Budget revenue

A. Taxes: Tax revenue is one of the major sources of funds for governments. There are plenty of types of taxes and some are very unique: Income tax, corporate tax, transaction tax, property tax, resource tax, ecological tax, religious tax, etc. Tax system of country has immense impact on development of that country. It depends on government to switch the burden of taxes to one of social groups. How this works?

We have mentioned before that generally, economy includes three groups of people according to their income level: rich class, middle class and poor class. Assume that government is in hardship and needs money. The biggest source of income is taxes. Government must decide which tax to increase. For example, corporate taxes are paid by corporates, businesses, business owners, thus rich class. Poor class and middle income class is not effected with that tax too much. So, increasing corporate tax will impact rich class. Nobody likes taxes! That is why when states increase taxes, the group that impacted by those taxes will of course feel discriminated. Tax burden will impact not only wealth level of that group (corporate tax for example will decrease profits of rich class) but also their future business decision. Too high corporate taxes will divert businesses and they will move their businesses out of country. Fewer businesses will open, and those already operating businesses will look for ways to get out of those heavy corporate taxes. The cases of tax evasion and money laundering will increase. Especially if government institutions are non-transparent and corruption levels are high, then increasing taxes might even worsen the budget situation than before. What if state decides to increase property taxes? Property owners will be hit, and generally it includes two groups: rich class and middle class. Those two groups are the main payers of property taxes. As we know, middle class is the main driver of economy,

hitting them will be hitting economy overall. Each tax has its own tax payer group, and reforms in that tax type will directly impact the well-being of the taxpayer group. Inefficient tax system favoring certain groups or discriminating other groups will lead to inefficiency, loss of human capital, slowing down of competition, social and economic inequality, market fragmentation, uneven development of regions, elimination of middle class, increase of poor class, distrust to government, unrest and general decrease of overall potential of economy. All tax types must be rational and fair, with positive long term impacts to economy and society. Otherwise, economic and social problems in future are guaranteed.

Simply by decreasing or increasing taxes, government can increase or decrease income and wealth of the population. Wealth level impacts aggregate demand directly as we have learned above. Taxes are a very sensitive instrument that is why governments try to make reforms in tax system as a last option. Again, nobody likes taxes, and nobody likes those who impose taxes! Tax increase reforms are very unpopular, that is why politicians try to touch that topic as a last reserve.

Now, let's come to the structural analysis of tax revenue. **The ratio of tax revenue to total budget revenue will show the level of economic development of that economy**. If $\left(\frac{\boldsymbol{Tax\ Revenue}}{\boldsymbol{GDP}}\right)$ ratio is high, it will show that main income of a country is from taxes. This will show dependence of economy to well-being of its people. When businesses are losing, state lose too. When businesses are winning, states win too. **This kind of dependence forces governments to be more transparent, more market oriented, competition fostering, respect human rights and compliance of strong rule of law**. Long word short, since state budget is mainly dependent on taxes, state does all "needed and required" for growth and development. Low ratio of $\left(\frac{\boldsymbol{Tax\ Revenue}}{\boldsymbol{GDP}}\right)$ will show that state budget is less dependent on taxes, that is why government will have less interest in increasing market efficiency in economy.

Tax types and tax calculation might be different from country to country as we have mentioned above. Recently, progressive taxing became more popular. Progressive taxing system is when tax burden increases parallel with income level of tax payer. This kind of tax levy leads to more fair tax burden distribution. Most of the countries use fixed tax system, when all tax rates are fixed regardless of your income level.

Taxes are also vital instrument in developing certain targeted businesses. For example, state that wants agricultural

sector develop faster can impose tax discount for businesses in agricultural sector, and also luring new investments into the sector. Tax discounts signals the market about new opportunities in the sector.

Top and Bottom countries according to $\left(\frac{Tax\ Revenue}{GDP}\right)$

	Highest	(%)
1	Austria	42.7
2	Belgium	47.9
3	Bosnia and Herzegovina	41.2
4	Croatia	36.7
5	Cyprus	39.2
6	Finland	54.2
7	France	47.9
8	Denmark	50.8
9	Greece	39
10	Germany	44.5
11	Israel	36.8
12	Hungary	39.1
13	Iceland	40.4
14	Italy	43.5
15	Norway	54.8
16	United Kingdom	34.4
17	Sweden	49.8
18	Spain	37.3
19	Slovenia	39.3
	Lowest	
1	Yemen	7.1
2	Laos	10.8
3	Ethiopia	11.6
4	Haiti	9.4
5	Myanmar	4.9
6	Bahrain	4.8
7	Cambodia	8
8	Chad	4.2
9	Republic of Congo	5.9
10	Equatorial Guinea	1.7
11	Guinea	8.2
12	Iran	6.1
13	Kuwait	1.5
14	Libya	2.7
15	Nigeria	6.1
16	United Arab Emirates	1.4
17	Sudan	6.3
18	Saudi Arabia	5.3
19	Oman	2

Exercise 1: *Kuwait is one of the richest countries according to GDP per capita ratio but as you can see from table above* $\frac{Tax\ revenue}{GDP}$ *ratio is very low. Why is that?*

Because main source of income Kuwait earns from petroleum (60% of its GDP), not from its tax collection.

Exercise 2: *By looking at above table, which government is more market oriented (high transparency, low bureaucracy, low corruption and efficient tax system) Norwegian or Nigerian? Why do you think so? Check your answer with official statistics.*

Indicators show us, Norway's government is more transparent, with low bureaucracy rate, corruption and efficient tax system than Nigeria's. Tax-to-GDP ratio of Norway is 54.8% while Nigeria's 5.7% (according to OECD.org statistics).

B. Fees: License fees, patent fees, custom duty fees, and all other fees are both source of revenue for government and also controlling tools of the economic sector. For example, state that wants to protect domestic producers imposes heave custom duty fees to imported goods belonging to the same class. This type of action is called ***protectionism,*** from word "to protect". Certain businesses are licensed due to financial, economical, safety and other strategic targets set by governments. Thus, governments take under "special control" certain sectors: Construction, pharmacy, leisure, gambling, defense, IT, etc. because they carry strategic importance for security and development of economy. Increasing fees will decrease income thus will impact negatively aggregate demand. Decreasing fees will increase income and lure in more businesses, and this will be a positive shock for aggregate demand.

C. Penalties: All countries have penalties for breaking rules and laws. Penalty amount generally depends on level of "crime". Harsh and heavy penalties will tighten overall discipline but at the same time it is going to limit "freedom of action" of people hugely. Too liberal penalty policies will instead create chaos. Governments must make sure that their penalty levels are constructed in such a way to increase ethical standard of population in long term, rather than punishing population today. High penalties decrease income of population and negatively impact aggregate demand dragging it to the left. Low penalties increase income of population but create a chaos.

D. Government investments: Governments are like any other businesses; they invest their surplus revenue. Government can invest onto debt obligations, purchase stock and shares of private companies, directly invest onto companies, etc. These investments bring dividends, returns, and profits. State company investments are considered government investments because it is a state company, as name mentions it, and all investment decisions are made by state authorization. Governments create sovereign wealth funds and use those funds for investments. Increase of revenue from investments gives governments opportunity to spend more. Government spending (purchase of goods and services from market) increases aggregate demand. Increased social spending increases income of population, which positively impacts demand curve dragging it to the right. Loss from investments decreases government revenue which leads to cutting budget spending. Cutting budget spending decreases aggregate demand dragging it to the left.

Countries by their sovereign wealth funds (in billions USD)

1	China	1,554.80
2	United Arab Emirates	1,298.70
3	Canada	1,270
4	Norway	1,063
5	Saudi Arabia	890
6	Singapore	764
7	Kuwait	524
8	Hong Kong	500
9	Qatar	320
10	Russia	178.38
11	Australia	161.85
12	South Africa	160
13	United States	150.8
14	Kazakhstan	127.6
15	South Korea	122.3
16	Iran	91
17	France	68.35
18	Libya	66
19	Brunei	40
20	Azerbaijan	39
21	Malaysia	34.9
22	Chile	24.1
23	Oman	24
24	New Zealand	22.7
25	East Timor	16.6
26	Bahrain	10.6
27	Ireland	8.5
28	Peru	7.9
29	Algeria	7.6

30	Brazil	7.3
31	Mexico	6
32	Botswana	5.7
34	Trinidad and Tobago	5.5
35	Angola	4.6
36	India	3.8
37	Colombia	3.5
38	Nigeria	2.9
39	Panama	1.2
40	Bolivia	1.2
41	Senegal	1

Exercise 3*: What do you know about Norway's sovereign wealth fund? Shortly.*

Norway invests its surplus revenue from petroleum to its GPFG (Government Pension Fund Global). It has over 1 USD trillion assets including 1.4% of global stock which makes this fund the largest of the world.

Exercise 4: *What do you know about Iran's sovereign wealth fund? Shortly.*

Sovereign wealth fund of Iran NDFI (National Development Fund of Iran) makes an investment to financial assets about 35 billion USD, to preserve revenue from Iran's oil and gas.

E. State company revenues: "State company" is a type of company where state is the majority stake holder (at least 51%). Revenues and profits of those companies are shared accordingly. The amount of state owned companies and value of them indicates how private businesses (privatization) are developed in economy. Huge amount of state companies with huge market capitalization show that private share of the economy is relatively weak. In economics it is called level of ***liberalization of market***. High amount of state companies potentially may lead to unfair competition in economy, and inefficient management of those companies, that is why economies with high amount of state owned companies are considered less competitive and less liberalized.

Top companies by their revenue (in millions USD)

1	Walmart	$514,405
2	Sinopec Group	$414,650
3	Royal Dutch Shell	$396,556

4	China National Petroleum	$392,976
5	State Grid	$387,056
6	Saudi Aramco	$355,905
7	BP	$303,738
8	ExxonMobil	$290,212
9	Volkswagen	$278,341
10	Toyota	$272,612
11	Apple	$265,595
12	Berkshire Hathaway	$247,837
13	Amazon	$232,887
14	UnitedHealth	$226,247
15	Samsung Electronics	$221,579
16	Glencore	$219,754
17	McKesson	$214,319
18	Daimler	$197,515
19	CVS Health	$194,579
20	Total	$184,106
21	China State Construction	$181,524
22	Trafigura	$180,744
23	Foxconn	$175,617
24	Exor	$175,009
25	AT&T	$170,756
26	ICBC	$168,979
27	AmerisourceBergen	$167,939
28	Chevron	$166,339
29	Ping An Insurance	$163,597
30	Ford	$160,338
31	China Construction Bank	$151,110
32	General Motors	$147,049
33	Mitsubishi	$145,243
34	Honda	$143,302
35	Costco	$141,576
36	Agricultural Bank of China	$139,523
37	Alphabet	$136,819
38	Cardinal Health	$136,809
39	SAIC Motor	$136,392
40	Walgreens Boots Alliance	$131,537
41	JPMorgan Chase	$131,412
42	Gazprom	$131,302
43	Verizon	$130,863
44	Bank of China	$127,714
45	Allianz	$126,779
46	AXA	$125,578
47	Kroger	$121,162
48	General Electric	$120,268
49	Fannie Mae	$120,101
50	Lukoil	$119,145

Exercise 5: *What do you know about Ping An Insurance? Shortly.*

Chinese company Ping An Insurance ranks as 2nd largest Insurance company worldwide in terms of bringing highest revenue in life and non-life insurance markets.

Exercise 6: *What do you know about UnitedHealth and CVS Health?*

UnitedHealth is one of the largest medical Insurance company in USA, which services more than 100 million clients not only in US but in other countries also. CVS Health is one of the biggest retail pharmacy provider in USA with over 6000 pharmacy locations.

Exercise 7*: What do you know about McKesson?*

American company McKesson is the largest medical technology, wholesale medical supplies and medical equipment provider mainly concentrated in Europe.

Exercise 8: *What do you know about State Grid?*

State Grid Corporation of electric utility is monopolistic corporation of China, the largest utility provider and power transmission manager worldwide. Also 5th largest by revenue among other companies.

Top companies (below) according to their market capitalization (value of company) in billions of USD

1	Saudi Aramco	2,100
2	Microsoft	1,470
3	Apple Inc.	1,440
4	Amazon	1,080
5	Alphabet Inc.	1,050
6	China National Petroleum Corporation	634
7	Alibaba Group	620
8	Berkshire Hathaway	570
9	Tencent	550
10	Samsung	529
11	Volkswagen Group	510
12	Toyota	478
13	JPMorgan Chase	470
14	Visa	460
15	LIC	450
16	State Grid Corporation of China	351

17	ExxonMobil	349
18	Daimler AG	313
19	Hyundai Motor Group	313
20	SoftBank	285
21	Chevron Corporation	266
22	BP	261
23	ONGC	240
24	Royal Dutch Shell	236
25	Total S.A.	224

Exercise 9: *By looking at table above, how many companies are in energy sector? How many of them in financial sector? How many of them in IT sector? How many of them in industrial sector?*

In energy sector: Royal Dutch Shell, Total S.A, ONGC, BP, Chevron corp., Exxon Mobil, State Grid Corporation of China, China National Petroleum corp., Saudi Aramco.
In industrial companies: Alibaba, Tencent, LIC, Hyundai Motor Group, Visa, Toyota, Volkswagen.
In IT sector: Samsung, Microsoft, Apple, Amazon, Alphabet.
In financial sector: SoftBank, JPMorgan Chase, Berkshire Hathaway.

Fiscal policy: Budget expenses

A. Social spending: Stipends for students, scholarships, support for handicapped people, single mothers, public kindergartens, bridges, roads, public schools, public universities, research subsidies, medical assistance for citizens, public transportation, athletes, arts and artists, museums, up keeping of historical sites, public swimming pool and stadiums, etc. these all called a social spending. Governments do this spending to increase welfare of population, to support poor class and middle class. Increase of social spending saves people's money, this increases aggregate demand shifting demand curve to the right. If governments decrease (or cut) social spending, aggregate demand decreases because people now will have to cover own social costs.

B. State employee salaries: State is the biggest employer almost in all economies that is why salary expenses of government are huge. Government can impact markets through its minimum wage policy or through hiring more employees. As government increases salaries or starts hiring more people this impacts aggregate demand positively because incomes of people increase. Decreasing salaries of state employees or laying them

off increases unemployment and decreases overall income of population, this will drag aggregate demand to the left.

C. **Purchases:** Government has many institutions, schools, entities and many other operations which need all kind of goods and services to support normal functioning of those state entities. To keep up operations states purchase goods and services from market, as any other non-governmental entity. Since state has huge number of entities to support, its purchases make huge impact on economy. Increases of government purchases gives positive shock to economy, shifting aggregate demand curve to the right. Decreasing purchases decreases demand.

D. **Investment spending:** Purchasing of debt obligations of foreign countries, direct or indirect investment to domestic and foreign economy are all investments and governments raise funds for this investment from budget surplus. Increase of investment spending increases future expectations of population and shifts demand curve to the right.

E. **Security and defense spending:** Domestic security, customs patrol, military and all related services are on government. Spending on those services take huge toll on budget.

Below are top military spenders (in billions of USD)

1	United States of America	684.6
2	China	181.1
3	Saudi Arabia	78.4
4	Russia	61.6
5	India	60.5
6	United Kingdom	54.8
7	France	52.3
8	Japan	48.6
9	Germany	48.5
10	South Korea	39.8
11	Brazil	27.5
12	Italy	27.1
13	Australia	25.5
14	Israel	22.6

Those above are the main spending targets of governments. Budget revenues sources and spending targets may change for each government but one thing is unarguable: Government is the biggest player in market, both in terms of earnings and spending which is done through its fiscal policy. No doubt that market is impacted by fiscal policy of this giant player.

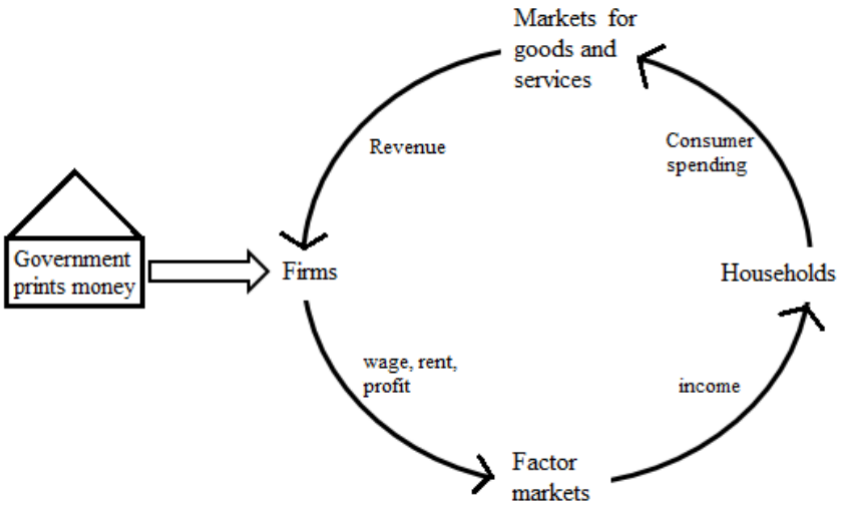
Monetary policy

When we talk about monetary policy, we talk about Central bank. We can summarize the main target of monetary policy of all central banks: **To provide enough liquidity to the market for long term sustainable growth and investment of an economy.** Central banks have many functions; here is the main one of them:

1) **Issue national currency (Money emission)**
2) **Provide financial safety and security of economy through issuing minimal financial standards for banks and other financial organizations**
3) **Supervision and audit of operations of all banks and financial organizations located in domestic market**
4) **Keep inflation rate in targeted zone**
5) **Increase reputation and trust of national currency though correctly applied exchange rate policies**
6) **Increase efficiency of banking sector**
7) **Provide and assist optimal growth of economy through maintaining efficient money depositing-lending policy of banks.**
8) **Many other functions.**

Money supplied to market is in tight control by Central banks (by governments) because of sensitive and extremely volatile nature of financial world. As we have learned in previous chapter saving (investment) is main "gasoline" of growth. Especially long term investments play huge role, but fluctuating inflation rates will scare investors and they will look for another investment area. As once one wise man said "Capital does not have a motherland". Stable financial system totally depends on how monetary policy is managed. "Bad" monetary policy will destroy economy faster than it was built. **Financially stable economy is economically stable economy!** Stable economy attracts businesses, investments and human capital while unstable economy "scares off" everything. In fact, a lot of developing and underdeveloped economies suffer exactly from that: outflow of capital from economy due to unstable financial system. Capital (Money) likes stability! Now let's learn how governments try to keep financial stability using monetary policy.

Money is circulated in economy. This money was emitted by central bank. Consumers earn salary. Spend certain part and invest or deposit to the bank the rest. Banks lend this deposit for new businesses. Businesses grow and produce things that consumers buy. All pay taxes and fees. Government uses this money for supporting development. **Managing money circulation is called monetary policy.**



What can be hard here? What can create a problem? Isn't it easy? Seems easy but it is not! These things distract money circulation in economy:

i. **Not all saved money is invested!** There might be tons of reasons why people and business do not invest saving: they do not trust banks, fewer opportunity for investment, low transparency in economy, etc. Especially in developing economies people trust financial system lesser. People better keep their savings under the pillow, rather than depositing it. This creates huge problem because huge bulk of emitted money is not in circulation anymore. **Not only omitted from circulation, but also not invested, simply sits there for nothing.** Distrust to financial system creates both investment and circulation problem.

ii. **Not everybody pays taxes!** Taxes are one of main resource of revenue for governments. **Inefficient tax system, corruption, low transparency of government institutions, bureaucracy, etc. these things create budget revenue problems for governments**. When government's revenue decreases they will have to either emit new money or take loans to pay off obligations. Emission creates inflation because more money is supplied to market with no reasons (emitted money is not backed by goods or services).

iii. **Underground economy!** High crime rates (especially drug trafficking, gambling, gun sales, prostitution, money laundering, corruption, and high amount of organized crime), low transparency of government institutions, bureaucracy, distrust to financial system, these things create underground economy, economy where a lot illegal money (earnings with no taxes paid) is circulated. **Underground economy is like a "black hole", will continue "sucking in" money from money**

circulation getting bigger and bigger in size unless governments fight against it.

Below is table of countries by their share of underground economy (%)

1	Zimbabwe	67
2	Haiti	56.38
3	Georgia	53.07
4	Nigeria	52.49
5	Gabon	52.01
6	Burma	50.99
7	Central African Republic	50.71
8	Benin	48.28
9	DR Congo	46.95
10	Guatemala	46.88
11	Bolivia	45.98
12	Madagascar	45.29
13	Liberia	43.67
14	Azerbaijan	43.66
15	Gambia	43.64
16	Thailand	43.12
17	Ukraine	42.9
18	El Salvador	42.6
19	Ivory Coast	42.4
20	Belize	42.29
21	Guinea	41.58
22	Peru	41.53
23	Swaziland	40.94
24	Comoros	40.92
25	Molodva	39.68
26	Nicaragua	39.51
27	Ghana	39.37
28	Tanzania	38.91
29	Bahamas	38.55
30	Libya	38.27
31	Tajikistan	37.73
32	Honduras	37.68
33	Eritrea	36.53
34	Armenia	35.96
35	Burundi	35.68
36	Sri Lanka	35.49
37	Angola	35.25
38	Brazil	35.22
39	Papua New Guinea	35.16
40	Reuplic of Congo	35.05
41	Giunea Bissau	34.94
42	Sierra Leone	34.18
43	Niger	34.12

44	Cambodia	3385
45	Russia	33.72
46	Senegal	33.68
47	Venezuela	33.63
48	Malawi	33.56
49	Kenya	33.43
50	Egypt	33.32
51	Zambia	32.99
52	Kazakhstan	32.82
53	Belarus	32.37
54	Lesotho	32.32
55	Cyprus	32.2
56	Uganda	31.88
57	Paraguay	31.66
58	Pakistan	31.62
59	Togo	31.49
60	Trinidad and Tobago	31.4
61	Equatorial Guinea	31.38
62	Mozambique	30.98
63	Tunisia	30.9
64	Solomon Islands	30.89
65	Kyrgyzystan	30.78
66	Brunei	30.44
67	Cape Verde	30.23
68	Nepal	30.22
69	Ecuador	30.18
70	Bosnia and Herzegovina	29.88
71	Burkina Faso	29.63
72	Mali	29.45
73	Malta	29.43
74	Lebanon	29.16
75	Taiwan	28.97
76	Cameroon	28.93
77	Yemen	28.81
78	Chad	28.76
79	Mexico	28.07
80	Rwanda	28.05
81	Philippines	27.97
82	Dominica Republic	27.6
83	Bangladesh	27.43
84	Turkey	27.13
85	Morocco	26.45
86	Greece	26.21
87	Albania	26.09
88	Guyana	26.07
89	Malaysia	25.75
90	Fiji	25.37
91	Colombia	25.25
92	Ethiopia	25.1

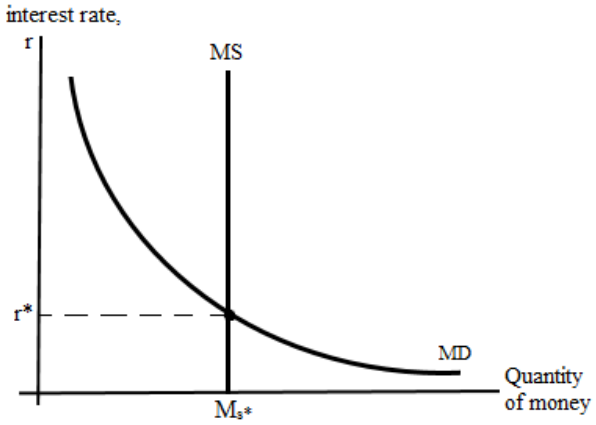
93	Laos	25
94	Argentina	24.99
95	Jamaica	24.97
96	United Araba Emirates	24.26
97	Botswana	23.99
98	Algeria	23.98
99	Oman	23.91
100	Suriname	23.8
101	Italy	22.97
102	Croatia	22.96
103	Romania	22.94
104	Spain	22.01
105	South Africa	21.99
106	Namibia	21.78
107	Indonesia	21.76
108	Kuwait	21.72
109	Bulgaria	20.83
110	Maldives	20.65
111	Hungary	20.49
112	Uruguay	20.38
113	Bhutan	20.28
114	Slovenia	20.21
115	South Korea	19.83
116	Syria	19.53
117	Costa Rica	19.24
118	Mauritus	19.23
119	Israel	19.18
120	Lithuania	18.65
121	Estonia	18.49
122	Iran	18.38
123	India	17.89
124	Portugal	17.82
125	Belgium	17.8
126	Polan	16.67
127	Bahrain	16.63
128	Latvia	16.62
129	Jordan	15.16
130	Norway	15.07
131	Vietnam	14.78
132	Denmark	14.7
133	Saudi Arabia	14.7
134	Finland	13.3
135	Mongolia	13.2
136	Chile	13.16
137	Qatar	13.08
138	Iceland	12.45
139	Hong Kong	12.39
140	China	12.11
141	Sweden	11.74

142	France	11.65
143	Slovakia	11.18
144	Czech Republic	10.47
145	Luxembourg	10.38
146	Ireland	9.58
147	Canada	9.42
148	Singapore	9.2
149	Austria	9.01
150	New Zealand	8.97
151	UK	8.32
152	Japan	8.19
153	Australia	8.1
154	Netherlands	7.83
155	Germany	7.75
156	USA	7
157	Switzerland	6.94

iv. **Independence of central banks:** No currency is backed by real value in the world. All current currencies circulating are fiat currencies. Fiat currency is a currency which is only backed by law. The law, which requires all to accept national currency as a medium of exchange in domestic economy. On itself, fiat money does not have any value; it is just a piece of cloth (you know that it is made of cotton right?). That is why in most countries central banks are independent of central executive government. Directors of central banks are appointed by Congress or Parliament, not by heads of executive office: Presidents and Premier Ministers! Directors of central banks report only to the Congress and Parliament. This eliminates opportunity of emitting more money for governments. If central executive governments had a chance to print money, this would have created a huge conflict of interest issue for governments. Instead of curbing government spending and finding more rational solution for budget deficit, or to any other economic or financial problem, governments would have just printed (emitted) more money and covered expenses. This would have accelerated inflation rates. Independence of central banks from governments forces governments to work more efficiently.

Now, let's talk about money supply. **Money supply is a process of changing quantity of money in market.** Supplier of money is government. Money demanders are consumers and businesses. Increasing money supply means increasing quantity of money in market by governments by using monetary and fiscal policies. Decreasing money supply means decreasing quantity of money in market by governments by using monetary and fiscal policies. Money is a good, and has the same characteristics of good; the only difference is that it is supplied by government only. Increase of supply will decrease

the value of the good. Value of money is interest rate. Banks offer loan, money and the price of loan is an interest rate. Thus, price of money is interest rate. High interest rates increase value of money; low interest rates decrease value of money. When governments increase supply of money, the value of money (or price of money) drops, money becomes cheap. Decreasing price (or value) of money means inflation. Money lost its purchasing power. Increasing price of money means deflation, purchasing power of money increased. Oversupplying of market with money will lead to inflation, undersupplying creates deflation.



***Exercise 10**: Interest rate (or price of money) depends on quantity of money supplied. Assume this is the function of interest rate:*

$$\frac{1000}{M_s} - 1 = i$$

*Where $\boldsymbol{M_S}$ is total quantity of supplied money (in Trillions TMT), **i** is interest rate in %. Graph this function and find out how will interest rate change if money supply increased from 200 Trillion TMT to 400 Trillion?*

***Exercise 11:** If money supply and interest rate function is in this form:*

$$M_S^{0.3}\, i^{0.4} = 20$$

Graph this function and find how interest rate will change if money supply is decreased from 2000 Trillion to 500 Trillion? ($\boldsymbol{M_S}$ is in Trillions, $\boldsymbol{M_S}$=2000 means Money supply is 2000 Trillion).

How governments change quantity of money supplied using monetary or fiscal policies?

Using monetary policy:

1. **Emitting (printing) money:** Increasing emission increases money supply.

2. **Central bank borrowing rate:** Increasing borrowing rates will increase value of money, and this will decrease money supply. When interest rates become high, people lower borrowing.

3. **Issuing bonds:** Governments sell debt obligations (bonds) and "drag money in" from the markets. Selling debt obligations decrease money supply. Buying debt obligations back by governments (repurchasing or even paying them off) increases money supply.

4. **Changing reserve rates of banks:** Central banks require all banks to keep certain portion of deposits for liquidity reasons this is called reserve rates in economics science. Central bank order to increases reserve rate will decrease money supply because banks will have lower loan amount. Decreasing reserve rates will increase money supply because banks will have more funds available now.

Using fiscal policy:

1. **Government spending:** Increase of budget spending (any spending: education, military, health, etc.) increases money supply in market. The word "spending" says it all here.

2. **Tax rates:** Increasing tax rates will decrease quantity of money left in market. Decreasing tax rates will increase quantity of money left in market.

3. **Fees, tariffs, penalties, etc.:** Increasing fees, tariffs, penalties, etc. will decrease quantity of money in market and increase budget revenue. Decreasing will have reverse effect.

Governments will change quantity of money in economy using either monetary or fiscal policy, or combination of them. It totally depends on government's decision.

All currencies are fiat money as we said before and only backed by local law. Laws are protected by government. That is why value of fiat currency in circulation is totally protected by governments. Protection of value of national currency is vital part of economic security strategy for governments because all wealth of nation is tied up to currency. In order to keep trust of people up high, governments (central banks) still buy gold and other precious metals. Gold reserves increase trust to governments, thus also

directly increasing trust to national currency of that economy. **Gold reserves increase creditworthiness of nations!**

Gold reserves in metric tons

1	United States	8,133.50
2	Germany	3,363.60
3	International Monetary Fund	2,814.00
4	Italy	2,451.80
5	France	2,436.00
6	Russia	2,299.20
7	China	1,948.30
8	Switzerland	1,040.00
9	Japan	765.2
10	India	654.9
11	Netherlands	612.5
12	Turkey	560.8
13	European Central Bank	504.8
14	Taiwan	422.4
15	Portugal	382.5
16	Kazakhstan	377.1
17	Uzbekistan	342.8
18	Saudi Arabia	323.1
19	United Kingdom	310.3
20	Lebanon	286.8
21	Spain	281.6
22	Austria	280

***Exercise 12**: By looking at above table of countries by their gold reserves, currencies of which countries you would have trusted the most? List 5 countries and tell why?*

US dollar, Euro, Japanese YEN, British pound and Australian Dollar. I would trust to the currencies of USA, Japan, Great Britain, Australia and Germany because these countries show their creditworthiness with their gold reserves and stable economic growth.

National currency is backed by local law in economy that is why local suppliers easily accept it as medium of exchange. What about foreign traders? If they accept local currency, how would they spend it in their own economy back home because there, this currency is not backed by anything? Each economy backs only its own currency. How international trade works then? Answer to this question is **exchange rates**. **Exchange rate is the price of one currency (national currency for example) in another currency (foreign currency).** Foreigner that came to our country first must exchange his national currency to our national currency and then

can go and easily trade. Assume that exchange rate of 1 USD=3.5 TMT. Then American with 1000 USD in his pocket first exchanges his currency (1000 USD=3500 TMT) to our currency and can easily spend it in our economy. How exchange rates are determined then? It is determined in two ways:

1) **Government**: Governments can put prices on their own currencies; this kind of determination is called **fixed exchange rate policy.** Governments directly control prices of their currencies. Fixed exchange rate policy is convenient for traders of both sides (domestic and foreign traders) because they both know that currency rates are fixed and do not fluctuate. At the same time, this puts huge financial burden on governments because they must now back up their fixed exchange rates with **reserve currencies** to satisfy demand. Reserve currencies are currencies that are trusted almost in all corners of the world and easily accepted as medium of exchange both by markets and governments. For example, foreign traders want to buy local good, so first they must go and buy local currency for fixed rates, here, demand for national currency occurs. It is easy to satisfy this demand because local currency is emitted here. Not much problem with exports we will say. What if local traders want to import goods from abroad? Then first they must buy foreign currency here (because they may not be able to find exchanges abroad) and then go for trade. Here, demand for foreign currency is created. Government must now satisfy this demand too, if it wants to keep fixed exchange rates. Countries that use fixed exchange rate policy keep huge amount of reserve currency stocks in their respected central bank vaults to satisfy money demand. Fixed exchange rate policy is very hard to implement if governments do not have guaranteed income in reserve currencies. If governments fail to satisfy local demand for foreign currency (reserve currency) **black market** will be created. **Black market** is "unofficial" or "illegal" exchange rate market.

Foreign exchange reserves in billions of USDs

1	China	3,101,692
2	Japan	1,378,239
3	Switzerland	848,398
4	Russia	569,800
5	India	513,254
6	Taiwan	488,690
7	Saudi Arabia	445,000
8	Hong Kong	442,300
9	South Korea	410,800
10	Brazil	345,706
11	Singapore	300,991
12	Germany	245,055

13	France	237,831
14	Thailand	241,000
15	Mexico	196,146
16	Italy	190,217
17	United Kingdom	179,225
18	Czech Republic	146,804
19	Israel	142,510
20	Indonesia	130,544
21	United States	129,264
22	Poland	112,943
23	United Arab Emirates	106,463
24	Malaysia	102,484
25	Philippines	88,996
26	Canada	87,322
27	Turkey	86,343
28	Iran[h]	86,000
29	Vietnam	81,145
30	Spain	74,037
31	Libya	71,629
32	Norway	69,139
33	Peru	68,150
34	Algeria	62,000
35	Australia	55,922
36	Colombia	55,000
37	Sweden	54,648
38	Denmark	54,251
39	Lebanon	53,100
40	South Africa	53,003
41	Azerbaijan	47,500
42	Iraq	47,020
43	Netherlands	44,859
44	Romania	42,654
45	Kuwait	40,600
46	Argentina	39,189
47	Qatar	37,729
48	Bulgaria	36,991
49	Chile	36,777
50	Nigeria	36,563
51	Egypt	36,254
52	Bangladesh	35,090
53	Kazakhstan	29,967
54	Belgium	29,676
55	Ukraine	28,500
56	Hungary	28,209
57	Portugal	27,053
58	Austria	26,443
59	Morocco	24,644
60	New Zealand	23,897
61	Macau, China	22,430

62	Croatia	21,304

2) **Market:** As we learned before, money is also a good. And as any other good it has a price. Second way of determination of price of a currency is done by markets. If international demand for your currency increases, price (and value) of your currency increases. If international demand for your currency decreases, price of your currency also decreases. When governments use this method of determination, it is called **floating exchange rate policy.** Floating exchange rates are less convenient for traders because rates will be in constant change. So, when demand for one currency increase and decrease? Demand for one currency increase and decrease according to demand for goods and services of that country. Demand for Japanese YEN increase if demand for Japanese goods and services increases. Demand for Japanese YEN decrease if demand for Japanese goods and services decrease. **Demand for your currency increase if demand for your goods and services increases.** High trade (export) volume, economic and financial stability, strong trade and diplomatic positions in international economic and political arena will increase national currency's reputation.

Currency composition of total foreign exchange reserves

1	US dollar	60.89%
2	Euro	23.54%
3	Japanese yen	5.70%
4	Pound sterling	4.62%
5	Chinese renminbi	1.96%
6	Canadian dollar	1.88%
7	Australian dollar	1.69%
8	Swiss franc	0.15%
9	Other currencies	2.56%

Exercise 13: *Assume exchange of 1 USD=90 YEN was last year. This year exchange rate is changed to 1 USD=100 YEN. How this exchange rate will impact prices of goods and services in both countries? (Also, assume annual inflation rate of USA is 3% and annual inflation rate of Japan is 5%).*

First of all, we must mark that both exchange rate fluctuations and inflation rate impacts the prices of goods and services in country. Easiest way to solve this question is to use vector mathematics. Let's for start take two prices and put them on one line, we will assume that last year prices were in equilibrium state. Not equal, but in equilibrium state and fluctuations change prices in different directions. First, let's start from exchange rate fluctuations. USD became more expensive relative to Japanese YEN. Last year Japanese trader paid 90 YEN to buy 1 USD, and this year he will

have to pay 100 YEN to buy 1 USD. How much value gained USD relative to YEN?

1$=90 YEN (last year)

1$=100 YEN (this year)

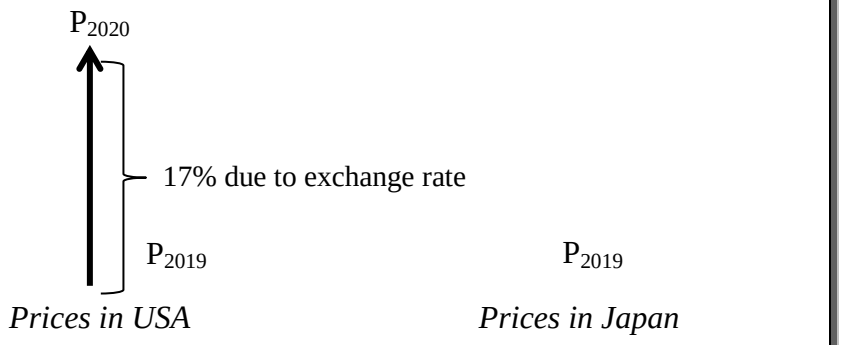
90 YEN → *100%*

100 YEN → *x%*

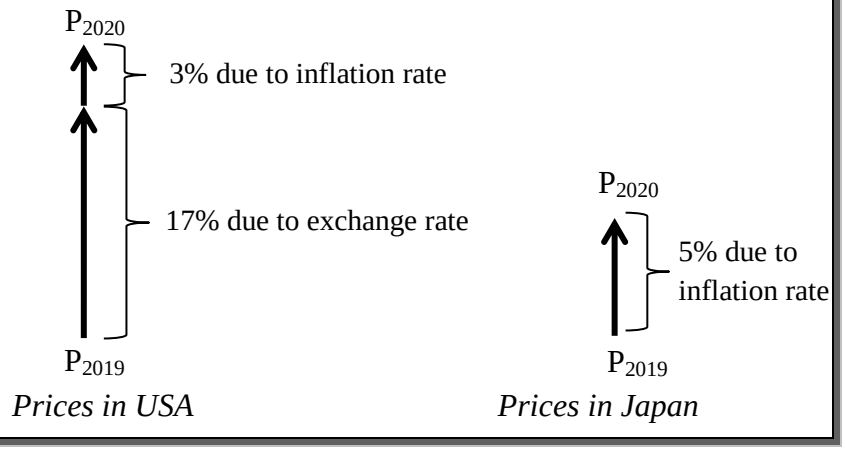
$x = \frac{100*100}{90} = 111$

USD gained 11% relative to YEN. USD became 11% more expensive relative to YEN.

When currencies gain value (become expensive) goods and services of that country also gains value. USA made goods and services also became more expensive compared to Japanese goods and services. USA made goods and services are 11% more expensive now compared to Japanese goods and services compared to last year. Let's show this using vector.



Now, that is not over, we also have inflations! Second thing we just add changes in price due to inflation. 3% inflation rate in USA means that prices of goods and services in USA increased by 3%. Inflation rate of Japan means the same. So, let's add inflation rate changes to our prices too:



Prices increased by 20% in total due to exchange rate fluctuation (17%) and inflation rate (3%) in USA. Prices increased by 5% in total due to exchange rate fluctuations and inflation of 5%. American made goods and services became 15% more expensive relative to Japanese goods and services this year. (20%-5%=15%)

Exercise 14: *Assume 1 EURO=20 Mexican PESOS. Exchange rates changed to 1 EURO=18 PESOS. Inflation rate in Eurozone is 5% and Mexico had actually a deflation of 3%. How these fluctuations impacted prices of goods and services in both countries?*

First let's calculate rate of change in exchange rates of currencies. As you can see from exchange rates, EURO became cheaper this year relative to PESO. How much cheap? EURO cost 20 PESOS last year, this year it costs only 18 PESOS per EURO:

1 EURO=20 PESO (last year)

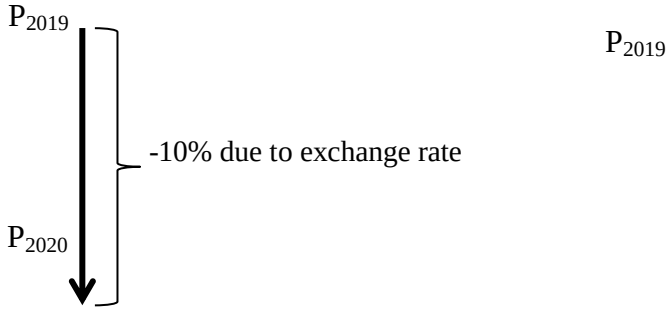
1 EURO=18 PESO (this year)

20 PESO → 100%
18 PESO → x%

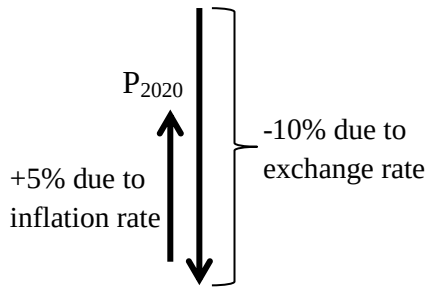
$x = \frac{18*100}{20} = 90$

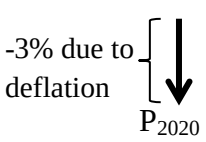
EURO dropped value by 10%. This means that prices of goods and services in Eurozone also became cheaper by 10%. Draw a vector:

Prices in Eurozone *Prices in Mexico*



Secondly, we add inflations. Inflation rate in Eurozone is 5% which means prices of goods and services increased by 5%. Deflation (currency increased purchasing power, we can buy more goods for lesser amount of money) rate in Mexico is 3% which means prices of goods and services in Mexico decreased by 3%. Let's add them to our vectors:





Prices of goods and services in Eurozone decreased by 5% (-10%+5%=-5%) this year. Prices of goods and services in Mexico decreased by 3% (3% deflation) this year. Mexican made goods and services are 2% more expensive this year relative to Euro made goods and services. (-3%-(-5%) =2%.

Homework:

I. Draw a graph where demand, short run supply and long run supply curves are in equilibrium state. New technological method of production was found (positive supply shock). Show on graph and tell me what will happen to unemployment and prices?

II. Which negative shock has worst impact on economy, negative demand shock or negative supply shock? Show on graph both shocks.

III. Draw a graph where demand, short run supply and long run supply curves are in equilibrium state. What will happen to economy (prices and unemployment rate) if new type of resources were found? Show changes in graph.

IV. Assume two neighbor countries A and B, they have no restrictions on capital and labor (capital and labor can easily travel in between countries). Wage rates in A is higher than B. Where will labor move?

V. In previous question, where will capital move if interest rate in country B is higher?

VI. Where will business move, if you know that tax rates in country A are lower?

VII. Underground economy in country B accounts around 40%, and it is 20% in country A. What do you think, to which country would organized crime had chosen to move if they had a chance to move?

VIII. Make a research about central banks of any 5 countries. Shortly write about them.

IX. Take any country and find out how are the dynamics of inflation rates in 30 years.

X. Take any country from African continent and make a research about central bank of it. Shortly write about it.

XI. Make a research about gold mines and gold production. Write the names of biggest gold mines.

XII. Money supply function is given below (in Trillion TMT). Graph this function and find out how interest rate will change if

money supply is decreased from 1500 Trillion TMT to 1000 Trillion TMT.

$$\frac{2000}{M_s} - 1 = i$$

XIII. Money supply function is given below (in Trillion TMT). Graph this function and find out when deflation of currency will start?

$$\frac{2000}{M_s} - 2 = i$$

XIV. Money supply function is given below (in Trillion TMT). Graph this function and find out how prices will change if money supply is decreased from 2000 Trillion TMT to 1000 Trillion TMT.

$$M_s^{0.3} i^{0.3} = 15$$

XV. Construct such a Money supply (or Money demand) function, so that 1% increase in money supply will decrease interest rate by 1%. Vice versa, 1% decrease in money supply must increase interest rate by 1%.

XVI. Exchange rate last year 1SOM=20 TENGE, this year 1SOM=24 TENGE. Annual inflation rate in Uzbekistan +7%, annual inflation rate in Kazakhstan +9%. How prices changed in two countries?

XVII. Exchange rate last year 1 USD=0.90 EURO, this year 1USD=0.95 EURO. Annual inflation rate in USA +4%, annual inflation rate in Eurozone +1%. How prices changed in two countries?

XVIII. Exchange rate last year 1 DINAR=9 TMT, this year 1 DINAR=8 TMT. Annual inflation rate in Tajikistan +5%, annual inflation rate in Turkmenistan -2%. How prices changed in two countries?

XIX. Exchange rate last year 1 RUBLE=94 YUAN, this year 1 RUBLE=100 YUAN. Annual inflation rate in Russia 0%,

annual inflation rate in China +10%. How prices changed in two countries?

XX. Exchange rate last year 1 BAHT=50 RUPEE, this year 1 BAHT=30 RUPEE. Annual inflation rate in Thailand +4%, annual inflation rate in India +10%. How prices changed in two countries?

XXI. In year 2000 1KG of apple was 3 SOMS, if Uzbekistan had constant annual inflation rate of 2%, then what will be price of the same 1 KG of apple in year 2030?

XXII. Japan increases its money supply by 1% annually. If total quantity of money in economy (2020) was 3000 Trillion YEN, what is going to be total quantity of money in circulation in year 2070?

XXIII. Money supply function is given below (in Trillion TMT). Graph this function and find out how money supply changed if interest rates increased from 8% to 20%?

$$M_s^{0.4} i^{0.5} = 100$$

XXIV. Governments are not afraid to spend (budget expenses) more than they earn (budget revenue) in crisis times, it is called **deficit spending** in economics science. Governments do that to smooth the business cycle. But it is very sensitive issue because if governments spend too much, later on they might get in trouble paying off the debt that was created by spending. Assume country A has annual budget revenue of 200 BLN TMT and annual budget expenses of 170 BLN TMT. It used **deficit spending** recently to come out of recession and now its budget revenue is increasing by 7% and budget spending is increasing by 3% annually. During **deficit spending** period government indebted 50 BLN TMT, and pays annual 10% interest on loan balance. If we assume that revenue and expenses will remain in constant growth, how long will it take country A to close all debt?

XXV. Exchange rate last year 1 RUBLE=30 YUAN, this year 1 RUBLE=27 YUAN. If Chinese businessman doing business in Russia exchanged 3 BLN YUAN to Russian RUBLE last year

and invested in Russia earned 10% profit wants to return to China, how much real profit he made from this investment?

XXVI. Kazakhstan needs loan in Swiss FRANK but in international exchange nobody is willing to sell FRANK to TENGE. So, Kazakhstan's government decides to purchase USD first and then purchase FRANK for USD. Exchange rates last year when Kazakhstan purchased USD then FRANK: 1 USD=9.5 TENGE, 1 FRANK=1.2 USD. This year exchange rates fluctuated a bit; 1 USD=10 TENGE, and 1 FRANK=1.1 USD. Did Kazakhstan lost or win from this fluctuation if last year Kazakhstan borrowed loaned 10 BLN FRANK with 6% annual interest on loan (also must be paid in FRANK)?

XXVII. Mexican businessman invested 1 BLN PESOS to both India and Pakistan but first PESOS we converted to national currencies of those countries. From investment in India Mexican investor earned 8% profit and from investment to Pakistan 10%. What was real profit of Mexican businessman if at the beginning of investment 1 PESOS=4.3 Indian RUPEES and 1 PESOS=6 Pakistani RUPEES (when he converted PESOS and invested), at the end when businessman wanted to convert back his total earnings to PESOS; 1 PESOS=4 Indian RUPEES and 1 PESOS=6.2 Pakistani RUPEES?

XXVIII. If you were a central bank chairman in one of developing countries, how would your reserve currency management tactics be to secure stability in financial sector?

XXIX. Japan's GDP is around 5 Trillion USD. If underground economy accounts for 9% of total economy, what is the size of illegal economy of Japan in terms of money?

XXX. Why all developed nations use **floating exchange rate** policy while many developing nations use **fixed exchange rate** policy?

XXXI. What are pros and cons of each exchange rate policy?

XXXII. Economists take inflation rates around 1% is as a "normal" inflation rate. Most of the economists actually think annual 1% inflation is better than annual 1% deflation. Why do they think so?

XXXIII. What is more preferable for export dependent nations, annual 3% inflation or annual 3% deflation?

XXXIV. Developed nations grow slower but their inflations rates are low too. Developing nations grow faster, and their inflation rate is high too. What kind of relationship do inflation and growth might have?

XXXV. What kind of economic problem does country might have if even after money supply was cut by central bank inflation is still rising?

XXXVI. Energy price increase is considered the worst of all economic shocks. Why do you think is that? Show impact of energy price hike on graph.

XXXVII. What do you know about BIS (Bank for International Settlements) and what has BIS to do with central banks of the world? Shortly.

XXXVIII. What is the most valuable currency in the world right now in terms of exchange rate?

XXXIX. What is the least valuable currency in the world right now in terms of exchange rate?

XL. China is the biggest holder of USD as a reserve currency. If USA is having annual inflation rate of 3%, how much China is losing from it?

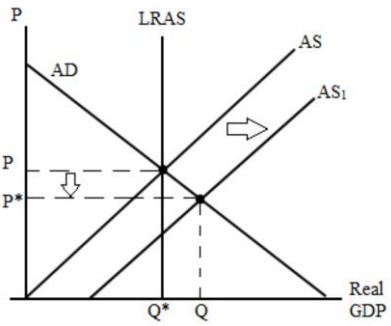
XLI. Which metals are considered “precious metals” and what are current prices of them (2020)?

XLII. Which stones are considered “precious stones” and what are current prices of them (2020)?

XLIII. Can unemployment, underground economy and inflation be eliminated at all? (0% unemployment rate, 0% underground economy and 0% inflation rate).

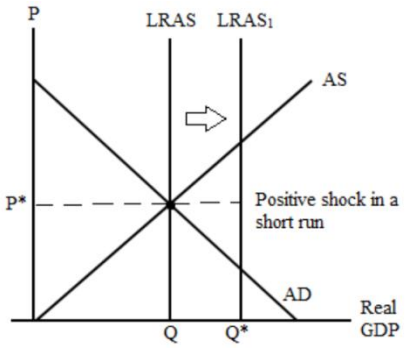
Solutions:

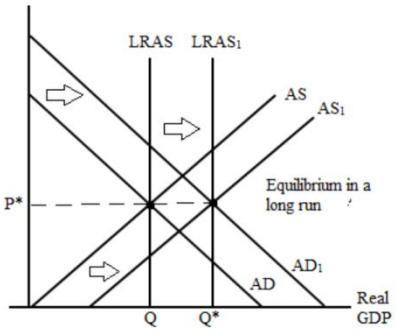
I. Prices and unemployment rate decrease due to increase in Aggregate Supply. When we are in long run equilibrium state, we have natural unemployment and natural inflation rate.



II. Negative supply shock has worse impact on economy, than negative demand, because in both cases they increase unemployment rate, but in negative supply shock in addition it increases aggregate price level in economy.

III. If new type of resources were found, in economy it means new input (or final good) for new goods and services. It increases potential output of economy, in a short run prices will not change, but unemployment will decrease.





IV. Labor will move to the country A due to higher wage rates, because all human beings rush on living better and wealthier.

V. People invest their capital where it brings higher return, consequently capital will move to the country B.

VI. All businesses will move to the country A, due to lower tax rate. Because lower tax rate means lower cost and higher profit!

VII. Organized crime would move to the country where higher opportunities to commit a crime, with higher underground

economy, higher crime rate, bureaucracy, corruption etc., which means to the country B.

VIII. Let's take top 5 the most influential central banks of the world:

1. The Fed-Central bank of United States also the strongest financial institution of the world. The Fed is considered to be independent because its decisions aren't ratified by the government or president. As the national currency of US is set as global currency, Fed constitutes the world's largest share of gold reserve worldwide. (8000 tons)
2. Bank of England- Bank of England- According to the financial system of Central bank of Great Britain, a lot of banks financial systems were based respectively. Its main functions are: Monitoring other banks and maintain monetary policy.
3. Bank of Japan-Central bank of Japan brings importance to the global economy with its rapidly growing domestic market. Functions of central bank of Japan are: maintaining financial stability, management of banknotes and enduring government security related operations.
4. Swiss National Bank- Central bank of Switzerland has mixed ownership (public and private), and its responsibilities are: maintaining financial stability and cashless payment transactions.
5. Reserve Bank of Australia-Reserve Bank of Australia-Central bank of Australia unlike Fed, make a decision with government authorities. Its responsibilities are: to keep stability of currency and full employment, regulate economic stability and financial development.

IX. Last 30-year inflation rate fluctuations of Switzerland (statista.com):

Year	Inflation rate
2020	-0.39%
2019	0.36%
2018	0.94%
2017	0.54%
2016	-0.43%
2015	-1.14%
2014	-0.01%
2013	-0.22%
2012	-0.69%
2011	0.23%
2010	0.69%

2009	-0.48%
2008	2.43%
2007	0.74%
2006	1.06%
2005	1.17%
2004	0.80%
2003	0.64%
2002	0.64%
2001	0.99%
2000	1.56%
1999	0.81%
1998	0.02%
1997	0.52%
1996	0.81%
1995	1.80%
1994	0.85%
1993	3.29%
1992	4.04%
1991	5.86%

X. Central Bank of Somalia was established in 1960. Main functions this bank implement are controlling exchange rate (Somali shilling), coordinates economic policy, fiscal policy and monetary policy.

XI. Some countries are top gold producers but not top gold miners. Table below shows Top 5 gold producing countries and gold mining companies:

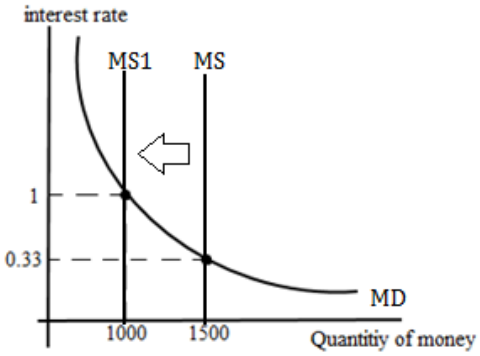
Gold producers	Gold Mining companies
China	Newmont Goldcorp (USA)
Australia	Barrick Gold (Canada)
Russia	AngloGold Ashanti (South Africa)
USA	Kinross Gold (Canada)
Canada	Newcrest Mining (Australia)

XII. When MS=1500, interest rate will be:

$\frac{2000}{1500} - 1 = 0.33\%$

When MS=1000, interest rate will be:

$\frac{2000}{1500} - 1 = 1\%$



XIII. $\frac{2000}{M_s} - 2 = i$

Deflation of currency will start when interest rate starts increasing.

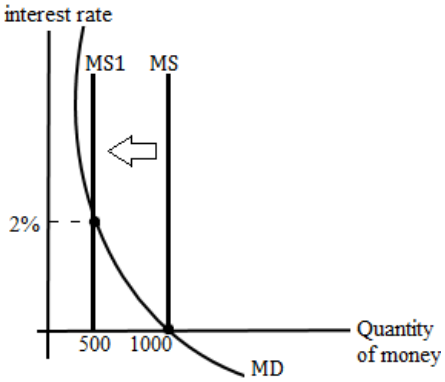
$\frac{2000}{M_s} - 2 > i$

$\frac{2000}{M_s} > 2$

$\frac{2000}{2} > M_s$

$1000 > M_s$

If quantity of money supplied is less than 100, deflation of currency will start.



XIV. Due to increase in interest rate, now the cost of the bank loan increases, quantity of people willing to borrow decrease, which decreases consumption therefore, aggregate demand decreases aggregate price level in economy.

$M_s^{0.3} i^{0.3} = 15$

$(2000)^{0.3} i^{0.3} = 15$

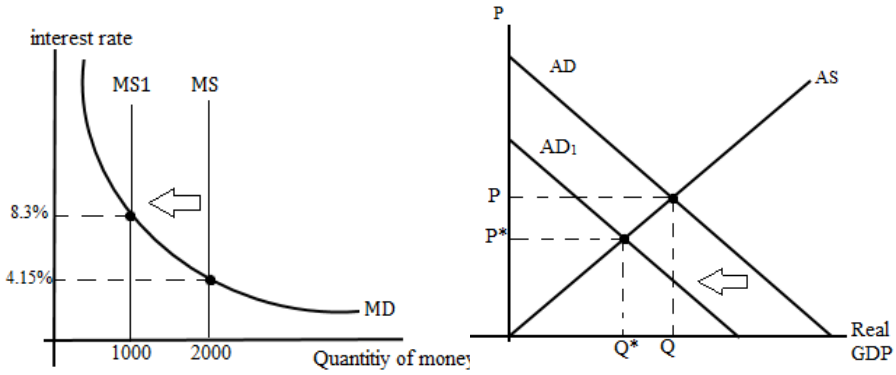
$(i^{0.3})^{3.33} = \left(\frac{15}{2000^{0.3}}\right)^{3.33}$

i=4.15%

$(1000)^{0.3}i^{0.3}=15$

$(i^{0.3})^{3.33}=\left(\frac{15}{2000^{0.3}}\right)^{3.33}$

i=8.3%



XV. $i_{t+1} = i_t - \left(\frac{MS_{t+1}-MS_t}{MS_t}\right) * 100$

Let's assume that after a certain period of time, say a year, money supply will increase for 2%, from 1TRLN to 1.02 TRLN. If today our interest rate is 4%, then after 2% change in money supply our interest rate must be 2%:

$i_{t+1} = 4 - \left(\frac{1.02-1}{1}\right) * 100 = 2\%$

XVI. 1 SOM=20 TENGE (last year)

1 SOM=24 TENGE (this year)

20 — 100%

24 — x%

$\frac{24*100}{20} = 120\%$

Uzbekistan (27%)

7% due to inflation

20% due to exchange rate

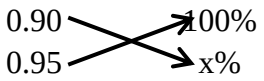
Kazakhstan (9%)

9% due to inflation

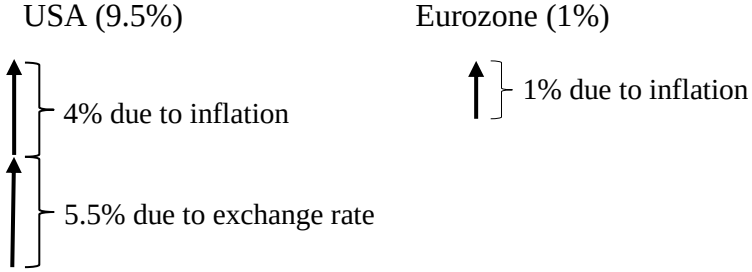
Due to inflation rate of 7% and 20% increase in price due to exchange rate overall increase in price level in Uzbekistan is 27%, while increase in price level in Kazakhstan is 9%.

XVII. 1 USD=0.90 EURO (last year)

1 USD=0.95 EURO (this year)



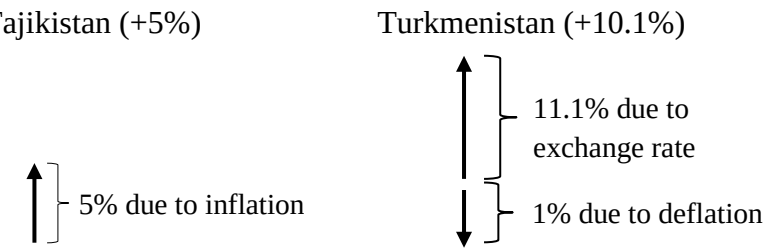
$x = \frac{0.95*100}{0.90}$ = 105.5%



Due to inflation rate of 4% and 5.5% increase in price due to exchange rate overall increase in price level in USA is 9.5%, while increase in price level in Eurozone is 1%.

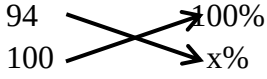
XVIII. 1 DINAR=9 TMT (last year)
1 DINAR=8 TMT (this year)

$x = \frac{8*100}{9}$ = 88.9%

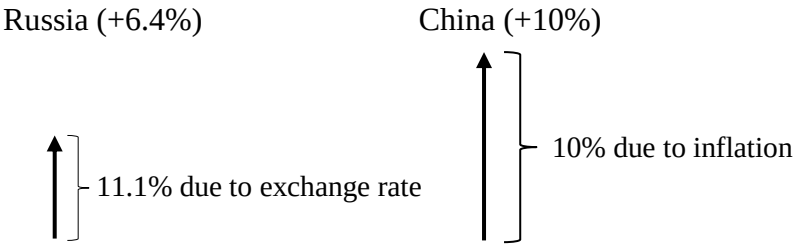


Due to deflation rate of 1% in Turkmenistan prices dropped by 1% and due to increase in currency value of 11.1% in increase exchange rate overall increase in price level in Turkmenistan is 10.1%, while increase in price level in Tajikistan 5%.

XIX. 1 RUBLE=94 YUAN (last year)
1 RUBLE=100 YUAN (this year)



$x = \frac{100*100}{94}$ = 106.4%

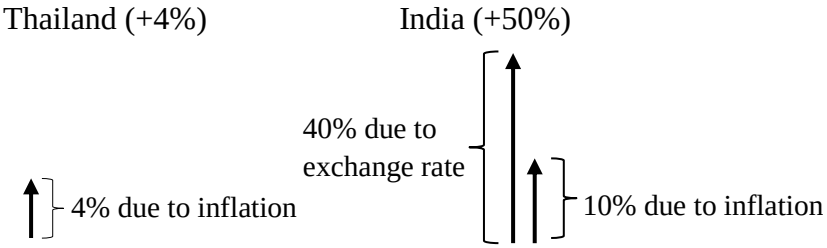


Due to inflation rate of 10% and overall increase in price level in China is 10%, while increase in price level in Russia due to exchange rate is 6.4%.

XX. 1 BAHT=50 RUPEE (last year)
1 BAHT=30 RUPEE (this year)

50 → 100%
30 → x%

$x = \frac{30*100}{50}$ = 60%



XXI. 2030-2000=30 years
$3*(1+0.02)^{30}$=5.43 SOMS per KG of apple

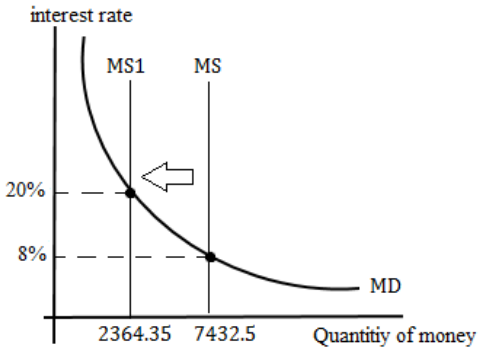
XXII. 2070-2020=50 years
Total quantity of money in circulation after 50 years
$=3000*(1+0.01)^{50}$=4933.89 Trillion YEN

XXIII. $M_s^{0.4}i^{0.5} = 100$
When interest rate is 8%, money supply will be:
$M_s^{0.4}(8)^{0.5} = 100$
$M_s^{0.4} = \frac{100}{(8)^{0.5}}$
$(M_s^{0.4})^{2.5} = \left(\frac{100}{(8)^{0.5}}\right)^{2.5}$
M_s= 7432.5 Trillion TMT
When interest rate increases to 20%, money supply will decrease to:
$M_s^{0.4}(20)^{0.5} = 100$

$M_s^{0.4} = \frac{100}{(20)^{0.5}}$

$(M_s^{0.4})^{2.5} = \left(\frac{100}{(20)^{0.5}}\right)^{2.5}$

M_s= 2364.3 Trillion TMT



XXIV. Budget revenue = 200 BLN TMT
Budget expense =170 BLN TMT
200*7%=14 BLN TMT
170*3%=5.1 BLN TMT
50%10%=5 BLN TMT
If from revenue we deduct expenses, we will have:
14-5.1=8.9 BLN TMT
And if pay off interest rate of loan:
8.9-5=3.9 BLN TMT
If with the rest 3.9 BLN TMT we pay off the principal of the debt yearly.

XXV. 1 RUBLE=30 YUAN (last year)
1 RUBLE= 27 YUAN (this year)
1 RUBLE → 30 YUAN
X RUBLE → 3 BLN YUAN

$X = \frac{3\ 000\ 000\ 000 * 1}{30}$ =100 000 000 RUBLE

100 000 000*(1+0.1) = 110 000 000 RUBLE
1 RUBLE → 27 YUAN
110 000 000 RUBLE → x YUAN

$X = \frac{110\ 000\ 000 * 27}{1}$ = 2 970 000 000 YUAN

2 970 000 000—30 000 000 000= - 30 000 000 (loss)

XXVI. 1 USD=9.5 TENGE (last year)
1 FRANK= 1.2 USD (this year)
1 FRANK → 1.2 USD
10 BLN FRANK → x USD

$x = \frac{1.2 * 10\,000\,000\,000}{1}$ =12 000 000 000 USD

1 USD → 9.5 TENGE
12 BLN USD → x TENGE

$X = \frac{12\,000\,000\,000 * 9.5}{1}$ = 114 BLN TENGE

now we calculate our loan of 10 BLN FRANK:

10 000 000 000*(1+0.06) =10 600 000 000 FRANK

1 USD=10 TENGE (last year)

1 FRANK= 1.1 USD (this year)

1 FRANK → 1.1 USD
10.6 BLN FRANK → x USD

$x = \frac{10\,600\,000\,000 * 1.1}{1}$ =11 660 000 000 USD

1 USD → 10 TENGE
11 660 000 000 → x TENGE

$x = \frac{11\,660\,000\,000 * 10}{1}$ = 116.6 BLN TENGE

116.6 BLN TENGE- 114 BLN TENGE= 2.6 BLN TENGE (profit).

XXVII. 1PESO=4.3 Indian RUPEES (last year)

1PESO=6 Pakistan RUPEES (last year)

We find how much Indian RUPEES we have:

1 PESO → 4.3 Indian RUPEES
1 BLN PESOS → x Indian RUPEES

Invest it with 8% interest rate:

$x = \frac{4.3 * 1\,000\,000\,000}{1}$ = 4 300 000 000 Indian RUPEES

4 300 000 000 Indian RUPEES*(1+0.08) =4 644 000 000 Indian RUPEES

Convert back our Indian RUPEES to Mexican PESOS with new exchange rate:

1 PESO → 4 Indian RUPEES (this year)
x PESO → 4 644 000 000

$x = \frac{1 * 4\,644\,000\,000}{4}$ =1 161 000 000 PESOS

1 161 000 000- 1 000 000 000=161 000 000 (real profit)

If we invest to Pakistan RUPEES:

1 PESO → 6 Pakistani RUPEES
1 BLN PESOS → x Pakistani RUPEES

$x = \frac{1\,000\,000\,000 * 6}{1}$ = 6 000 000 000 Pakistani RUPEES

Invest it with 10% interest rate:

6 000 000 000*(1+0.1) =6 600 000 000 Pakistani RUPEES

Convert back our Pakistani RUPEES to Mexican PESOS:

1 PESO → 6.2 Pakistani RUPEES (this year)

x PESO → 6 600 000 000 Pakistani RUPEES

$x = \frac{1 * 6\ 600\ 000\ 000}{6.2}$ = 1 064 516 129 PESOS

1 064 516 129 - 1 000 000 000=64 516 129 PESOS (real profit)

XXVIII. Developing countries use fixed exchange rate policy. So the best way is to keep in reserve vault foreign currencies to secure stability in financial sector. In case if demand for your currency increases we take money from the vault and give it to the market, and if demand for foreign currency increases we (state) sell government bonds, thus decrease money supply and appreciate currency.

XXIX. Illegal size =5 Trillion*9%= 450 BLN

XXX. In floating exchange rate policy, exchange rate is determined by supply and demand of the relative currencies, and this policy is used by developed countries because demand for their currency is almost always high, which makes float exchange rate stable (around 1-2% volatility), in addition monetary system is prudent with low underground economy. For using floating exchange rate, financial system must be developed. Developing nation's currency is very sensitive to relative foreign currency in the market, thus developing nations use fixed exchange rate policy and keep reserve currencies in vaults to keep exchange rate stable.

XXXI. Advantages of floating exchange rate policy:
Low reserve currency, monetary system is autonomic, less risk for speculative attack.
Disadvantage of floating exchange rate policy: firms are uncertain about future price level which leads to less long term investment.
Advantages of fixed exchange rate: firms and investors are certain about future prices which lead to higher long term investment and inflation rate is controlled.
Disadvantages of fixed exchange rate: huge amount of different currencies must be kept in vault which loses value year by year, due to no exchange rate fluctuations there is high probability of speculative attack.

XXXII. For mainly exporting countries inflation is better than deflation because goods for foreigners become more expensive, if their currency is appreciated, which can lead to lower demand for domestically produced goods. But if a country more imports, then, deflation of domestic currency is better than inflation because for them foreign goods become cheaper.

XXXIII. For export dependent countries annual inflation of 3% is better because for foreigners' domestic goods become cheaper if domestic currency is depreciated.

XXXIV. Developed nations' money circulation and monetary policy is more prudent and transparent, low underground economy which in turn saves central banks from printing money. While developing nations have money circulation, corruption bureaucracy problems which distract money circulation transparency. Money flows into the black hole and government starts printing money which leads to increase price level.

XXXV. If even after central bank cuts money supply, inflation is rising, this economy might have underground economy problems. Illegal money flow, money laundering, black markets create such kind of economic problems.

XXXVI. Increase in energy prices can be due to decrease of supply of energy or due to increase in demand for energy. Taking from demand increase side: number of population is growing year by year, thus, dependence for the energy is also growing which increases global demand and price for oil. From decrease in supply side: natural resources are limited, and big economies that affect to the global market prices like US, China are main producers of oil. If drop in price affects to them, it affects to the global economy too.

XXXVII. Bank of International Settlements is a global financial institution purpose of which is to coordinate central banks and maintain global monetary and financial stability. Owned by central banks with the membership of 62 countries such as Argentina, Australia, Brazil etc.

XXXVIII. The most valuable currency in the world is Kuwaiti DINAR in terms of US dollar.1 DINAR=3.29$.

XXXIX. Iranian Rial is the least valuable currency in the world in terms of USD. 42105 Irania Riyal =1$.

XL. As value decreases by 3% due to inflation, China is losing 3% of total reserved currency in USD. (Not quantity!).

XLI. Top 7 precious metals:

Precious Metals	Price in USD (per kg)
Rhodium	276,496.24
Palladium	63,979.99
Gold	55,950.34
Iridium	52,566.47
Platinum	27,488.89
Ruthenium	8,680.70
Silver	587.39

XLII. Top 7 precious stones:

Precious stones	Price in USD (per carat)
Blue Diamond	3.93 million
Jadeite	3 million
Pink Diamond	1.19 million
Ruby	1.18 million
Emerald	305000
Alexandrite	70000
Musgravite	35000

XLIII. Unemployment, inflation and underground economy can never be eliminated at all. It is impossible to make them 0%, due to imperfection of economy. In addition all 3 variables are correlated, by eliminating inflation rate, unemployment rate will increase thus crime rate will also be up high. For the growth of any economy natural inflation rate is appropriate, so we will have natural unemployment rate thus we will have crime too.

Chapter 7: Basic Finance 1

When we hear "finance" first thing that comes to our mind is banks. That is normal and actually that is true. Banks play major role in financial system, I would even say that banks play main role. Banks always played important role throughout the history of humanity in development of economy and society as a whole. When I say banks, I mean lending institutions. Back in times they might not had banking institutions in a form as we know it today, but never the less lending and borrowing was present, that is for sure. Lending was done by land owners, lords, emperors, wealthy merchants and other people and borrowers were (as today) the same businesses, states, regular peasants, and even kings and queens. There are many important functions of banking institutions but I will count the most important ones and quickly pass to the most widely used calculations in banking industry:

1) **Resources of funds:** What if there were no banks? If businesses needed funds they would have to look for investors themselves. Interest rates would have been high and very diverse, lenders would have had a problem with adverse selection (fraudsters and other bad borrowers), and many more. Instead of that we have a one place where capital flows in as deposits and everybody can go and borrow. Wealthy banks mean tons of opportunity for business.

2) **Turning savings into investment:** As we have learned before, turning savings into investment is the primary function of banks. Development of banking system and strong trust between clients and banks will create amazing atmosphere of professional relationship. Instead of holding cash and just keeping everything under the pillow people will trust to banks and leave savings there. As we know, investment is the driver of economy, and banks are main tunnels channeling savings to investment. People do not have a lot of time and professional education for research of investment opportunities and evaluation of this investment for long term. All this financial calculations and technical side of investments are hard to grasp for ordinary people. Banks will do that job. They will look for investment opportunities, they will make research and calculations, and they will measure all pros and cons, and finally come to conclusion. Like a magician pulling off a rabbit from hat, banks must pull investment returns from savings. How do they do that? They use **interest rates** for that. They lure in both depositors with interest rates and borrowers. Banks will come up with all sorts of deposit and borrowing services and these services can even be very personalized, thus interest rates might be fitted to client's needs. These packages of financial services are called **financial goods** in economics

science. Banks sell **financial goods.** Loan is a financial good, and the price of good is interest rate. When banks need funds they increase deposit rates, when banks want to increase borrowing they decrease borrowing rates. By playing with interest rates banks make profit. Deposit rates are always smaller than borrowing rates, otherwise how would banks make money? Banks make profit from difference in between deposit and borrowing rates. You can also think of it this ways: Banks purchase money from depositors, and sell money to borrowers. Bank's purchasing price of money is deposit rates, and banks selling price is borrowing rates. They make profit from difference of those two prices of money. The bigger the difference the bigger the profit. So, let's learn what kinds of calculations are used by banks in the process of lending and borrowing.

Past, present and future values of money

Not only for banks, but for all people knowing the future value of loan, investment or profit is extremely vital. We invest today to earn money tomorrow; we want to know how much we will earn. In order to make analysis, look for better options if it is needed. Aside from trade, in banking industry, you have an opportunity to know exactly how much your earning be let's say 4 years later from now when purchasing financial good (deposit for example). Bankers use mathematical methods for that and let's see how they do that.

Exercise 1: *You deposited 40000 TMT for 5 years at local bank for annual simple interest rate of 8%. How much money you will have after 5 years?*

Assume we invested at the beginning of the year, so at the end of the year we will have:

40000+40000*(0.08) =43200 TMT

At the end of second year:

43200+43200*(0.08) =46656 TMT

The rest three years are:

46656+46656*(0.08) =50388.48 TMT
50388.48 +50388.48 *(0.08) =54419.5584 TMT
54419.5584 +54419.5584 *(0.08) =58773 TMT

We will have 58773 TMT in our account after 5 years. We made 18773 TMT from our investment of 40000 TMT in 5 years.

Now, there is easier method for doing this calculation. Let's take 40000=A, then we can write first year earning as:

40000+40000*(0.08) =A+A*(r) =A (1+r) *(total money in account after first year)*
(r) = interest rate

Let's take A (1+r) =B, *then we can write second year's earning as:*

B+B*(r) =B (1+r) *(after second year)*

Let's take B (1+r) =C, *then we can write third year's earning as:*

C+C*(r) = C (1+r) *(after third year)*

Let's take C (1+r) =D, *then we can write fourth year's earning as:*

D+D*(r) = D (1+r) *(after fourth year)*

Let's take D (1+r) =E, *then we can write fifth year's earning as:*

E+E*(r) =E (1+r) *(after fifth year)*

Now, replace E with D:

D (1+r) =E then E (1+r) = D (1+r) (1+r) =D$(1+r)^2$

Now, replace D with C:

C (1+r) =D then D $(1+r)^2$= C(1+r) $(1+r)^2$= C$(1+r)^3$

Do the same with B and A and we will come to:

A $(1+r)^5$

Investment of initial amount A with simple annual interest rate 8% for five years will bring total revenue of:

A $(1+r)^5$

In our example A=40000, r=0.08 (8%) then we will have:

$40000\ (1+0.08)^5 = 58773$

This result is equal with our first solution!

This method of calculation is called finding **future value** and **present values** in economics science. We have found future value of investment. Now let's do some exercise with past and present values:

Exercise 2: *You have 200000 TMT in your bank account. That amount is the total revenue of investment done 10 years ago with simple annual interest rate of 6%. What was the initial investment amount?*

We know that:

$$A\ (1+r)^{n} = B$$

A=initial investment, r=interest rate, n=number of years and B=future value. We can substitute from our exercise A=unknown, r=0.06 (6%), n=10 (years), B=200000, then:

$$A\ (1+r)^{n} = B$$

$$A\ (1+0.06)^{10} = 200000$$

$$A\ (1.79) = 200000$$

$$A = 111732\ TMT$$

Initial investment was 111732 TMT 10 years ago.

Exercise 3: *What must be deposit rate to turn my 30000 TMT into 100000 TMT in 7 years?*

$$A\ (1+r)^{n} = B$$

A=initial investment, r=interest rate, n=number of years and B=future value. We can substitute from our exercise A=30000, r=unknown, n=7 (years), B=100000, then:

$$30000\ (1+r)^{7} = 100000$$

$$(1+r)^{7} = 3.33$$

Take ln of both sides of equation:

$$\ln\left((1+r)^{7}\right) = \ln(3.33)$$

$$7 * \ln(1+r) = 1.20$$

$$\ln(1+r) = 0.17$$

$$ln_e(1+r) = 0.17$$

$$1+r = e^{0.17}$$

1+r=1.185

R=0.185

Deposit rate=18.5%

Exercise 4: *I deposited 50000 TMT at local bank with special deal: Every year my deposit rates will increase by 1%, and initial rate is 7%. How much money in my account I will have after 10 years?*

$$A\ (1+r)^{n} = B$$

A=initial investment, r=interest rate, n=number of years and B=future value. We can substitute from our exercise A=50000, r=7% initial rate and adds 1% each year, n=10 (years), B=unknown, then after first year:

30000 (1+0.07)

Second year:

30000(1+0.07) (1+0.08)

Third year:

30000(1+0.07) (1+0.08) (1+0.09)

And so on for ten years:

30000(1+0.07) (1+0.08) (1+0.09)… (1+0.16)=88803 TMT

Exercise 5: *I saved my money under pillow. My total saving is 100000 TMT. If annual inflation rate is constant 3% in economy, what will be my saving's purchasing power after 6 years?*

Now, first of all, what does inflation mean? Inflation means losing purchasing power of currency or increase of prices (same thing!). Let's first calculate how much purchasing power will be lost after one year:

A(inflation rate) =lost purchasing power*
A is initial saving
A-A(inflation rate) =total purchasing power left*
We can also rewrite above equation as:
A-A(inflation rate) =A*(1-inflation rate) =purchasing power*
Then:

100000(1-0.03) =97000 TMT

This means that this year 100000 TMT has 97000 TMT worth of purchasing power compared to last year. Last year you could buy 100000 TMT worth of goods and services with 100000 TMT. But

this year, with 100000 TMT in hand, you can only buy 97000 TMT worth of goods and services. Money lost its purchasing power due to inflation. So, next year money continues losing its purchasing power:

$$100000(1-0.03)\ (1-0.03) = 94090\ TMT$$

After two years, with your 100000 TMT in hand, you can only purchase 94090 TMT worth of goods and services. Long word short, let's mark inflation rate=ir, initial saving as A, then if we assume that inflation rate is stable then after n=number of years, purchasing power of initial saving is B. Then:

$$A\ (1-ir)^n = B$$

In our exercise A=100000 TMT, ir=0.03 (3%), n=6. Then:

$$A\ (1-ir)^n = B$$
$$100000\ (1-0.03)^6 = B$$
$$B = 83297\ TMT$$

Constant annual inflation rate of 3% for 6 years will leave my 100000 TMT initial saving with purchasing power of 83297 TMT. My savings will lose 100000-83297=16703 TMT of purchasing power in 6 years.

Exercise 6: *What if inflation fluctuates every year? How do we do calculation in that case? Assume ir=3% first year, and 3.5%, 4.4%, 5%, 3.2% and 2% each year?*

First of all, we know that:

$$A\ (1-ir)^n = B$$

Purchasing power after one year:

$$100000\ (1-0.03)$$

Second year's purchasing power:

$$100000\ (1-0.03)\ (1-0.035)$$

Third years:

$$100000\ (1-0.03)\ (1-0.035)\ (1-0.044)$$

And so on:

$$100000\ (1-0.03)\ (1-0.035)\ (1-0.044)\ (1-0.05)\ (1-0.032)(1-0.02) = 80646\ TMT$$

Due to fluctuating inflation rates each year my initial savings of 100000 TMT will have only 80646 TMT of purchasing power left.

50000 TMT was deposited at local bank with 8% annual interest rate, but this interest rate is paid semi-annually, which means once in six months. After six months, bank will add 4% to your base

amount and next six months it will add rest 4% of all amounts in bank. Thus, we know that:

$$A\ (1+r)^n = B$$

But now bank wants to divide interest rate to two, pay you once in a six month. So operation will go this way:

$$A\left(1+\frac{r}{2}\right) = 50000\left(1+\frac{0.08}{2}\right) = 52000\text{TMT}$$

This amount will be in your account after six months. Now, let's add next six months:

$$52000\left(1+\frac{0.08}{2}\right) = 54080$$

TMT will be in your account in one year. Long word short we can write it:

$$50000\left(1+\frac{0.08}{2}\right)\left(1+\frac{0.08}{2}\right) = 50000\left(1+\frac{0.08}{2}\right)^2 = 54080$$

We can rewrite our well known formula as:

$$A\ (1+r)^n = B$$

$$A\left(1+\frac{r}{2}\right)^{nt} = B$$

Where:

(A) is initial deposit
(r) is annual interest rate,
(t) is number of times your annual interest rate will be broken down in a year, compounded in a year
(n) number of years
(B) is future value

Exercise 7: *I have deposited 1 MLN TMT 9 years ago at local banks with annual interest rate of 10% compounded quarterly. How much money do I have in my account right now?*

So, let's see numbers:

A=1 MLN, r=10%, t=4 (quarterly means once in a three month, so compounded 4 times a year), n=9 (years). So:

$$A\left(1+\frac{r}{t}\right)^{n*t} = B$$

$$1000000\left(1+\frac{0.1}{4}\right)^{9*4} = 2432535\ \text{TMT}$$

1 MLN TMT invested 9 years ago for 10% annual interest rate compounded quarterly will make me owner of 2432535 TMT today. What if I invested with simple annual interest rate?

$$A\,(1+r)^{n} = B$$
$$1000000\,(1+0.1)^{9} = 2357947 \text{ TMT}$$

If I had invested with simple 10% annual interest rate, without compounding, I would have earned only 2357947 TMT, which is 2432535-2357947=74588 TMT less.

Exercise 8: *30000 TMT deposited with 8% annual interest rate compounded daily for 5 years. How much money will be in my account 5 years later?*

Here, A=30000, r=8%, t=365 (compounded daily means our annual interest rate is broken to 365 parts a year!), n=5. Let's put it in formula:

$$A\left(1+\frac{r}{t}\right)^{n*t} = B$$
$$30000\left(1+\frac{0.08}{365}\right)^{5*365} = 44753 \text{ TMT}$$

What if compounded weekly? Then t=52 (52 weeks in a year!):

$$A\left(1+\frac{r}{t}\right)^{n*t} = B$$
$$30000\left(1+\frac{0.08}{52}\right)^{5*52} = 44740 \text{ TMT}$$

What if compounded monthly? Then t=12 (12 months in a year!):

$$A\left(1+\frac{r}{t}\right)^{n*t} = B$$
$$30000\left(1+\frac{0.08}{12}\right)^{5*12} = 44695 \text{ TMT}$$

What if compounded quarterly? Then t=4 (4 quarters in one year!):

$$A\left(1+\frac{r}{t}\right)^{n*t} = B$$
$$30000\left(1+\frac{0.08}{4}\right)^{5*4} = 44578 \text{ TMT}$$

What if compounded semi-annually? Then t=2 (semi-annual means twice a year!):

$$A\left(1+\frac{r}{t}\right)^{n*t} = B$$

$$30000\left(1+\frac{0.08}{2}\right)^{5*2} = 44407 \text{ TMT}$$

What if simple annual interest rate? Then t=1:

$$A\left(1+\frac{r}{t}\right)^{n*t} = B$$

$$30000\ (1+0.08)^{5} = 44080 \text{ TMT}$$

As you can see from above calculations, compounding increases returns (revenue!).

I want to take your attention to daily compounding from above equation:

$$A\left(1+\frac{r}{t}\right)^{n*t} = B$$

$$30000\left(1+\frac{0.08}{365}\right)^{5*365} = 30000(1.4918)$$

$$(1.4918)= e^{rn}=e^{0.08*5}$$

In finance, when we deal with daily compounding, (e^{rn}) is used. r=annual interest rate, n=number of years. Mathematically (e=2.7...).

Exercise 9: *I borrowed 10000 TMT for 40 days from local bank with annual interest rate 9% compounded daily. How much money do I have to pay back in total?*

Here, A=10000, r=9%, n=number of years, in our exercise we only loan money for 40 days, in years it is going to be $\frac{40}{365} = 0.11$*, so n=0.11, then:*

$$A(e^{rn}) = 10000\ (e^{0.09*0.11}) = 10099 \text{ TMT}$$

I will have to return in total 10099 TMT. Interest that I paid is only 99 TMT for this loan.

Exercise 10*: 20000 TMT was deposited at the beginning of 2015 for 9% annual interest rate compounded monthly for 5 years (until beginning of 2020). Annual inflation rate in year 2015 was 6%, in 2016 it was 7%, in year 2017 it was 9%, in year 2018 it was 5%, and in year 2019 it was 4%.*
A) Find total revenue from deposit
B) Find purchasing power of money at the beginning of 2020

A) From the formula we already know

$$A\left(1+\frac{r}{t}\right)^{n*t} = B$$

$$20000\left(1+\frac{0.09}{12}\right)^{5*12} = 31313 \text{ TMT}$$

B) Purchasing power of money:

PP=31313 (1-0.06) (1-0.07) (1-0.09) (1-0.05) (1-0.04) =22718 TMT

Exercise 11: *What if in above question, deposit rates were not stable too, and were increasing with 1% annually?*

A) From the formula we already know

$$A\left(1+\frac{r}{t}\right)^{n*t} = B$$

$$20000\left(1+\frac{0.09}{12}\right)^{12}\left(1+\frac{0.10}{12}\right)^{12}\left(1+\frac{0.11}{12}\right)^{12}\left(1+\frac{0.12}{12}\right)^{12}\left(1+\frac{0.13}{12}\right)^{12} = 34574 \text{ TMT}$$

B) Purchasing power of money:

PP=34574 (1-0.06) (1-0.07) (1-0.09) (1-0.05) (1-0.04) =25084 TMT

Exercise 12: *Client deposited 2 MLN TMT at local bank for 4 years with annual interest rate of 8% compounded monthly. Bank lent the money to its business client for four years with annual interest rate of 12% compounded quarterly. How much profit bank made from this deal?*

Let's calculate how much money must bank return to its client after four years (total cost of bank):

$$A\left(1+\frac{r}{t}\right)^{n*t} = B$$

$$2000000\left(1+\frac{0.08}{12}\right)^{4*12} = 2751332 \text{ TMT}$$

Let's now calculate how much revenue made bank from lending this money (total revenue of bank):

$$A\left(1+\frac{r}{t}\right)^{n*t} = B$$

$$2000000\left(1+\frac{0.12}{4}\right)^{4*4} = 3209412 \text{ TMT}$$

Total profit of bank is:

$$\prod = TR - TC$$

$$\prod = 3209412 - 2751332 = 458080 \text{ TMT}$$

Bank's profit from these deals is 458080 TMT.

Fixed payments

Banks use fixed payment methods very often, because it is convenient for bank clients. Total loan is divided to periods and client pays equal amount every period. For example, client borrows money for buying a car. Client assures that he can close this loan in 3 years with monthly payments. So, total of 12*3=36 payments. Bank gives the rate and calculates how much client will be paying every month and they sign the deal. This is called fixed payment method.

$$L = \frac{C}{(1+\frac{r}{t})^1} + \frac{C}{(1+\frac{r}{t})^2} + \frac{C}{(1+\frac{r}{t})^3} + \frac{C}{(1+\frac{r}{t})^4} + \ldots + \frac{C}{(1+\frac{r}{t})^{n*t}}$$

Where this formula came from? This formula comes from the rationale that if we did not pay monthly payments, our total loan amount in n years would have been:

$$L\left(1+\frac{r}{t}\right)^{n*t} = B$$

Which is a familiar formula we already know; the problem is that loan is not paid once, it is paid in ***n*t*** periods (or any other agreed time period) and this will lower down the loan burden for client. How it will lower down the loan burden? Let's calculate it using first method: paying off debt in one last payment. Assume we made a deal to pay all the loan in the end, no payments will be made till

the end of the contract. How much will client owe to the bank at the end of n^{th} year?

$$L\left(1+\frac{r}{t}\right)^{n*t} = B$$

At the end of n^{th} year deal client will close loan deal with B TMT. Let's play with this formula for finding second method: fixed payments:

$$L\left(1+\frac{r}{t}\right)^{n*t} = B$$

We can also write it as:

$$L = \frac{B}{\left(1+\frac{r}{t}\right)^{n*t}}$$

Since we are not paying everything at once but by periodically, then we just divide B to periods:

$$L= \frac{C}{(1+\frac{r}{t})^1} + \frac{C}{(1+\frac{r}{t})^2} + \frac{C}{(1+\frac{r}{t})^3} + \frac{C}{(1+\frac{r}{t})^4} + \dots + \frac{C}{(1+\frac{r}{t})^{n*t}}$$

We can write this:

$$L=C\ [\frac{1}{(1+\frac{r}{t})^1} + \frac{1}{(1+\frac{r}{t})^2} + \frac{1}{(1+\frac{r}{t})^3} + \frac{1}{(1+\frac{r}{t})^4} + \dots + \frac{1}{(1+\frac{r}{t})^{n*t}}]$$

This is equal to:

$$L=C\ [(\frac{1}{1+\frac{r}{t}})^1 + (\frac{1}{1+\frac{r}{t}})^2 + (\frac{1}{1+\frac{r}{t}})^3 + (\frac{1}{1+\frac{r}{t}})^4 + \dots + (\frac{1}{1+\frac{r}{t}})^{n*t}]$$

Let's say $\left(\frac{1}{1+\frac{r}{t}}\right) = a$ and since (***r, t*** > 0), **0<a<1**;

We can write above equation as:

$$L=C\ (\frac{1}{1+\frac{r}{t}})[1 + (\frac{1}{1+\frac{r}{t}})^1 + (\frac{1}{1+\frac{r}{t}})^2 + (\frac{1}{1+\frac{r}{t}})^3 + \dots + (\frac{1}{1+\frac{r}{t}})^{(n*t-1)}]$$

Replace $\left(\frac{1}{1+\frac{r}{t}}\right) = a$*:*

$$L=C*a*(1+a+a^2+a^3 \dots a^{n*t-1})$$

From mathematical series we know that:

$$(1+a+a^2+a^3+\ldots+a^{n*t-1}) = \left(\frac{1-a^{n*t}}{1-a}\right)$$

Rewrite above formula:

$$L=C*a*(1+a+a^2+a^3 \ldots a^{n*t-1})$$

$$\mathbf{L=C*a*\left(\frac{1-a^{n*t}}{1-a}\right)}$$

Replace $\left(\frac{1}{1+\frac{r}{t}}\right) = a$:

$$\mathbf{L=C*\left(\frac{1}{1+\frac{r}{t}}\right)*\left(\frac{1-\left(\frac{1}{1+\frac{r}{t}}\right)^{n*t}}{1-\left(\frac{1}{1+\frac{r}{t}}\right)}\right)}$$

This is our final formula used for fixed payments!

That is where this formula is coming from! As you will see later on, when we divide loan payments to periods our loan burden decreases!

Exercise 13: *Assume you borrowed 400000 TMT from a bank with annual interest rate of 10% compounded monthly with the deal that you will repay it with monthly fixed payments in 5 years. What is your monthly fixed payment amount?*

Let's assume we used first method: paying off debt with one last payment. Then at the end of fifth year we would have paid:

$$L\left(1+\frac{r}{t}\right)^{n*t} = B$$

$$400000\left(1+\frac{0.1}{12}\right)^{5*12} = = 658123 \text{ TMT}$$

At the end of 5th year deal client will close loan deal with 658123 TMT if uses first method: paying off debt with one single payment.

*Now, let's see how things will change when second method used: debt is closed by fixed payments. There will be total of 5*12=60 fixed amount payments to close 400000 TMT loan which is also interest bearing, 10% annual with monthly compounded. So, bank's equation is going to be like this:*

$$L=C*\left(\frac{1}{1+\frac{r}{t}}\right)*\left(\frac{1-\left(\frac{1}{1+\frac{r}{t}}\right)^{n*t}}{1-\left(\frac{1}{1+\frac{r}{t}}\right)}\right)$$

Let's first find $\left(\frac{1}{1+\frac{r}{t}}\right)=\left(\frac{1}{1+\frac{0.1}{12}}\right)=0.9917$ then:

$$400000=C*(0.9917)*\left(\frac{1-(0.9917)^{5*12}}{1-(0.9917)}\right)$$

$$400000=C*(0.9917)*\left(\frac{1-(0.9917)^{60}}{1-(0.9917)}\right)$$

$$400000=C*(0.9917)*\left(\frac{1-(0.606)}{1-(0.9917)}\right)$$

$$400000=C*(0.9917)*\left(\frac{0.394}{0.0083}\right)$$

$$400000=C*(0.9917)*(47.47)$$

$$C=8496 \text{ TMT}$$

*Client's monthly fixed payment will be 8496 TMT. He must make 60 payments until he closes the debt. So 8496*60=509760 TMT which is way below than first method, 658123 TMT.*

Exercise 14: *1 MLN TMT is loaned with 5% annual inflation rate compounded monthly be paid in 10 years with monthly fixed payments. What will be monthly fixed payment? What if loan duration shortened to 8 years? What is loan duration shortened to 5 years?*

We know that:

$$\mathbf{L=C*\left(\frac{1}{1+\frac{r}{t}}\right)*\left(\frac{1-\left(\frac{1}{1+\frac{r}{t}}\right)^{n*t}}{1-\left(\frac{1}{1+\frac{r}{t}}\right)}\right)}$$

Let's first find $\left(\frac{1}{1+\frac{r}{t}}\right)=\left(\frac{1}{1+\frac{0.05}{12}}\right)=0.9958$ then:

$$1000000=C*(0.9958)*\left(\frac{1-(0.9958)^{10*12}}{1-(0.9958)}\right)$$

$$1000000=C*(0.9958)*\left(\frac{1-(0.60)}{1-(0.9958)}\right)$$

$$1000000=C*(0.9958)*(95.24)$$

$$C=10544 \text{ TMT}$$

When n=8 years:

$$1000000=C*(0.9958)*\left(\frac{1-(0.9958)^{8*12}}{1-(0.9958)}\right)$$

$$1000000=C*(0.9958)*\left(\frac{1-(0.6676)}{1-(0.9958)}\right)$$

$$1000000=C*(0.9958)*(79.14)$$

$$C=12690 \text{ TMT}$$

When n=5 years:

$$1000000=C*(0.9958)*\left(\frac{1-(0.9958)^{5*12}}{1-(0.9958)}\right)$$

$$1000000=C*(0.9958)*\left(\frac{1-(0.7768)}{1-(0.9958)}\right)$$

$$1000000=C*(0.9958)*(53.14)$$

$$C=18900 \text{ TMT}$$

Exercise 15: *Monthly fixed payment is 5000 TMT and payment duration is 7 years, with annual interest of 10% compounded quarterly. What is my total loan amount?*

We know that:

$$L = \frac{C}{(1+\frac{r}{t})^{1}} + \frac{C}{(1+\frac{r}{t})^{2}} + \frac{C}{(1+\frac{r}{t})^{3}} + \frac{C}{(1+\frac{r}{t})^{4}} + \ldots + \frac{C}{(1+\frac{r}{t})^{n*t}}$$

But before, our monthly payment and monthly compounding matched. In this exercise our t=4. For this kind of calculations our formula is modified to:

$$L=\frac{C}{(1+\frac{r}{t})^{\frac{1*t}{12}}} + \frac{C}{(1+\frac{r}{t})^{\frac{2*t}{12}}} + \frac{C}{(1+\frac{r}{t})^{\frac{3*t}{12}}} + \frac{C}{(1+\frac{r}{t})^{\frac{4*t}{12}}} + \ldots + \frac{C}{(1+\frac{r}{t})^{\frac{12*n*t}{12}}}$$

$$\mathbf{L=\frac{C}{(1+\frac{r}{t})^{\frac{1*t}{12}}} + \frac{C}{(1+\frac{r}{t})^{\frac{2*t}{12}}} + \frac{C}{(1+\frac{r}{t})^{\frac{3*t}{12}}} + \frac{C}{(1+\frac{r}{t})^{\frac{4*t}{12}}} + \ldots + \frac{C}{(1+\frac{r}{t})^{n*t}}}$$

We can rewrite it as:

$$L=C\left[\frac{1}{(1+\frac{r}{t})^{\frac{1*t}{12}}} + \frac{1}{(1+\frac{r}{t})^{\frac{2*t}{12}}} + \frac{1}{(1+\frac{r}{t})^{\frac{3*t}{12}}} + \frac{1}{(1+\frac{r}{t})^{\frac{4*t}{12}}} + \ldots + \frac{1}{(1+\frac{r}{t})^{n*t}}\right]$$

This is equal to:

$$L=C\left[\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}+\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{2*t}{12}}+\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{3*t}{12}}+\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{4*t}{12}}+\ldots+\left(\frac{1}{1+\frac{r}{t}}\right)^{n*t}\right]$$

Let's say $\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}=a$ and since (***r, t*** > 0), **0<a<1**;

We can write above equation as:

$$L=C\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}\left[1+\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}+\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{2*t}{12}}+\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{3*t}{12}}+\ldots+\left(\frac{1}{1+\frac{r}{t}}\right)^{n*t-1}\right]$$

Replace $\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}=a$:

$$L=C*a*(1+a+a^2+a^3+...+a^{n*t-1})$$

From mathematical series we know that:

$$(1+a+a^2+a^3+...+a^{n*t-1})=\left(\frac{1-a^{n*t}}{1-a}\right)$$

Rewrite above formula:

$$L=C*a\mathbf{*(1+a+a^2+a^3+..+a^{n*t-1})}$$

$$\mathbf{L=C*a*\left(\frac{1-a^{n*t}}{1-a}\right)}$$

Replace $\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}=a$:

$$L=C\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}*\left(\frac{1-\left(\frac{1}{1+\frac{r}{t}}\right)^{n*t}}{1-\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}}\right)$$

This is our final formula used for fixed payments with all types of compounding!

C = fixed payment each period in TMT
r=interest rate bank charging for loan
t=compounding, t=12 for monthly, t=6 bi-monthly compounding,

t=4 for quarterly compounding, t=2 semi-annual compounding, t=1 for simple annual compounding.

12-denominator of $\boldsymbol{t}$, shows how many times fixed payments are paid to bank in a year. If denominator is 12 it means fixed payments are paid to bank monthly, if denominator is 6 it means fixed payments are paid to bank bi-monthly. 4 means quarterly, 2 means semi-annually, 1 means once in a year.

Let's put the numbers from exercise:
C=5000, r=10%, t=4 (quarterly!), n=7, then let's find *a* first:

$$\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}} = a \rightarrow \left(\frac{1}{1+\frac{0.1}{4}}\right)^{\frac{1*4}{12}} = 0.9918$$

$$\mathbf{L=C*a*\left(\frac{1-a^{n*t}}{1-a}\right)}$$

$$L=5000*(0.9918)*\left(\frac{1-(0.9918)^{7*4}}{1-(0.9918)}\right)$$

$$L=5000*(0.9918)*\left(\frac{1-(0.9918)^{28}}{1-(0.9918)}\right)$$

$$L=5000*(0.9918)*(25.12)$$

$$L=124580 \text{ TMT}$$

Total loan amount is approximately 124580 TMT.

Exercise 16: *Total loan=2 MLN TMT, monthly fixed payment=8000 TMT, loan duration n=10 years. What is the loan interest rate if we assume that it was simple interest rate?*

Now for solution of this type of problems we need mathematical programs, but we can actually use iteration method and can get very close to real answer. Here is how we do it:

$$L=C*\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}*\left(\frac{1-\left(\frac{1}{1+\frac{r}{t}}\right)^{n*t}}{1-\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}}\right)$$

$$\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}} = a = \left(\frac{1}{1+r}\right)^{\frac{1}{12}}$$

Then:

$$\mathbf{L=C*a*\left(\frac{1-a^{n*t}}{1-a}\right)}$$

$$2000000=8000*\left(\frac{1}{1+r}\right)^{\frac{1}{12}}*\left(\frac{1-\left(\frac{1}{1+r}\right)^{\frac{10*12}{12}}}{1-\left(\frac{1}{1+r}\right)^{\frac{1}{12}}}\right)$$

$$250=\left(\frac{1}{1+r}\right)^{\frac{1}{12}}*\left(\frac{1-\left(\frac{1}{1+r}\right)^{10}}{1-\left(\frac{1}{1+r}\right)^{\frac{1}{12}}}\right)$$

Try r=10% first:

$$\left(\frac{1}{1+0.1}\right)^{\frac{1}{12}}*\left(\frac{1-\left(\frac{1}{1+0.1}\right)^{10}}{1-\left(\frac{1}{1+0.1}\right)^{\frac{1}{12}}}\right) = 77 \text{ do not work}$$

Try r=5%:

$$\left(\frac{1}{1+0.05}\right)^{\frac{1}{12}}*\left(\frac{1-\left(\frac{1}{1+0.05}\right)^{10}}{1-\left(\frac{1}{1+0.05}\right)^{\frac{1}{12}}}\right) = 95 \text{ do not work}$$

After couple of tries you will find out that answer is 1% <r<2%

What I want to point out here is that as long as we do understand where numbers come from, the calculation must not be a problem.

Refinancing

Refinancing means changing the old loan deal to new one, with different interest rate or different time period. Banks and clients use it very often. Reason to changes might be anything starting from intentions of client to close debt faster due to economic crisis, where banks had to go to "easing" for clients otherwise most of the clients were about to be bankrupt. Especially in mortgage loan deals collateral is property itself, and for banks in case if client goes bankrupt, taking collateral property and selling it might be much costlier than loan contract modification (refinancing). Changing economic conditions such as inflation rate hikes, crisis, natural catastrophes (earthquake, flooding, etc.) will force banks and clients to change old loan deal with new one. But the details of the deal will benefit only one side, either bank or client. At the end of the day there must be only one winner. Both sides cannot win, it is zero sum game.

Exercise 17: *3 MLN TMT was loaned for 20 years with bimonthly compounded annual interest rate of 8%. Find monthly fixed payments.*

$$L=C*\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}*\left(\frac{1-\left(\frac{1}{1+\frac{r}{t}}\right)^{n*t}}{1-\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}}\right)$$

$$3000000=C*\left(\frac{1}{1+\frac{0.08}{6}}\right)^{\frac{1*6}{12}}*\left(\frac{1-\left(\frac{1}{1+\frac{0.08}{6}}\right)^{20*6}}{1-\left(\frac{1}{1+\frac{0.08}{6}}\right)^{\frac{1*6}{12}}}\right)$$

3000000=C*(0.9934) *(120.75)

C=25000 TMT

After 100th payment, bank offered a new 20 year loan with monthly compounded annual interest rate of 5%. Find out which fixed rate payment will make borrower better off.
*Now, with old rates borrower were have to make 12*20=240 payments and each payment is 25000 TMT. Let's calculate left loan amount after 100th payment:*

$$25000*\left(\frac{1}{1+\frac{0.08}{6}}\right)^{\frac{1*6}{12}}*\left(\frac{1-\left(\left(\frac{1}{1+\frac{0.08}{6}}\right)^{\frac{1*6}{12}}\right)^{100}}{1-\left(\frac{1}{1+\frac{0.08}{6}}\right)^{\frac{1*6}{12}}}\right)=1822362$$

New loan amount: 3000000-1822362=1177638; new r=5% and loan period is 20 years. Let's use well known formula again:

$$L=C*\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}*\left(\frac{1-\left(\frac{1}{1+\frac{r}{t}}\right)^{n*t}}{1-\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}}\right)$$

$$1177638=C*\left(\frac{1}{1+\frac{0.05}{12}}\right)^{\frac{1*12}{12}}*\left(\frac{1-\left(\frac{1}{1+\frac{0.05}{12}}\right)^{20*12}}{1-\left(\frac{1}{1+\frac{0.05}{12}}\right)^{\frac{1*12}{12}}}\right)$$

1177638= C*(150.75)

C=7812

Now, if you accept this new refinancing offer you will have to pay 7812*240=1874880 TMT over 20 years!
That is a great deal! If you have kept old deal you had 3500000 TMT left to pay!

If you do not accept deal you pay:
25000*240=6000000 TMT

If you accept deal you pay:
(25000*100) + (7812*240) = 4374880 TMT

So how to find if refinancing deal is good or bad? Shortest way of determining if you are better off with refinancing offer or not, you must divide left amount of previous loan to number of periods of new offer that is how optimal fixed payment is determined. If new offer comes with high fixed payment offer, then borrower is worse off if accepts. If new offer comes with lower fixed amount than determined one, then borrower is better off if accepts! In previous example:

Total left debt=3500000 TMT

New refinancing offer for 20 years, thus total number of periods are 20*12=240.

Then $\frac{3500000}{240}$ =14583 TMT

Borrower must accept any refinancing deal offer if it comes below 14583 TMT monthly fixed payments over 20 years. What if new refinancing deal is offered but period is only 12 years? Easy, again turn it to payment periods: 12* 12=144 total payment periods. Then:

Then $\frac{3500000}{144}$ =24306 TMT

If bank agrees any fixed payment below 13467 TMT, then borrower must accept new refinancing deal. If bank demands higher fixed payment, then reject the deal!

Exercise 18: *In above exercise, what must monthly compound annual interest rate be if new deal was offered with fixed monthly payment of 14583 TMT for next 20 years?*

$$L = C * \left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}} * \left(\frac{1-\left(\frac{1}{1+\frac{r}{t}}\right)^{n*t}}{1-\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}}}\right)$$

$$3500000 = 14583 * \left(\frac{1}{1+\frac{r}{12}}\right)^{\frac{1*12}{12}} * \left(\frac{1-\left(\frac{1}{1+\frac{r}{12}}\right)^{20*12}}{1-\left(\frac{1}{1+\frac{r}{12}}\right)^{\frac{1*12}{12}}}\right)$$

$$240 = \left(\frac{1}{1+\frac{r}{12}}\right) * \left(\frac{1-\left(\frac{1}{1+\frac{r}{12}}\right)^{240}}{1-\left(\frac{1}{1+\frac{r}{12}}\right)}\right)$$

Again, let's use our iteration method. Try r=3% first:
This does not work because right side of equation comes up to 180. Lower down interest rate to 2.5%. Does now work! Right side of equation equals to 186. Lower down to 1.75% then; answer is 202, which is pretty close! Try 1.5%; right side comes up 208. Try 1%; does not work comes up 213. Try 0.5%; very close, right side equals to 230! Try 0.25%; right side is 235. Las time try 0.2%; and it works!

Annuity

It is in human nature to save. Saving (investing) by fixed amount for certain period of time is called annuity. Client wants to save some money for college education of his kids, for vacation, for purchasing a house or anything, but client wants to do it with fixed amount of money monthly (or yearly, any time client wishes) from his earnings, 100 TMT or 500 TMT per month. Banks offer this type of service too. Let's explore how they do it technically:

Assume you have a daughter and she is only 3 years old. You want to save some money for her college so that when she hits 18 you would not have any problem sending her to college. You go to local bank and local bank agrees to deduct each month from your salary 500 TMT for next 15 years and deposit it to your savings account with 6% annual interest rate compounded monthly. How much money you will be able to save after 15 years?

First of all, 15 years is 15*12=180 month and 180 fixed amount payments. Let's start from first one; first payment of 500 TMT will be earning interest for 180 months:

$$500\ (1+\frac{0.06}{12})^{12*15}=1227\ TMT$$

Second 500 TMT will earn interest for 179 months!

$$500\left(1+\frac{0.06}{12}\right)^{179}=1221\ TMT$$

Third 500 TMT will earn interest for 178 months!

$$500\left(1+\frac{0.06}{12}\right)^{178}=1215\ TMT$$

So on. Let's add all earnings:

$$500\ (1+\frac{0.06}{12})^{180}+500\ (1+\frac{0.06}{12})^{179}+500\ (1+\frac{0.06}{12})^{178}+\ldots+500\ (1+\frac{0.06}{12})^{1}$$
$$=Saving$$

We can take 500 TMT's out of parenthesis:

$$500\left[(1+\frac{0.06}{12})^{180}+(1+\frac{0.06}{12})^{179}+(1+\frac{0.06}{12})^{178}+\ldots+(1+\frac{0.06}{12})^{1}\right]$$

Let's say $(1+\frac{0.06}{12})=a$ and take it out of parenthesis too:

$$500*a*[a^{179}+a^{178}+a^{177}+a^{176}\ldots+a+1]$$

From mathematical series we know that:

$$[a^{179}+a^{178}+a^{177}+a^{176}\ldots+a+1]=\left(\frac{1-a^{180}}{1-a}\right)$$

We can rewrite:

$$500*a*(\frac{1-a^{180}}{1-a})$$

$$\left(1+\frac{0.06}{12}\right)=a=1.005$$

Then:

$$500*(1.005)*\left(\frac{1-(1.005)^{180}}{1-(1.005)}\right)=146136\ TMT$$

At the end of the 15 years by just saving 500TMT per month you would have been able to save 146136 TMT. If you saved yourself any banking services under the pillow, you would have saved only 500*180=90000 TMT.

General formula for annuity (monthly fixed amount is saved) is (we already derived it above in exercise!):

$$A * \left(1 + \frac{r}{t}\right)^{\frac{t}{12}} * \left(\frac{1-\left(1+\frac{r}{t}\right)^{n*t}}{1-\left(1+\frac{r}{t}\right)^{\frac{t}{12}}} \right)$$

A = fixed amount saved each period in TMT
r=interest rate (deposit rate) bank offers for depositing in bank
t=compounding, t=12 for monthly, t=6 bi-monthly compounding, t=4 for quarterly compounding, t=2 semi-annual compounding, t=1 for simple annual compounding. 12-denominator of t, shows how many times savings is deposited to bank. If denominator is 12 it means savings are deposited monthly, if denominator is 6 it means savings are deposited bi-monthly. 4 means quarterly, 2 means semi- annually, 1 means once in a year.

Exercise 19: *How much I will be able to save if I deposit monthly 1000 TMT for 10% annual interest rate compounded quarterly for 10 years?*

$$A * \left(1 + \frac{r}{t}\right)^{\frac{t}{12}} * \left(\frac{1-\left(1+\frac{r}{t}\right)^{n*t}}{1-\left(1+\frac{r}{t}\right)^{\frac{t}{12}}} \right)$$

$$1000 \left(1 + \frac{0.1}{4}\right)^{\frac{4}{12}} * \left(\frac{1-\left(1+\frac{0.1}{4}\right)^{10*4}}{1-\left(1+\frac{0.1}{4}\right)^{\frac{4}{12}}} \right)$$

1000*(1.008)*(210) =211680 TMT

*In 10 years I will be able to save approximately 211680 TMT. If I did not use bank's annuity service, I would have been able to save only 1000*120 (month in 10 years) =120000 TMT.*

Top 100 banks according to their assets (in BLN USD)

1	China, Industrial and Commercial Bank of China	4,324.27
2	China, Construction Bank	3,653.11
3	China, Agricultural Bank of China	3,572.98
4	China, Bank of China	3,270.15
5	Japan, Mitsubishi UFJ Financial Group	2,892.97
6	United Kingdom, HSBC	2,715.15
7	United States, JPMorgan Chase	2,687.38
8	United States, Bank of America	2,434.08

9	France, BNP Paribas	2,429.26
10	France, Crédit Agricole	2,256.72
11	Japan, Japan Post Bank	1,984.62
12	Japan, SMBC Group	1,954.78
13	United States, Citigroup Inc.	1,951.16
14	United States, Wells Fargo	1,927.26
15	Japan, Mizuho Financial Group	1,874.89
16	Spain, Banco Santander	1,702.61
17	France, Société Générale	1,522.05
18	United Kingdom, Barclays	1,510.14
19	France, Groupe BPCE	1,501.59
20	China, Postal Savings Bank of China	1,467.31
21	Germany, Deutsche Bank	1,456.26
22	China, Bank of Communications	1,422.63
23	Canada, Royal Bank of Canada	1,116.31
24	United Kingdom, Lloyds Banking Group	1,104.42
25	Canada, Toronto-Dominion Bank	1,102.04
26	China, Merchants Bank	1,065.25
27	Italy, Intesa Sanpaolo	1,057.82
28	Japan, Norinchukin Bank	1,011.14
29	Netherlands, ING Group	1,000.72
30	United States, Goldman Sachs	992.97
31	China, Industrial Bank (China)	976.79
32	France, Crédit Mutuel	976.14
33	Switzerland, UBS	972.18
34	Italy, UniCredit	960.21
35	China, China Minsheng Bank	959.63
36	United Kingdom, Royal Bank of Scotland Group	957.6
37	China, Shanghai Pudong Development Bank	950.01
38	China, CITIC Bank	904.02
39	United States, Morgan Stanley	895.43
40	Canada, Scotiabank	872.62
41	Switzerland, Credit Suisse	812.91
42	Spain, Banco Bilbao Vizcaya Argentaria	782.16
43	United Kingdom, Standard Chartered	720.4
44	Australia, Commonwealth Bank	688.4
45	China, Everbright Bank	679.81
46	Canada, Bank of Montreal	665.2
47	Netherlands, Rabobank	662.77
48	Australia and New Zealand, Banking Group	661.72
49	Germany, DZ Bank	627.31
50	Finland, Nordea	622.66
51	Australia, Westpac	611.47
52	Australia, National Australia Bank	571.34
53	China, Ping An Bank	565.72
54	Denmark, Danske Bank	564.83
55	India, State Bank of India	561.54

56	Japan, Resona Holdings	549.51
57	Japan, Sumitomo Mitsui Trust Holdings	509.28
58	Canada, Canadian Imperial Bank of Commerce	495.99
59	United States, U.S. Bancorp	495.43
60	Russia, Sberbank of Russia	482.53
61	South Korea, Shinhan Bank	478.5
62	Germany, Commerzbank	478.4
63	United States, Truist Financial Corp	473.08
64	South Korea, KB Financial Group Inc	449.15
65	Spain, CaixaBank	439.25
66	Singapore, DBS Bank	430.45
67	Japan, Nomura Holdings	425.5
68	China, Huaxia Bank	422.74
69	Netherlands, ABN AMRO Group	420.89
70	United States, PNC Financial Services	410.3
71	Brazil, Itaú Unibanco	407.37
72	United States, Capital One	390.37
73	United States, The Bank of New York Mellon	381.51
74	China, Bank of Beijing	374.97
75	South Korea, Nonghyup Bank	369.92
76	Singapore, OCBC Bank	365.57
77	Brazil, Banco do Brasil	365.51
78	South Korea, Hana Financial Group	365.1
79	Brazil, Banco Bradesco	345.21
80	China, Guangfa Bank	343.26
81	Sweden, Handelsbanken	328.59
82	Belgium, KBC Bank	327.87
83	Brazil, Caixa Econômica Federal	321.68
84	Norway, DNB ASA	317.75
85	South Korea, Woori Bank	313.54
86	United Kingdom, Nationwide Building Society	307.45
87	China, Bank of Shanghai	306.04
88	Sweden, SEB Group	305.79
89	France, La Banque postale	304.88
90	Singapore, United Overseas Bank	300.68
91	Japan, Fukuoka Financial Group	296.58
92	Germany, Landesbank Baden-Württemberg	287.99
93	Austria, Erste Group	275.72
94	South Korea, Industrial Bank of Korea	275.54
95	Germany, BayernLB	266.27
96	Qatar, National Bank	259.48
97	China, Zheshang Bank	258.63
98	Sweden, Swedbank	257.79
99	Switzerland, Raiffeisen Gruppe	256.43
100	Spain, Banco Sabadell	251.1

3) **Create money:** When money is under pillow, they are out of circulation. When money is in bank (as a deposit or long term investment, etc.) money creation process starts. This is how process works: Assume you deposit 1000 TMT in your account at local bank. Bank has an obligation to keep certain portion of deposits for liquidity reservations put by central bank. Let's say it is 10% of all deposits. The rest of money, 900 TMT, bank use for earning revenue. Assume your neighbor needs money, 900 TMT for three months, and she applies for loan and bank lends the money. Your neighbor spends it to cover needs. Here, your 1000 TMT deposit money created a 900 TMT for somebody else. Your unused (unneeded for some time money) is used in circulation creating incomes and returns for somebody else. In efficient economy, all savings are turned to investments. More investments mean faster growth and development. Economies must strive for achieving efficient financial system.

Exercise 20: *Assume Central bank obliges to keep all banks 10% of all deposits in vault for liquidity safety reasons. This is called reserve rates. Thus, banks can use only 90% of all deposits for gaining revenue. Theoretically, if we assume that economy is 100% efficient, thus all savings are turned into investments, then find out how much money banks will create from 1 BLN TMT deposit?*

1 BLN TMT deposited to first bank will have to keep 10% of it and use the rest, 900 MLN TMT for lending. Lenders take loan money and pay off their debts from local supermarket. Supermarket manager deposits all income to deposit account (do not forget that we assumed economy to be perfectly efficient, all money is in banks), and banks again can use 90% of deposited money for lending and so on. We can show it mathematically as:

Total money created by banks from 1 BLN TMT:

1BLN+1BLN*(90%) +900MLN*(90%) +810MLN*(90%) + 729MLN*(90%)...we can shortly write it as:

$$1\ BLN*[1+(0.9)+(0.9)^2+(0.9)^3+(0.9)^4+\dots\ ] = 1BLN*\left(\frac{1-(0.9)^n}{1-(0.9)}\right)$$

this formula comes from mathematical series:

$$B+B*a+B*a^2+B*a^3 \dots * B*a^{n-1}=B*\left(\frac{1-(a)^n}{1-(a)}\right)$$

*We can say here that **n** is so large and continues that it goes to infinity, thus n→∞. Also, since in our example a<0 (a=0.9) then we use limits to prove that:*

$\lim_{n\to\infty}\left(\frac{1-(a)^n}{1-(a)}\right)=\frac{1}{1-a}$ because as n→∞, then $(0.9)^n \to 0$

Then our formula:

$$1\text{BLN}*\left(\frac{1-(0.9)^n}{1-(0.9)}\right) \to 1\text{BLN}*\frac{1}{1-(0.9)}=\frac{1\text{BLN}}{0.1}=10\text{BLN}$$

In perfectly efficient economy with efficient financial system (where all savings are turned to investment) banks will create 10 BLN TMT from only 1 BLN TMT initial deposit.

***Exercise 21:** Central bank reserve rates in country A with efficient economy is 8%. How much money will be created in economy if initial investment is 20 BLN SOMS?*

$$20\text{ BLN}*[1+(0.92)+(0.92)^2+(0.92)^3+...]=20\text{BLN}*\left(\frac{1-(0.92)^n}{1-(0.92)}\right)$$

$\lim_{n\to\infty}\left(\frac{1-(0.92)^n}{1-(0.92)}\right)=\frac{1}{1-0.92}$ because as n→∞, then $(0.92)^n \to 0$

$$20\text{BLN}*\left(\frac{1}{1-(0.92)}\right)=\frac{20\text{BLN}}{0.08}=250\text{BLN SOMS}$$

In perfectly efficient economy with efficient financial system (where all savings are turned to investment) banks will create 250 BLN SOMS from only 20 BLN SOMS of initial deposit.

***Exercise 22:** An island has perfectly efficient financial system and total quantity of money in circulation is 100 BLN USD. What is the money base (initial deposit) if reserve rate in an island is 20%?*

By formula we derived above:

$$\text{Money created (money in circulation)}=\frac{Initial\ deposit\ (money\ base)}{reserve\ ratio}$$

$$100\text{BLN}=\frac{Initial\ deposit\ (money\ base)}{0.2}$$

Initial deposit = 20 BLN USD.

4) **Clients' database:** It might sound much unexpected but banks know the best about financial situation of people in

> economy. They know the trend, they know flow of money, and they know profitability of sectors because they are right in the middle of it. In the middle of financial crossroads. Clients borrow, depositors invest, businesses rise up and go bankrupt and almost all of them deal with banks. No wonder all prestigious banks also make not bad money on consultation services. Banks know the last rumors, all gossips and upcoming financial reforms because these topics touch financial interests of everybody. Banks are the first ones who make reforms when they feel the need.

Financial sector is very lucrative and volatile by its nature. Especially when concerning banks. Panic is the biggest enemy of financial sector that is why governments must always keep one hand in the pulse of financial market and immediately clarify issues arising in financial market. Moral hazard problem arising from "unknown" is very dangerous in financial world. Loss from panic is more dangerous, hazardous and continuous compared to any other loss that is why building trust between financial institutions, government and people are vital for building strong and sustainably growing financial market. Strong channel must be built in between that trio: government, financial institutions and people. Transparency is a key for financial development. It is a fact that banks play main role in whole financial system, so stability of whole financial system strongly depends on stability of banking sector. Recently, not only financial stability in local market became a big deal, but also global financial stability was the main issue of our global world. We cannot deny the fact that all countries depend on each other: depend financially, ecologically, economically, politically, etc. For sustainable financial growth, we need financial stability everywhere, in all market. That is why, Bank for International Settlements (BIS, Headquarters in Basel, Switzerland) Basel Committee had come up with Basel Accords set of recommendations for banks around the world to increase trust and liquidity. These recommendations concerns reserve rates of banks, investments, holding risky assets and so on. This is amazing, and I strongly believe that world must move in that direction; in the direction of financial cooperation. This will increase financial security, stability and growth. Never the less, rate of financial crimes increases! Money laundering, hacking, financing of terrorism, etc. increasing, and majority of those crimes were accomplished through banking system. This poses serious questions and challenges for our global world. What must be done to increase efficiency of banks; to make them more transparent, less risky and more economic friendly? I am saying economic friendly because in mortgage crisis of 2008, the term "too big to fail" was popping up in media. Some banks had grown so big that their failure threatened whole financial system. How big banks must be allowed to grow. Enough big to threaten whole system?

How financial market must be structured to deal with this type of problems? And how many more is awaiting us in the future?

Homework:

I. Loan amount 2 MLN TMT, annual interest rate 4% compounded monthly. How long it will take to make it 3 MLN TMT?

II. 1 MLN TMT invested to a project which brings 10 % annual return. Bank gives annual 8% interest compounded daily. Which one is better investment option?

III. You loaned 3 MLN TMT 8 years ago and paying fixed rate of 37500 TMT per month. If loan was for 20 years, what is bank's annual interest rate if annual interest is simple?

IV. You loaned 2.5 MLN 9 years ago, and paying 18000 TMT fixed monthly payment for 20 years. If bank offers you to refinance your old loan with new one for 15 years, what must be new monthly fixed payment in order this deal to be fair?

V. My son is growing up, and I need to save some money for his college education. I have only 10 years left till his college age comes. Minimum college fees are 150000 TMT and banks are giving maximum of 7% annual interest rate compounded quarterly. What is my monthly fixed amount of saving must be?

VI. I need 200000 TMT in 5 years and I have 50000 TMT. Annual simple interest rate is 14%, annual monthly compounded 10%, and daily compounded is 8%. Whivh investment option will reach 200000 TMT first?

VII. I deposited to bank 2 MLN for three years with bi-monthly compounded interest of 8%, and bank lend that money for 10% daily compounded rate for three years. How much profit bank made?

VIII. 40 days later I returned 45000 TMT (interest is included). What is my initial loan amount if bank charged 7% annual interest rate compounded daily?

IX. If I deposit monthly 2000 TMT for 9 years for compounded daily annual interest rate of 9%, will that be enough to buy 300000 TMT priced car?

X. I and my brother, we both deposited our money at the same bank. I deposited 2 MLN TMT and my brother deposited 2.2 MLN. In 8 years I had exactly 4 MLN in my account and my brother had 4.5 MLN. We know that we both had

same rates, but compounding was different. What is rate and compounding?

XI. How many Basel Accords are there?

XII. 5 MLN TMT is loaned for 25 years for 32000 TMT monthly fixed payment with annual simple interest rate X. 10 years later, a new deal was offered to pay off old loan: 20 year loan offer for 25000 monthly fixed payment. Firstly, find approximate interest rate of first deal and secondly tell if new refinancing deal is good or bad?

XIII. Assume efficient financial system where all saving turned into investments. If 30 BLN TMT is initial deposit to bank, how much money will be created if reserve rates are 10%?

XIV. Total money in circulation is 2 TRLN TMT; government wants to increase quantity of money in circulation to 3 TRLN TMT. How much money must be deposited to banks to increase money circulation if reserve rates are 12%?

XV. Total money in circulation is 5 TRLN TMT, and inflation rate is 10% in economy. Central bank knows that (from experience) to decrease inflation by 1%, total money in circulation must be decreased by 4%. To increase inflation by 1% money supply must be increased by 4%. In economy where reserve rate is 8%, how Central bank can decrease inflation rates from 10% to 2% using monetary policy?

XVI. I deposited 500000 TMT for 8 years for annual interest rate of 6% compounded daily. If annual inflation rate is constant 3%, find purchasing power of my deposit at the end of 8^{th} year.

XVII. How governments can fight against “natural volatility” of financial markets?

XVIII. Which one is the biggest enemy of financial world: panic or inflation?

XIX. Assume your revenue is 1 MLN TMT and your total cost is 0.6 MLN TMT. If revenues are rising with constant rate of 5% and total cost is with 10%, how many years later you will start making losses?

XX. Why do banks use refinancing?

XXI. Assume you have a “dream bike” which is priced around 0.4 MLN TMT and price is rising with annual constant rate

of 2%. You plan to purchase this bike by saving 10000 TMT per month at local bank which gives 12% annual interest rate compounded monthly. How long will take till you save enough to purchase this bike?

XXII. Price of only 1 banana is 1 TMT. What will be price of banana 100 years later if prices are rising by constant annual rate of 1%?

XXIII. Total quantity of money in circulation in both countries is equal but inflation rates are different. Why do you think is that?

XXIV. Reserve rates and inflation rates in two countries are equal but total quantity of money in circulation is different. Why do you think is that?

XXV. Annual interest rate is 8% compounded semi-annually, I want to save total of 800000 TMT in 8 years. What must be my monthly fixed saving plan?

XXVI. Annual interest rate is 8% compounded semi-annually, I want to save total of 800000 TMT in 8 years. What must be my initial deposit?

XXVII. I deposited 1 MLN TMT for six years with fluctuating market interest rate compounded weekly. Interest rates are (5.5%), (6.2%), (7%), (7.5%), (8.8%) and (9%). What is my total earning after six years?

XXVIII. At 2 TRLN TMT in money circulation inflation rate is 0. Increase of 1% of quantity of money in circulation will increase inflation by 0.3%. State plans to keep inflation rate around 2%. If reserve rate in banks is 15%, how much money must be deposited initially to keep the targeted inflation rate?

XXIX. When I borrow money from banks for fixed annual interest rate, and then inflation hikes up, who is the winner, me or a bank (borrower or lender)?

XXX. During a deflation in economy, who are winners, borrowers or lenders?

XXXI. Total quantity of money in circulation is equal in two countries; does this mean that sizes of both economies are equal?

Solutions:

I. 3MLN TMT=2 MLN TMT*$(1+\frac{0.04}{12})^{12*n}$

$\frac{3\text{ MLN TMT}}{2\text{ MLN TMT}} = (1+\frac{0.04}{12})^{12*n}$

$1.5 = (1+\frac{0.04}{12})^{12*n}$

$\ln 1.5 = 12*n*\ln(1+\frac{0.04}{12})$

$12n = \frac{\ln 1.5}{\ln(1+\frac{0.04}{12})}$

12n=121.8

n=10 years

II. FV=1 000 000*(1+0.1) =1 100 000 (if we invest to the project)

FV=1 000 000* $e^{0.08*1}$=1 083 287 (if we invest to the bank)

Investing to the project with 10% return is better.

III. Loan amount=3 MLN TMT

Monthly fixed payment=37500TMT

Simple interest rate: t=1

$$\left(\frac{1}{1+\frac{r}{t}}\right)^{\frac{1*t}{12}} = a = \left(\frac{1}{1+r}\right)^{\frac{1}{12}}$$

$$3000000 = 37500*\left(\frac{1}{1+r}\right)^{\frac{1}{12}}*\left(\frac{1-\left(\frac{1}{1+r}\right)^{20}}{1-\left(\frac{1}{1+r}\right)^{\frac{1}{12}}}\right)$$

Try r=15%:

$$80 = \left(\frac{1}{1+r}\right)^{\frac{1}{12}}*\left(\frac{1-\left(\frac{1}{1+r}\right)^{20}}{1-\left(\frac{1}{1+r}\right)^{\frac{1}{12}}}\right) = 80$$

IV. C=18000 TMT

Total loan amount paid after 20 years=18000*20*12= 43200000

Total loan amount left after 9 years: 43200000-(9*18000*12) =2376000 (left to pay)

$\frac{2376000}{15*12}$ = 13200

If new coupon payment is 13200, then the new deal will be fair.

V. $150000 = S * \left(1 + \frac{0.07}{4}\right)^{\frac{4}{12}} * \left(\frac{1-(1+\frac{0.07}{4})^{4*10}}{1-(1+\frac{0.07}{4})^{\frac{4}{12}}}\right)$

150000=S*(1.0058) *(172.7)
S=863

VI. 200000=50000*(1+0.14) n

$\ln(\frac{200000}{50000})$= n*ln(1+0.14)

n= $\frac{\ln(\frac{200000}{50000})}{\ln(1+0.14)}$= 10.58

200000=50000*$\left(1 + \frac{0.1}{12}\right)^{12*n}$

$\ln(\frac{200000}{50000})$= 12*n*ln(1+$\frac{0.1}{12}$)

12*n= $\frac{\ln(\frac{200000}{50000})}{\ln(1+\frac{0.1}{12})}$= 167

n= 13.92

200000=50000*$e^{0.08*n}$

$\ln(\frac{200000}{50000})$= 0.08*n*ln e

n= $\frac{\ln(\frac{200000}{50000})}{0.08}$= 17.33

(investment with 14% annual interest rate will reach 200000 first)

VII. 2000000*(1+ $\frac{0.08}{6}$)6*3=2538469.3 TMT
2000000*$e^{0.1*3}$ =2699717.6 TMT
Profit of the bank is: 2699717,6 - 2538469,3=161248.3 TMT

VIII. 45000=PV*$e^{0.07*\frac{40}{365}}$

PV= $\frac{45000}{e^{0.07*\frac{40}{365}}}$= 44656 TMT is my initial amount

IX. $S = 2000\left(1 + \frac{0.09}{365}\right)^{\frac{365}{12}} * \left(\frac{1-(1+\frac{0.09}{365})^{365*9}}{1-(1+\frac{0.09}{365})^{\frac{365}{12}}}\right)$

S=2000*(1.0075) *(166.36)
S=335215.4
Yes, it will be enough to buy 300000 TMT priced car.

X. 4 MLN = 2MLN*$(1+i)^8$

ln2=8ln*(1+i)

$\frac{\ln 2}{8}$= ln *(1+i)

0.0866= ln *(1+i)

$e^{0.0866}$=1+i

(1.09-1) =i=0.0905 or 9.05%

4.5 MLN = 2.2 MLN*$\left(1+\frac{0.0905}{t}\right)^{8*t}$

From iteration method we find that it is daily compounding.

XI. Basel Accord is regulations which set Basel Committee on Banking Supervision to regulate credit risk. There are 3 Basel accords.
Pillar1: Minimum capital requirement.
Pillar2: Requires supervisory review process.
Pillar3: Requirement of market discipline.

XII. C=32000
L=5000000
N=25 years

$$\frac{5000000}{32000} = 156.25 = \left(\frac{1}{1+r}\right)^{\frac{1}{12}} * \left(\frac{1-\left(\frac{1}{1+r}\right)^{25*1}}{1-\left(\frac{1}{1+r}\right)^{\frac{1}{12}}}\right)$$

We try 6.3%

$$\left(\frac{1}{1+0.063}\right) = 0.9407$$

$\left(\frac{1}{1+0.063}\right)^{\frac{1}{12}} * \left(\frac{1-\left(\frac{1}{1+0.063}\right)^{25*1}}{1-\left(\frac{1}{1+0.063}\right)^{\frac{1}{12}}}\right)$ = 153 which is very close to 156.25

After 10 years our loan is: 32000*25*12- 32000*10*12= 5760000 TMT
20*12=240 new payments

$\frac{5760000}{240}$ = 24000 TMT

To benefit from refinancing our new coupon must be less than 24000 TMT while new coupon is 25000 TMT which makes loss of 240*(25000-24000) =240000 TMT.

XIII. $30BLN*(\frac{1-(0.9)^n}{1-(0.9)})$

$30BLN*\left(\frac{1}{1-(0.9)}\right) = \frac{30\ BLN}{0.1}$ = 300 BLN TMT

XIV. $\frac{x}{0.12} = 1$ TRLN TMT

1 000 000 000 000*0.12=x

x = 120 000 000 000 TMT must be deposited to increase money in circulation.

XV. 1% deflation leads to 4% decrease of money supply. To decrease inflation rate by 8%, we must decrease money supply by 32%.
Central bank can decrease money supply by 32%, which is: 5 000 000 000*32%=1 600 000 000 TMT by selling government bonds, decreasing deposit rates or increasing borrowing rates.

XVI. $500000*e^{8*0.06}$ =808037

$808037*(1-0.03)^8$=633293.6 TMT (is purchasing power of 500000 after 8 years)

XVII. As was said above in the chapter, "governments must always keep one hand in the pulse of financial market and immediately clarify issues arising in financial market" to fight against "natural volatility" of financial markets. Developed countries like US avoid it through their white house speaker, which presses publicly what is going on every day in financial markets.

XVIII. Panic is the biggest enemy of financial world. Moral hazard problem arising from "unknown" is very dangerous in financial world, which can lead to bank run, inflation hikes and eventually to crisis. Loss from panic is very dangerous

XIX. $1\ 000\ 000*(1+0.05)^n - 600\ 000(1+0.1)^n < 0$

$\ln(\frac{1000000}{600000}) < n \ln(\frac{1+0.1}{1+0.05})$

$n > \frac{\ln(\frac{1000000}{600000})}{\ln(\frac{1+0.1}{1+0.05})} > 10.98$

After approximately 11 years I will start making a loss.

XX. When economic conditions worsen, borrowers may get into trouble in paying off loans. Bankruptcies are not good for

banks either. That is why banks may offer deals for troubled clients. It is called refinancing deal.

XXI. If I don't want to deposit my money into the bank and if we assume that price of the bike remains constant, then I will buy this bike after:

10000*12*t=400000

T= $\frac{400000}{120000}$= 3.3 years

If I deposit to the bank with 12% interest rate compounded monthly and with monthly payment of 10000 TMT, then after 3.3 years I will have:

$$P = 100000\ \left(1+\frac{0.12}{12}\right)^{\frac{12}{12}} * \left(\frac{1-\left(1+\frac{0.12}{12}\right)^{12*3.3}}{1-\left(1+\frac{0.12}{12}\right)^{\frac{12}{12}}}\right) = 487779$$

TMT

And after 3.3 years price of the bike will be:
FV =400000(1+0.02)$^{3.3}$=427012 TMT
After 3.3 years I can buy this bike, in addition I will have my savings:
487779-427012=60767 TMT

XXII. $1(1+0.01)^{100} = 2.7$ TMT

XXIII. Because of different factors like population quantity, underground economy, different monetary policy distracts money circulation to be the same.

XXIV. Money supply must be according to money demand in economy. Each country has different money demand function. Size of economies is different.

XXV. $$800000 = C * \left(1+\frac{0.08}{2}\right)^{\frac{2}{12}} * \left(\frac{1-(1+\frac{0.08}{2})^{2*8}}{1-(1+\frac{0.08}{2})^{\frac{2}{12}}}\right)$$

800000=C*(1.0065) *(134.3)

C=5918.34 TMT

XXVI. $$800000 = \text{PV} * \left(1+\frac{0.08}{2}\right)^{2*8}$$

PV= $\frac{800000}{(1+\frac{0.08}{2})^{2*8}}$= 427126.5

XXVII. FV=1000000*

$$*\left(1+\frac{0.055}{52}\right)^{52}\left(1+\frac{0.062}{52}\right)^{52}\left(1+\frac{0.07}{52}\right)^{52}$$

$$*\left(1+\frac{0.075}{52}\right)^{52}\left(1+\frac{0.08}{52}\right)^{52}*\left(1+\frac{0.09}{52}\right)^{52}$$

=1552211.5TMT

XXVIII. 1% increase in MS → 0.3% increase in inflation rate

X% MS → 2% inflation rate

X * increase in MS= $\frac{1*2}{0.3}$ =

6.67% increase in money supply

Money in circulation = $\frac{Initial\ deposit\ (money\ base)}{reserve\ rate}$

2 TRLN TMT+2 TRLN*6.67%=2.1334 TRLN TMT

0.1334TRLN TMT= $\frac{Initial\ deposit\ (money\ base)}{reserve\ rate}$

0.1334 TRLN* 0.15=0.02001 TRLN TMT

XXIX. After borrowing money from the bank, if inflation hikes up borrower is a winner, because banks will probably increase borrowing rates to catch up with inflation rates. The ones that borrowed before rate reforms will win.

XXX. During deflation in economy lenders are winners. Due to increase of purchasing power of money lenders will probably decrease interest rates, all those borrowed before decreasing interest rates are losers.

XXXI. Quantity of money in circulation does not tell anything about the size of economy.

Chapter 8: Basic Finance 2

The main function of financial institutions is turn savings into investments. Banks attract surplus money with deposit rates and create loans (investment into economy). Insurance companies provide insurance services and collect premiums and invest it into economy. Pension funds use pension contributions to invest them for long term. Stock and derivative exchange markets are the main source of investment funds for corporations. That is why if economies are planning to grow, they must develop their financial markets and must provide all required for development of financial institutions because they are the main mechanism of economy turning savings into investment. As we have talked before, investment is the core of growth and development. Instead of studying finance like other books teach it, I chose to teach (learn) finance through learning functions of financial institutions. What do they do, and how do they do it? We have learned about banking sector and what type of calculations they use when providing banking services. In this chapter we will learn about insurance services and pension system.

Insurance

People are risk averse by nature. Risk averseness means that people would not want to take irrational risks: risks that have high probability of occurrence and very low payback. Even if people take risks, we want to take rational risk: risk that has low probability of occurrence and high payback. Insurance service is built on exactly on that feeling of human being. People will be ready to pay, just not to face that risk. We are ready to pay not to lose our property, health, or our dire lives, if we face with such a risk. Insurance services provide exactly that: it transfers risk from **insured** onto itself (**insurer**) for certain payment which is called *premium* in professional insurance world. All technical aspects (hazards, payment, duration of insurance coverage, beneficiaries, etc.) will be mentioned in insurance contract in between insured and insurer. *Insurance policy* issued by insurance company aftermath of signed agreement will be the main legal document providing insured with financial security. *Adverse selection* and *Moral hazard* problem arise for insurance companies. *Adverse selection* problem arises for insurer because insured knows better the „real" situation than insurer, and insured can benefit from that. In order to eliminate adverse selection problem insurer, requires insured to pass from certain tests (especially in health insurance) or sign up special questionaries' forcing insured to "play fair". To overcome *moral hazard* problem arising due to behavior changes of insured after signing insurance contract, insurers put "conditions" in contract mentioning when and which hazards will

not be covered by policy. *Deductibles* and *co-insurance* are also used to encourage insured to behave "normally". Deductibles are amount of hazard which will be covered by insured. *Co-insurance* is same as deductibles but in percentage terms. Almost all insurance contracts today have deductibles or use co-insurance terms (especially in property and health insurances) and insurance companies never cover all hazards by 100%. Generally, insurance companies cover 80% of hazard only, and that is called full coverage. Insured who knows that in case of damage, he will have to cover 20% of all lost. This makes insured behave carefully even if he has purchased insurance coverage effectively eliminating moral hazard problem for insurer. Insurance sector is full of legal battles because insurance companies will try to decrease the hazard amount while insured will want to be compensated more generously. That is why for future insurance specialists (agents or brokers) having strong legal background is big plus, because technical calculations in insurance sector are not that much as legal details. Insurance business built on four fundamental principles: **Principle of indemnity, Principle of insurable interest, Principle of subrogation and Principle of utmost good faith. Principle of indemnity** states that insured must not profit from hazard, thus, insurer will cover no more than actual loss. If your nose had been broken broken before purchasing of health insurance and you were hospitalized due to car accident where your ribs were broken and if you decide to fix your nose too, insurance company will not cover your nose surgery, because this will break the principle of indemnity. Insurance company will not allow you to gain profit from accident. If you had a car accident and your 14 inch rims broken in accident, insurance company will not replace them with better rims, they will replace them with almost identical one. Otherwise you would have profited from accident. **Principle of insurable interest** states that insured must have personal interest in insured, otherwise insurance will not be allowed because it is going to be a gamble. Purchasing insurance policy will be impossible if you cannot prove that you have personal interest in insured. Insurance company, for example, will not allow me to insure my neighbor's property if I do not own some share from his property. If they were allowing me, later I can burn down my neighbor's house and could have earned some decent profit. I am not allowed to purchase health insurance on somebody who is not my relative or my employee, because later on I can simply do something to that person and as a beneficiary can make some good money on it. No insurance company will sell you insurance coverage if you do not prove them that you have interest (share) in that property or a person. **Principle of subrogation** states that after covering the losses of insured, insurer has a right to recover own losses from third parties. If neighbor's dog did bite your leg and you were hospitalized, insurance company will be obliged to cover certain portion of hazard by contract but after covering hazard, insurance

company will have full right to sue your neighbor down. You will not be able to interfere into the case because your insurance contract will state the principle of subrogation. Even if third party was your close relative (for example your brother accidentally hit your car), after covering its own part of loss insurance company will have a right to sue your close relative. The only way to stop legal process will be compensating insurance company losses (covering your loss yourself if close person to you is in the case). **Principle of utmost good faith** states that all parties must inform the other party and be honest regarding any change of situation, conditions or anything that can lead to hazard or losses. There are many other legal aspects of insurance business which no doubt would have been extremely interesting for future legal specialists (lawyers) but I just wanted to state the core and go for technical part which is more interesting for economists.

Insurance companies collect monthly premiums and invest them into real estate, stock, bonds and other dividend bearing investments (same calculations as annuity!) They do not only act as insurance companies but partially as investment banks. Matter fact, biggest portion of profits earned is coming from investment rather than insurance business for most of the insurance companies. Present value calculations, annuity like calculations, and probability mathematics are widely used in insurance sector. We are going to see technical calculations regarding life, health and property insurance. Calculations will be divided in two components: compensation and premiums. We will see how in life, health and property insurance premiums and compensations are calculated.

Life insurance

The difference of life insurance from other insurance types is that the in life insurance insured's survivors will benefit from this insurance (beneficiaries will be paid) not the insured herself. The life insurance purchasers are the ones who have dependents. Single person with no close relatives or loved ones have no point of buying life insurance, because only survivors will benefit from this insurance.

x	l_x	d_x	q_x	D_x	N_x	C_x	M_x	L.E.
0	100,000	1,260	0.012600	100,000,000	1,992,208.86	1,200.00	5,132.91	74.4
1	98,740	90	0.000932	94,038.10	1,892,208.86	83.45	3,932.91	74.3
2	98,648	64	0.000649	89,476.64	1,798,170.76	55.29	3,849.46	73.4
3	98,584	49	0.000497	85,160.57	1,708,694.12	40.31	3,794.18	72.4
4	98,535	40	0.000406	81,064.99	1,623,533.55	31.34	3,753.87	71.5
5	98,495	36	0.000366	77,173.41	1,542,468.56	26.86	3,722.53	70.5
6	98,459	33	0.000335	73,471.62	1,465,295.15	23.45	3,695.66	69.5
7	98,426	30	0.000305	69,949.52	1,391,823.53	20.31	3,672.21	68.5
8	98,396	26	0.000264	66,598.29	1,321,874.01	16.76	3,652.90	67.6
9	98,370	23	0.000234	63,410.18	1,255,275.73	14.12	3,635.14	66.6
10	98,347	19	0.000193	60,376.53	1,191,865.55	11.11	3,621.02	65.6
11	98,328	19	0.000193	57,490.35	1,131,489.02	10.58	3,609.92	64.6
12	98,309	24	0.000244	54,742.13	1,073,998.67	12.73	3,599.34	63.6
13	98,285	37	0.000376	52,122.63	1,019,256.54	18.69	3,586.61	62.6
14	98,248	52	0.000529	49,621.92	967,133.91	25.01	3,567.92	61.7
15	98,196	67	0.000682	47,233.95	917,511.99	30.69	3,542.91	60.7
16	98,129	82	0.000836	44,954.03	870,278.04	35.78	3,512.21	59.7
17	98,047	94	0.000959	42,777.58	825,324.01	39.06	3,476.44	58.8
18	97,953	102	0.001041	40,701.49	782,546.43	40.36	3,437.38	57.8
19	97,851	110	0.001124	38,722.96	741,844.94	41.46	3,397.01	56.9
20	97,741	118	0.001207	36,837.55	703,121.97	42.36	3,355.56	56.0
21	97,623	124	0.001270	35,041.03	666,284.42	42.39	3,313.20	55.0
22	97,499	129	0.001323	33,330.02	631,243.39	42.00	3,270.81	54.1
23	97,370	130	0.001335	31,700.88	597,913.37	40.31	3,228.81	53.2
24	97,240	130	0.001337	30,151.00	566,212.49	38.39	3,188.50	52.2
25	97,110	128	0.001318	28,676.85	536,061.49	36.00	3,150.11	51.3
26	96,982	126	0.001299	27,275.29	507,384.63	33.75	3,114.12	50.4
27	96,856	126	0.001301	25,942.72	480,109.35	32.14	3,080.37	49.4
28	96,730	126	0.001303	24,675.21	454,166.63	30.61	3,048.23	48.5
29	96,604	127	0.001315	23,469.59	429,491.42	29.38	3,017.61	47.6
30	96,477	127	0.001316	22,322.60	406,021.84	27.99	2,988.23	46.6
31	96,350	130	0.001349	21,231.64	383,699.23	27.28	2,960.24	45.7
32	96,220	132	0.001372	20,193.32	362,467.60	26.38	2,932.96	44.7
33	96,088	137	0.001326	19,205.35	342,274.28	26.08	2,906.58	43.8
34	95,951	143	0.001490	18,264.73	323,068.92	25.92	2,880.50	42.9
35	95,808	153	0.001597	17,369.06	304,804.19	26.42	2,854.57	41.9
36	95,655	163	0.001704	16,515.54	287,435.13	26.80	2,828.16	41.0
37	95,492	175	0.001833	15,702.29	270,919.58	27.41	2,801.35	40.1
38	95,317	188	0.001972	14,927.15	255,217.30	28.04	2,773.95	39.1
39	95,129	203	0.002134	14,188.30	240,290.14	28.84	2,745.91	38.2
40	94,926	220	0.002318	13,483.83	226,101.85	29.76	2,717.07	37.3
41	94,706	241	0.002545	12,811.98	212,618.02	31.05	2,687.31	36.4
42	94,465	264	0.002795	12,170.83	199,806.04	32.39	2,656.26	35.5
43	94,201	288	0.003057	11,558.88	187,635.20	33.66	2,623.87	34.6
44	93,913	314	0.003344	10,974.80	176,076.33	34.95	2,590.21	33.7
45	93,599	343	0.003665	10,417.24	165,101.53	36.36	2,555.26	32.8
46	93,256	374	0.004010	9,884.83	154,684.29	37.76	2,518.91	31.9
47	92,882	410	0.004414	9,376.36	144,799.46	39.42	2,481.15	31.0
48	92,472	451	0.004877	8,890.45	135,423.10	41.30	2,441.73	30.1
49	92,021	495	0.005379	8,425.80	126,532.64	43.17	2,400.44	29.3
50	91,526	540	0.005900	7,981.41	118,106.84	44.85	2,357.27	28.4

51	90,986	584	0.006419	7,556.49	110,125.43	46.19	2,312.43	27.6
52	90,402	631	0.006980	7,150.47	102,568.94	47.53	2,266.23	26.8
53	89,771	684	0.007619	6,762.44	95,418.47	49.07	2,218.70	26.0
54	89,087	739	0.008295	6,391.34	88,656.03	50.49	2,169.63	24.1
55	88,348	797	0.009021	6,036.50	82,264.69	51.86	2,119.13	24.4
56	87,551	856	0.009777	5,697.19	76,228.19	53.05	2,067.27	23.6
57	86,695	919	0.010600	5,372.84	70,531.00	54.24	2,014.22	22.8
58	85,776	987	0.011507	5,062.75	65,158.16	55.48	1,959.98	22.0
59	84,789	1,063	0.012537	4,766.18	60,095.41	56.91	1,904.50	21.3
60	83,726	1,143	0.013676	4,482.32	55,329.23	58.38	1,847.58	20.5
61	82,581	1,233	0.014931	4,210.49	50,846.91	59.87	1,789.21	19.8
62	81,348	1,324	0.016276	3,950.12	46,636.42	61.23	1,729.34	19.1
63	80,024	1,415	0.017682	3,700.79	42,686.30	62.32	1,668.11	18.4
64	78,609	1,502	0.019107	3,462.24	38,985.51	63.00	1,605.79	17.7
65	77,107	1,587	0.020582	3,234.37	35,523.27	63.40	1,542.78	17.0
66	75,520	1,674	0.022166	3,016.95	32,288.90	63.69	1,479.38	16.3
67	73,846	1,764	0.023888	2,809.60	29,271.95	63.92	1,415.69	15.7
68	72,082	1,864	0.025859	2,611.89	26,462.35	64.33	1,351.78	15.1
69	70,218	1,970	0.028055	2,423.19	23,850.47	64.75	1,287.45	14.4
70	68,248	2,083	0.030521	2,243.05	21,427.28	65.20	1,222.70	13.8
71	66,165	2,193	0.033144	2,071.04	19,184.23	65.37	1,157.50	13.2
72	63,972	2,299	0.035938	1,907.04	17,113.19	65.27	1,092.13	12.6
73	61,673	2,394	0.038818	1,750.96	15,206.15	64.73	1,026.86	12.1
74	59,279	2,480	0.041836	1,602.85	13,455.19	63.86	962.13	11.5
75	56,799	2,560	0.045071	1,462.66	11,852.34	62.78	898.26	11.0
76	54,239	2,640	0.048673	1,330.22	10,389.68	61.66	835.48	10.5
77	51,599	2,721	0.052734	1,205.22	9,059.46	60.53	773.81	9.9
78	48,878	2,807	0.057429	1,087.30	7,854.24	59.47	713.29	9.4
79	46,071	2,891	0.062751	976.05	6,766.94	58.33	653.82	8.9
80	43,180	2,972	0.068828	871.24	5,790.89	57.11	595.49	8.5
81	40,208	3,036	0.075507	772.64	4,919.65	55.56	538.37	8.0
82	37,172	3,077	0.082777	680.29	4,147.00	53.63	482.81	7.6
83	34,095	3,083	0.090424	594.26	3,466.72	51.18	429.18	7.2
84	31,012	3,052	0.098414	514.79	2,872.45	48.25	378.00	6.8
85	27,960	2,999	0.107260	442.02	2,357.66	45.15	329.76	6.5
86	24,961	2,923	0.117103	375.82	1,915.64	41.91	284.60	6.1
87	22,038	2,803	0.127189	316.01	1,539.82	38.28	242.69	5.8
88	19,235	2,637	0.137094	262.68	1,223.81	34.30	204.41	5.5
89	16,598	2,444	0.147247	215.88	961.12	30.27	170.11	5.2
90	14,154	2,246	0.158683	175.32	745.24	26.50	139.84	4.9
91	11,908	2,045	0.171733	140.48	569.92	22.98	113.34	4.7
92	9,863	1,831	0.185643	110.81	429.44	19.59	90.36	4.4
93	8,032	1,608	0.200199	85.94	318.63	16.39	70.77	4.2
94	6,424	1,381	0.214975	65.47	232.68	13.40	54.39	4.0
95	5,043	1,159	0.229824	48.94	167.22	10.71	40.98	3.8
96	3,884	945	0.243306	35.90	118.27	8.32	30.27	3.7
97	2,939	754	0.256550	25.87	82.37	6.32	21.95	3.5
98	2,185	587	0.268650	18.32	56.60	4.69	15.63	3.4
99	1,598	448	0.280350	12.76	38.18	3.41	10.94	3.3
100	1,150	335	0.291304	8.75	25.42	2.43	7.53	3.2

Source: Based on R. Muksian (2003). *Mathematics of Interest Rates, Insurance, Social Security, and Pensions*. Prentice Hall, Upper Saddle River, NJ.

Above is the Commutation table created by using US. Internal Revenue Service's data and used by US based life insurance companies for various calculations. This is not a universal table because data collected only from mortality rates in US! Each country must collect their own mortality rate data by ages and insurance companies can use those data for finding premiums and probabilities of risk. We are going to use this table for insurance calculations. Explanation of table:

(x) = *age of a person in years. Table starts from the birth (x=0) of sample size 100000 people, and goes to (x=100).*

(l_x) = *number of people alive at age* **x**

(d_x) = *number of people who died between ages* d_x *and* d_{x+1}, $(d_x = l_x - l_{x+1})$

(q_x) = *mortality rate, probability that person will die in between ages* x *and* $x+1$. $(q_x = \frac{d_x}{l_x})$

(p_x) = *probability of living of a person aged* x *to age* k *(out of context, it is not in table but very useful.* $(p_{x,k} = \frac{l_k}{l_x})$

(D_x) = *present value of 1 USD (or TMT) for each person alive at age* x. $(D_x = \frac{l_x}{(1+r)^x})$ (r=5%, 0.05 is used in all calculations)

(N_x) = *present value of all annuity payments for all persons alive at each age group from* x *up to the* ***100*** *years of age. Thus:*

$$N_x = D_x + D_{x+1} + D_{x+2} + D_{x+3} + D_{x+4} + \dots D_{100}$$

$$\sum_{x=k}^{k=100} D_x = N_x$$

(C_x) = *present value of 1 USD (or TMT) which was paid to beneficiaries of people who died at age* x. $(C_x = \frac{d_x}{(1+r)^{x+1}})$

(M_x) = *represents 1 USD (or TMT) payment for all people that eventually going to die but still alive at age* x. *Thus:*

$$M_x = C_x + C_{x+1} + C_{x+2} + C_{x+3} + C_{x+4} + \dots C_{100}$$

$$\sum_{x=k}^{k=100} C_x = M_x$$

Exercise 1*: How many people will die at age 25?*

We use $(d_x = l_x - l_{x+1})$ *find from table:*

$$d_{25} = l_{25} - l_{25+1} = 97110 - 96982 = 128$$

Exercise 2: *What are the chances that 60 years old person will live to age 90?*

Use ($\boldsymbol{p_{x,k} = \frac{l_k}{l_x}}$), first find alive people at age 60 from table, l_{60}= 83726, and then alive people at age 90, l_{90}= 14154. Then, probability of a 60 years old person living reaching age 90 is approximately 17%:

$$\boldsymbol{P_{60,90} = \frac{l_{x+1}}{l_x} = \frac{l_{90}}{l_{60}} = \frac{14154}{83726} = 0.17}$$

Exercise 3: *What is probability that I will die at age 36?*

Use $\mathbf{q_x = \frac{d_x}{l_x}}$, which is $\mathbf{q_x = \frac{l_x - l_{x+1}}{l_x}}$

$$\boldsymbol{q_{36} = \frac{l_{36} - l_{37}}{l_{36}} = \frac{95655 - 95492}{95655} = 0.0017}$$

Probability of me dying at age 36 is 0.17%.

Exercise 4: *Where $\boldsymbol{D_{70}}$ comes up from table?*

Use $\boldsymbol{D_x = \frac{l_x}{(1+r)^x}}$, then $\mathbf{D_{70} = \frac{l_{70}}{(1+r)^{70}} = \frac{68248}{(1+0.05)^{70}} = 2243.05}$

Exercise 5: *If annuity payment for people in cohort of 60 years is 1000 TMT, then find present values of those payments at age back to zero (annual payments start at age 60, then paid until death).*

Use

$$\sum_{x=k}^{k=100} \boldsymbol{D_x = N_x}$$

$$\sum_{x=60}^{k=100} (1000)D_{60} = (1000)N_{60} = (1000) * (55329.23)$$

$$= 55329230$$

This equation must be understood this way: assume at age 60 people who purchased life insurance start paying 1000 TMT annuity payments monthly so that when they die their relatives will be paid face value of insurance contract. Insurance companies try to add all annuity payments paid and calculate their present value.

Exercise 6: *Find $\boldsymbol{C_{80}}$ and how it was calculated.*

Use $\boldsymbol{C_x} = \frac{\boldsymbol{d_x}}{(\mathbf{1}+\boldsymbol{r})^{x+1}}$, then:

$$\mathbf{C_{80}} = \frac{\boldsymbol{d_{80}}}{(\mathbf{1}+\mathbf{0.05})^{80+1}} = \frac{\mathbf{2972}}{(\mathbf{1.05})^{81}} = \mathbf{57.11}$$

Present value of 1 TMT paid to beneficiaries of people who died at age 80 is 57.11 TMT today.

Exercise 7: *If payment for each person who dies at age group 70 is 2000 TMT, find accumulated of paid for all.*

Use

$$\sum_{\boldsymbol{x=k}}^{\boldsymbol{k=100}} \boldsymbol{C_x} = \boldsymbol{M_x}$$

$$\sum_{x=70}^{k=100} (2000\,TMT)C_{70} = (2000\,TMT) * M_{70}$$

$$= (2000) * (1222.7) = 2445400$$

Top insurance companies and their total assets (in BLN USD)

1	Axa	France	1,011.40
2	Allianz	Germany	994.3
3	Prudential Financial	United States	815.1
4	Nippon Life	Japan	710.9
5	Berkshire Hathaway	United States	707.9
6	MetLife	United States	687.5
7	Japan Post Insurance	Japan	666.6
8	Prudential plc	United Kingdom	646.2
9	Legal & General	United Kingdom	625.7
10	Ping An Insurance	China	620.4
11	Assicurazioni Generali	Italy	590.5
12	China Life Insurance	China	579.4
13	Manulife Financial	Canada	550.3
14	Aviva	United Kingdom	546.9
15	JA Kyosai	Japan	529.2
16	Dai-ichi Life	Japan	504.6
17	American International Group	United States	491.8
18	CNP Assurances	France	475.7

19	Aegon N.V.	Netherlands	449.7
20	Life Insurance Corporation	India	449
21	Amundi	France	437.9
22	Zurich Insurance Group	Switzerland	395.3
23	Meiji Yasuda Life	Japan	379.9
24	Sumitomo Life	Japan	341.1
25	New York Life Insurance Company	United States	339.1

***Exercise 8**: I am 35 years old and I want to receive 300000 TMT when I reach age of 50, how much do I need to invest today? (r=5%).*
Now, normally we did this:

$$A\,(1+r)^{15}=300000\ TMT$$
$$A\,(1.05)^{15}=300000\ TMT$$
$$A=146420\ TMT$$

But now, we now that probability of being alive at 50 years of age for 35 years old grown up man is not a 100%. Let's solve this using probability now:

$$A*(1+0.05)^{15}=300000*\mathbf{(p_{35,\,50})}$$
$$A*(1+0.05)^{15}=300000*\mathbf{\left(\frac{l_{50}}{l_{35}}=\frac{91526}{95808}=0.955\right)}$$
$$A*(2.0789)=300000(0.955)$$
$$A=137813\ TMT$$

In insurance world there is no guarantee that you will live to age 50, so always probability is part of the calculation.

Whole life insurance policy

Whole life insurance policy is when insurer is obliged to pay face value of insurance policy to survivors of insured upon his death. Assume $\mathbf{B_x}$ to be a net single premium (which is fully paid after signing of insurance contract) which is a sum of mathematical expectations that face value would be paid to insured's beneficiaries. Mathematical expectation is product of probability of death ($\boldsymbol{q_x}$) and present value of benefits($\frac{1}{(1+r)^n}$) . Thus:

$$B_x = q_x\left(\frac{1}{(1+r)^1}\right) + q_{x+1}\left(\frac{1}{(1+r)^2}\right) + q_{x+2}\left(\frac{1}{(1+r)^3}\right)\ldots$$

We know that $\boldsymbol{q_x} = \frac{\boldsymbol{d_x}}{\boldsymbol{l_x}}$, then:

$$B_x = \left(\frac{d_x}{l_x}\right)\left(\frac{1}{(1+r)^1}\right) + \left(\frac{d_{x+1}}{l_x}\right)\left(\frac{1}{(1+r)^2}\right) + \left(\frac{d_{x+2}}{l_x}\right)\left(\frac{1}{(1+r)^3}\right)\ldots$$

Let's say $\left(\frac{1}{(1+r)^n}\right) = v^n$:

$B_x = \left(\frac{d_x v^{n+1}}{l_x v^n}\right) + \left(\frac{d_x v^{n+2}}{l_x v^n}\right) + \left(\frac{d_x v^{n+3}}{l_x v^n}\right) \ldots = \frac{d_x v^{n+1} + d_x v^{n+2} + d_x v^{n+3} \ldots}{l_x v^n}$

We also know that $\mathbf{C_x} = \frac{d_x}{(1+r)^{x+1}} = \boldsymbol{d_x v^{n+1}}$ and $\mathbf{D_x} = \frac{l_x}{(1+r)^x}$ then:

$$\mathbf{B_x} = \frac{C_x + C_{x+1} + C_{x+2} \ldots}{D_x}$$

$$\mathbf{B_x} = \frac{M_x}{D_x}$$

If we want to solve for certain face value ***F***, then net single premium **(cost of insurance=B_x)** for *x* aged person who wants to leave his beneficiaries **F** amount of money after his death is equal to:

$$\mathbf{B_x} = \mathbf{F}\left(\frac{M_x}{D_x}\right)$$

Exercise 9*: 35 years old man wants to leave his family 1MLN TMT after his death, how much is his net single premium (what is the cost of this insurance)?*

$$B_x = F\left(\frac{M_x}{D_x}\right)$$

$$B35 = 1000000\left(\frac{M_{35}}{D_{35}}\right)$$

$$B35 = 1000000\left(\frac{2854.57}{17369.06}\right) = 164348\ TMT$$

Net single premium for him is going to be 164348 TMT. Not everybody can afford to pay 164348 TMT immediately there, which is why companies divide this sum to whole annuity, thus insured will pay fixed amount annually and his beneficiaries will receive face value of ***F.*** *Annual premium for above 35 years old person who wants to leave 1 MLN TMT for his love ones is found by formula:*

$$\mathbf{P_x} = \mathbf{F}\left(\frac{M_x}{N_x}\right)$$

$$P_{35} = 1000000\left(\frac{M_{35}}{N_{35}}\right)$$

$$P_{35} = 1000000\left(\frac{2854.57}{304804.19}\right) = 9365\ TMT$$

Annual payment for 35 years old man who wants to leave 1 MLN TMT after his death will have to pay annually 9365 TMT until his death.

Term life insurance policy

Insurer will pay survivors of insured if only insured dies within time specified period of time mentioned in the contract. That is why it is called term life insurance policy. If insured wants to pay all premium immediately (**B)** for the face value of insurance contract (**F)**, net single premium, then:

$$\mathbf{B_{x,n}=F\left(\frac{M_x-M_{x+n}}{D_x}\right)}$$

Where (**B)** is net single premium, (**x**) represents age, (**n)** term of insurance in years, (**F**) represent face value of insurance policy, (**M)** represents 1 USD (or TMT) payment for all people that eventually going to die but still alive at age ***(x),*** **(D)** present value of 1 USD (or TMT) for each person alive at age (**x)**.

Exercise 10: *40 years old Meret wants to buy term life insurance for 20 years with face value of 1 MLN TMT. What is the cost of this insurance (net single payment)?*

$$B_{x,n}=F\left(\frac{M_x-M_{x+n}}{D_x}\right)$$

$$B_{40,20}=1000000\left(\frac{M_{40}-M_{40+20}}{D_{40}}\right)$$

$$B_{40,20}=1000000\left(\frac{2717.07-1847.58}{13483.13}\right)$$

$$B_{40,20}=64487 \text{ TMT}$$

Cost of insurance will be 64487 TMT if premium paid immediately. What if Meret prefers paying this insurance with annuities, thus paying fixed amount per year? If Meret prefers paying with annuities, then:

$$\mathbf{P_{x,n}=F\left(\frac{M_x-M_{x+n}}{N_x-N_{x+n}}\right)}$$

$$P_{40,20}=F\left(\frac{M_{40}-M_{60}}{N_{40}-N_{60}}\right)$$

$$P_{40,20}=1000000\left(\frac{2717.07-1847.58}{226101.85-55329.23}\right)$$

$$P_{40,20}=5091 \text{ TMT}$$

If Meret prefers paying premiums in terms of annuities, then he will have to pay 5091 TMT annually.

Property and liability insurance

Fire, earthquake, car accident, vandalism, accidental injury, are examples of property and liability insurance. Insurance business is developed in some countries while in others it is not, does this mean that people do not value their property or they are not afraid of casualties? No, the main reason of development of insurance business relies on economic development of that society. In underdeveloped countries where houses are from trees and could be built in two days, will you have a reason to insure it against any risk? In economy where medical services are not developed at all and you still use "auntie's methods" of healing, will health insurance grow? Not, of course. Poor people does not have anything much to lose, but rich people have. In developed economies where wealth and health is valued highly, insurance business will flourish. Now you understand why insurance business in Uganda is incomparable to the same in South Korea.

One of the main problems of insurance companies dealing with property and liability insurance is reimbursement. Reimbursement is replacement of damaged property or financial coverage of hazard incurred. To do this, insurance companies must assess value of hazard correctly; otherwise they will either hurt insured (which might end up with lawsuit) or make a loss. Two things impact the value of property: Inflation and depreciation! Market value of property might go up and down with price fluctuations. Depreciation is natural loss of value of property due to wear and tear. So, when assessing a loss/hazard, insurance companies use this information regarding each property. While inflation rates will impact the prices of almost all properties equally, depreciation rates are different for each type of property. Some properties like furniture have 5-10 years of lifespan, while some properties like ships and steel-concrete buildings have 40-50 years of lifespan.

Generally, two methods are used for depreciation calculations: linear and non-linear. Linear method is simply dividing the value of property to the life span of that property and assuming that each year property will lose exactly that amount of value.

Exercise 11: *A new car was bought for 100000 TMT and factory guarantees life span of this car to be at least 10 years. Then, each year this car will lose 10% of initial value, thus 10000 TMT. What will be value of this car in 6 years? It is going to be 40000 TMT. It*

lost 10000 TMT each year for 6 years; in total it lost 60000 TMT from its initial value.

$$\mathbf{V_t = V_i\,(1\text{-}de*t)}$$

Where V_t= value of property at time (t), V_i = initial value of property, de = depreciation rate, (t) = number of years. In above example:

$$V_t = V_i\,(1\text{-}de*t)$$

$$V_6 = (100000)\,(1\text{-}(0.1)*6)$$

$$V_6 = 40000 \text{ TMT}$$

Non-linear depreciation method is method which depreciation rates are discounted not from initial value as we did in linear deprecation method, but from the last year's value of property. Let's use non-linear depreciation for above example of 100000 TMT worth car. Again assume depreciation rate is 10% but non-linear. One year later the value of car will be:

$$V_1 = V_i\,(1\text{-}(0.1)) = 100000(1\text{-}0.1) = 90000 \text{ TMT}$$

At the end of second year the value will drop to:

$$V_2 = V_1\,(1\text{-}(0.1)) = 90000(1\text{-}(0.1) = 81000 \text{ TMT}$$

Now, instead of 90000 TMT, let's replace it with our initial equation. Then:

$$V_2 = V_i\,(1\text{-}(0.1))\,(1\text{-}(0.1)) = V_i\,(1\text{-}(0.1))^2$$

If we continue, we will find formula for calculating value of property using non-linear method:

$$\mathbf{V_t = V_i(1\text{-}de)^t}$$

If we use non-linear method of depreciation in above example, after 6 years the value of the car is going to be

$$V_t = V_i\,(1\text{-}(0.1))^t$$

$$V_6 = 100000(1\text{-}(0.1))^6$$

$$V_6 = 53144 \text{ TMT}$$

When assessing value of property if only depreciation is used, this is called Actual Cash Value calculation.

***Exercise 12**: Initial value of transatlantic cargo ship is 5 TRLN. TMT find actual cash value of this ship 30 years later using two*

methods of depreciation rate calculation if depreciation rate of ship is 3% annually.

Linear method: $\mathbf{ACV=V_i\ (1\text{-}de*t)}$

ACV=5 TRLN TMT (1-(0.03) *(30))

ACV =0.5 TRLN TMT

Non-linear method: $\mathbf{ACV =V_i\ (1\text{-}de)^t}$

ACV =5TRLN TMT $(1-(0.03))^{30}$

ACV =2 TRLN TMT

Now let's see how inflation rates are impacting procedure of assessment. As we all know, inflation means rising prices. Price is value of property. Rising prices increase market value of property, and thus amount of reimbursement. While depreciation decreases the book value of property, inflation increases market value of property. Deflation means decreasing prices so in case of deflation, market value of property will fall. All in all, depreciation rate impacts the book value of property while inflation impacts only market value. **When both depreciation and inflation impact were used in assessment of value of property, it is called Replacement Value.** It is called replacement value because sometimes insured and insurer might mention in contract that insurer may have a right to replace the lost property instead of compensating it with cash payment. Replacement will occur with almost exactly same property. We can show both depreciation and inflation in one formula by:

Linear method: $\mathbf{RV=V_i(1\text{-}de*t)(1+ir)^t}$

Non-linear method: $\mathbf{RV=V_i(1\text{-}de)^t\ (1+ir)^t}$

The reason why inflation rates are used in assessment of property value is that insurance companies will have to reimburse/replace hazard by market value by the principle of indemnity.

Exercise 13: *Initial value of transatlantic cargo ship is 5 TRLN. TMT and this ship were insured. Ship sank 30 years later during storm. Using both methods of depreciation rate calculations and inflation rate, find total replacement value. (Depreciation rate is 3% and inflation rate 4% annually.)*

Linear method: $\mathbf{RV= V_t=V_i(1\text{-}de*t)(1+ir)^t}$

$RV= V_i(1\text{-}de*t)(1+ir)^t$

$RV=5(1\text{-}0.03*30)(1+0.04)^{30}$

RV=1.6 TRLN TMT

Non-linear method: $\mathbf{RV=V_t=V_i(1-de)^t\,(1+ir)^t}$

$RV=V_i(1-de)^t\,(1+ir)^t$

$RV=5(1-0.03)^{30}\,(1+0.04)^{30}$

RV=6.5 TRLN TMT

When we used non-linear method of depreciation calculation the replacement value of ship actually increased over the years, the reason behind is inflation rate. Inflation rate (price) increased faster than depreciation rate.

Deductibles

Deductibles are lump sum amount of compensation to be paid by insured himself. The difference between deductibles and co-insurance is that deductibles are lump sum amount while co-insurance is a share (in percentage) of compensation for hazard.

***Exercise 14**: A house was damaged by flooding for 50000 TMT. Replacement value of the house is 500000 TMT but insurance was bought for 400000 TMT with 5000 TMT deductible clause for each event. How much the owner of the house will be reimbursed for this loss?*

If house was fully destroyed by event, then insurance company would have only reimbursed the insured by 400000 TMT, the face value of insurance. Up to that point, up to 400000 TMT damage, insurance will have to cover the loss but for each case, insurance company has deductible clause of 5000 TMT from each reimbursement. Thus, total damage 50000 TMT- 5000 TMT=45000 TMT will be reimbursed by insurance company.

***Exercise 15**: What if damage was inflicted for 4000 TMT?*

Again, insurance company must reimburse insured for up to 400000 TMT damage, but company has 5000 TMT deductible clause for each event. So, 4000-5000=-1000 TMT. Which means insurance is not paying anything if damage is lesser than 5000 TMT.

Deductibles are used to discourage insured from filing claims for little hazards. This saves time and resources of company, otherwise even for 50 TMT hazard insurance companies would have to respond. This would have cost more than damage itself for insurance companies. Deductibles and co-insurance are control mechanisms of insurance companies against moral hazard. When

insured knows that insurance policy will not cover full damage and that there are deductibles at place for miserable hazard, insured will act the most honest way possible. General formula for total reimbursement is:

$$R = \frac{Insurance\ coverage\ purchased * (inflicted\ damage\ amount - deductible)}{(co-insurance\ share) * (Replacement\ value)}$$

$$\boldsymbol{R = \frac{I * (L - D)}{j * RV}}$$

***Exercise 16**: In above example, **(I)** =400000 TMT, **(L)** =50000 TMT, **(D)** =5000 TMT, **(J)** = 0.8 (co-insurance, 20% to 80%, 20% will be covered by insured, 80% will be covered by insurer), **(RV)**=500000 TMT. Then:*

$$R = \frac{I * (L - D)}{j * RV}$$

$$R = \frac{400000 * (50000 - 5000)}{0.8 * 500000} = 45000\ TMT$$

***Exercise 17:** A ship with market value of 2 MLN TMT was insured for 1.4 MLN TMT and 10000 TMT deductibles for each event were also in contract. Storm in the ocean caused 1 MLN TMT damage on ship. Calculate how much insurance company must reimburse? Suppose contract also had co-insurance clause of 20% to 80%*

$$R = \frac{I * (L - D)}{j * RV}$$

$$R = \frac{1400000 * (1000000 - 10000)}{0.8 * 2000000} = 866250\ TMT$$

Since owner of the ship did not buy full insurance (80% of 2 MLN would have been 1.6 MLN) but bought lesser amount of coverage, that is why he has been reimbursed for lower amount of total damage. If he had purchased insurance coverage for 80% then, almost all damage would have been covered by insurance company, except deductibles of course:

$$R = \frac{I * (L - D)}{j * RV}$$

$$R = \frac{1600000 * (1000000 - 10000)}{0.8 * 2000000} = 990000\ TMT$$

Exercise 18: *Business building in the center of the city has grown up in market value due to the rapid financial growth and development of that part of the city. Building had insurance coverage up to 20 MLN TMT and 50000 TMT deductible clause for each case occurred, but the market value of the building was 50 MLN TMT. Arson incident caused 18 MLN TMT damage to the building. How much will insurance company reimburse insured?*

$$R = \frac{I * (L - D)}{j * RV}$$

$$R = \frac{20000000 * (18000000 - 50000)}{0.8 * 50000000} = 8975000\ TMT$$

Insurance company will reimburse only 8975000 TMT, because insurance was bought only for 20 MLN and that is only 40% of real market value of house.

Especially for buildings and houses insurance coverage will not exceed 80% of the replacement value of that building, because it is considered that 20% of the market value comes from the location of the building rather than material cost. It is very normal to see the most of the buildings buying only coverage for 80% of the total damage.

Pension funds

Society is developing and growing. Back in times old people were taken care of by their young siblings. Parents looked after their children and children paid up their "social debt" when their parents were old. Growing and developing society constantly needs energy, young energy, that is why opportunity cost for young and energetic people spent time looking after old parents is very high now. Parents know that, society is conscious of this issue. That is why people save, save for "old times". Save to be less dependent to somebody else, save to live independently. Nowadays people live much longer than before. A lot of countries are passing the 70 mark in life expectancy data. This makes even harder for younger generation to look after their "long living" parents financially. Pension funds solve that problem. You make little contributions when you are young, so when you get old you receive pension payments. Your pension payments will depend on amount you contributed when you were young: more you contributed, more money you will get when you are old. Pension funds solve two problems: first one is a social problem concerning seniors, second one is economical because large amount of contributions make investment possible. As we know, investment is the engine of growth and development. Pension funds are vital in society and in economy. Pension funds, as any other financial institution face

many risks: market risk, inflation risk, operational risk, investment risk, etc. and I want to talk about the ones that I strongly believe are the most important ones:

1) **Inflation risk:** Inflation is the biggest enemy of all investments, and inflation is the biggest risk of pension funds. Economy where high inflation is witnessed, consumption goods are heavily purchased. Instead of investing for future, consumers prefer consuming it today, because value of money is higher today than tomorrow. Successfully functioning pension system needs very low and stable inflationary environment.

2) **Generation or population demographics:** It is fact that as economies developing, opportunity cost of having kids are increasing. Thus babe becomes a luxury good! Babies require huge amount of time, health and money from parents to be. Number of babies per family decreases as economy continues development. Japan, Russia and Germany are one of examples of nations having demographic problems. Citizens make few babies, and population of seniors is increasing faster than population of youngsters. This creates huge problem for pension funds because they close pension payments of seniors with pension contributions of young generation. If pension payments exceed contributions, then pension fund will have financial difficulty in meeting its obligations. Developed economies try to solve their demographic problems with migration. In fact, it is in the best interest of developed economies having migrants come in. Even though developed economies seem like they do favor for "migrants" by letting them in, in fact they need migrants for their economy to function properly. Young working people both lower down prices and make huge contributions to economy.

3) **Unemployment:** Pension contributions depend on employment. Economies with high unemployment level or with high level of underground economy will have social and economic problems. Seniors that worked "unofficially" (underground economy) who were not able to save for their retirement will depend on their relatives and siblings. There is no guarantee that they will look after them, or look after them properly. That is a social problem, which again will fall on the "shoulders" of government.

Top pension funds according to assets (in BLN USD)

1	Federal Old-age and Survivors Insurance Trust Fund (Social Security)	$2,719
2	Australian National Superannuation Scheme	$1,857

3	Government Pension Fund of Norway	$1,046
4	Government Pension Investment Fund (Japan)	$1,103
5	Civil Service Retirement and Disability Fund (USA)	$932
6	Military Retirement Fund (USA)	$813
7	Thrift Savings Plan (TSP) (USA)	$546
8	National Pension Service (NPS) (South Korea)	$462
9	Stichting Pensioenfonds ABP (ABP) (Netherlands)	$499
10	Canada Pension Plan and CPP Investment Board	$317
11	Caisse de dépôt et placement du Québec (Canada)	$299
12	California Public Employees' Retirement System (USA)	$370
13	National Social Security Fund (China)	$251
14	Central Provident Fund (Singapore)	$208
15	California State Teachers' Retirement System (USA)	$206
16	Employees Provident Fund (Malaysia)	$185
17	Stichting Pensioenfonds Zorg en Welzijn (Netherlands)	$183
18	Ontario Teachers' Pension Plan	$176
19	AFP (Chile)	$160
20	Employees' Provident Fund Organisation (India)	$128
21	Russian National Wealth Fund	$125
22	Arbejdsmarkedets Tillægspension (Denmark)	$119
23	Government Employees Pension Fund (South Africa)	$112
24	Caixa de Previdencia dos Funcionários do Banco do Brasil	$80
25	State Teachers Retirement System of Ohio (USA)	$76
26	AGIRC - ARRCO (USA)	$70
27	Pensions Reserve Fund (France)	$56
28	National Pension Reserve Fund (Ireland)	$30

Let's speak technical aspect of pension funds, thus how do they do money? Pension funds collect pension contributions from those who are under official retirement age which is different in all economies, and uses certain portion of it to pay off pension payments for retirees and invest the rest of the portion. Generally, pension payments are ranging from 50% to 80% of average wage. Pension contributions depend on contributor but if it generally does not exceed 5% of average wage. Just by looking at these numbers we know that in order for survival of this financial institution, pension contributors must well over exceed the number of pension payment receivers.

***Exercise 19**: Average market wage is 3000 TMT, average pension contribution is 5% of wage, and average pension payee is 60 % of average wage. If this pension fund has 50000 retirees in an account, how many contributors must this fund have to keep this system afloat?*

*60% of 3000 TMT is 1800 TMT. 50000 retirees are receiving 1800 TMT each month: 50000*1800=90000000 TMT. Average contributor contributes 5% of 3000 TMT, 150 TMT monthly. Then,*

just to keep pension fund afloat there must be 90000000/150=600000 more than 600000 contributors. Why more than? Because if it is less than 600000 pension fund will not be able financially meet its obligations. If it is exactly 600000 then how would company cover operation and other costs? How would company earn profit if all funds go to meet payments? Company must have more than 600000 contributors to be afloat. Pay attention to numbers, 50000 retirees+600000 contributors=650000 number of population. 50000 are around 8% of population. We can say that 8% of population's retirements are paid by 92% of working population.

Exercise 20: *Assume I am 30 years old and my monthly wage is 3000 TMT. My monthly contribution for pension plan is 6% of my wage. My wage rises annually by 10% and retirement age is 65, then find out how much in total I contributed to my pension plan?*

*6% of 3000TMT is 180 TMT and this amount contributed monthly for 12 months, 180*12=2160 TMT annual contribution. Now, since wage is rising by 10% annually, same will happen to total contributions, 2160 TMT*1.1=2376 second year's total contribution. It goes that way:*

$$2160 + 2376 + 2613.6 \ldots \text{(35 years)}$$

$$2160 + 2160(1.1) + 2160(1.1)(1.1) + 2160(1.1)(1.1)(1.1) \ldots$$

$$2160(1 + (1.1) + (1.1)^2 + (1.1)^3 \ldots (1.1)^{34})$$

$$2160 \left(\frac{1-(1.1)^{35}}{1-1.1}\right) = 585412 \text{ TMT}$$

In 35 years, total my contribution for my retirement plan will be around 585412 TMT. Assume average wage after 35 years is around 15000 TMT and my retirement plan pays around 70% of average wage. 50% of 15000 TMT is around 7500 TMT. How much do I need to live just to spend my own contributions?

$$7500*X=585412$$
$$X=78 \text{ month (around 6.5 years)}$$

We must not forget that pension funds do not just collect contributions but they also invest and earn profits. In this example we totally omitted interest rates earned by pension fund, inflation rates and other factors. This exercise was simply to present how pension system works.

Homework:

I. Using above given mortality table of US, what is the probability of 25 years old person reaching age 90?

II. What is the probability that I will die in between ages 50 and 51? (if we assume that I am US citizen)

III. How do you think this data was created?

IV. Wepa is 20 years old and wants to receive 500000 TMT when he reaches age of 60, how much does he need to invest today? (r=5%) and use probability from mortality table used in chapter.

V. I am 35 and assume my first grandkid will be born 20 years later from now. Kids start recognizing people when they are 2 years old (my grandson will recognize me and remember me after age 2 only, up to that age kid's memory will not be fully functioning yet). What are my chances of playing with my grandson at age 3? What are the chances of my grandson seeing me at age 58 (he is 3)? Who has more chances?

VI. Mahri is 40 years old married women. She wants to leave 500000 TMT after she is dead to her family. What is cost of this insurance?

VII. In above example, assume Mahri cannot pay all premium at once, she wants to pay it annually. Calculate her annual premiums for life insurance with face value of 500000 TMT.

VIII. Merjen was paying annual premium of 13000 TMT starting at age 28. How much insurance payments her family will receive when she is dead?

IX. Atahan is a miner and that is a pretty risky job. He wants to buy a term life insurance covering his next ten years. What is the cost of this term life insurance for 10 years with 100000 TMT face value, if you know that Atahan is 45 years old?

X. What if Atahan cannot pay premium of term life insurance at once (net single payment) and he prefers paying it with annuities, how much annually must Atahan contribute for this insurance policy?

XI. Fighter jet plane has 25 years of service lifespan if maintained properly. If you know that and price of one plane is 20 MLN TMT, what is the book value of this plane after 10 years?

XII. Painting, if maintained properly, loses its material quality by 0.5% annually (colors blur, palette gets old, etc.) but if it is a classic painting, prices rise on average by 5% annually. What is the original price of the Picasso's 1925's painting which priced 90 MLN TMT today (2020)?

XIII. A yacht with replacement value of 300 MLN TMT was insured for 100 MLN TMT only, deductible of 2 MLN TMT. Due to fire on board yacht was damaged for 80 MLN TMT. How much insurance company will reimburse? Suppose contract also had co-insurance clause of 20% to 80%.

XIV. An office building with insurance of 50 MLN TMT had inflicted damage for 20 MLN TMT. The insured was reimbursed 19900000 TMT for damages. If deductible clause is 100000 TMT, what is the replacement value of this building? Suppose contract also had co-insurance clause of 20% to 80%

XV. Will insurance company reimburse above damaged yacht for 25 MLN TMT? Why?

XVI. Can I insure a Mickey Mouse?

XVII. In what cases can your aunty purchase property insurance on your house?

XVIII. My niece accidentally hit my car which was fully insured and had 5000 TMT deductible clause for each case. Total damage from hit was 400 TMT (paint was scratched a bit). Can I apply for insurance damage compensation? Will I receive any reimbursement?

XIX. What if damage was for 8000 TMT, will I be reimbursed? What will happen then?

XX. Jemile had a health insurance which covers 90% of all costs incurred up to 200000 TMT. (Co-insurance 10% to 90%). Deductibles are also included in contract and are 3000 TMT for each case. Recently Jemile had a bad car accident where she had to spend 19 days in hospital. Each day spent at hospital cost 1000 TMT, daily shots and pills are for 2000 TMT and other therapy expenses were in total of 7000 TMT. How much of those costs will be reimbursed by insurance company and how much will be paid by Jemile herself?

XXI. Why in certain countries insurance business is developed and in others it is not?

Solutions:

I. $P_{25.90} = \frac{l_{x+1}}{l_x} = \frac{l_{90}}{l_{25}} = \frac{14154}{97110} = 0.1457\ or\ 14.57\%$

II. $q_x = \frac{l_x - l_{x+1}}{l_x}$

$q_{36} = \frac{l_{50} - l_{51}}{l_{50}} = \frac{91526 - 90986}{91526} = 0.0059\ or\ 0.59\%$

III. This data was created on the base of official information of annual death register. By taking sample of 100000 people, they collect annual quantity of death that die at different ages and the approximate data is created.

IV. $A*(1+0.05)^{40}=500000*\left(\frac{l_{60}}{l_{20}} = \frac{83726}{97471} = 0.857\right)$

A*7.04=428500

A=60866.5 TMT

V. Probability of me to live up to 58 years old:

$P_{35,58} = \frac{l_{x+1}}{l_x} = \frac{l_{58}}{l_{35}} = \frac{85776}{95808} = 0.895$ or 89.5%

Probability of my grandson to live up 3 years old:

$P_{0,3} = \frac{l_{x+1}}{l_x} = \frac{l_3}{l_0} = \frac{98584}{10000} = 0.986$ or 98.6% (grandson has more chances)

VI. $B_{40}=500000\left(\frac{M_{40}}{D_{40}}\right)$

$B_{40}=500000\left(\frac{2717.07}{13483.83}\right) = 100752.9$ TMT

VII. $P_x= F\left(\frac{M_x}{N_x}\right)$

$P_{40}=500000\left(\frac{M_{40}}{N_{40}}\right)$

$P_{40}=500000\left(\frac{2717.07}{226101.85}\right) = 6008.5$ TMT

VIII. $P_{28}=A * \left(\frac{3048.23}{454166.63}\right) = 13000$

$A= \left(\frac{13000}{0.0067116996}\right) = 1936916.2$ TMT

IX. $B_{x,n}=F\left(\frac{M_x - M_{x+n}}{D_x}\right)$

$B_{45,10}=(100000)\left(\frac{M_{45}-M_{45+10}}{D_{45}}\right)$

$B_{45,10}=(100000)\left(\frac{2555.26-2119.13}{10417.24}\right)$ = 4186.6 TMT

X. $P_{x,n}=F\left(\frac{M_x-M_{x+n}}{N_x-N_{x+n}}\right)$

$P_{45,10}=F\left(\frac{M_{45}-M_{55}}{N_{45}-N_5}\right)$

$P_{45,10}=100000\left(\frac{2555.26-2119.13}{165101.53-82264.69}\right)$ = 526.49 TMT

XI. Linear method: $ACV=V_i(1-de*t)$
ACV=20000000 TMT (1-(0.04) *(25)) = 0 TMT

Non-linear method: $ACV =V_i(1-de)^t$
ACV =20000000 TMT $(1-(0.4))^{10}$=6973568.8 TMT

XII. Linear method: $RV=a(1+0.05)^{95}$ = 90000000TMT

$a=\frac{90000000}{(1+0.05)^{95}}$ = 873492

Non-linear method: $RV=V_t=V_i(1-de)^t(1+ir)^t$
$RV=90000000(1-0.005)^{95}(1+0.05)^{95}$=5 759 948 387.7 TMT

XIII. $R=\frac{I*(L-D)}{j*RV}$

$R=\frac{100000000*(80000000-2000000)}{0.8*300000000}$ = 32500000 TMT

XIV. $R=\frac{I*(L-D)}{j*RV}$

$19900000=\frac{50000000*(20000000-100000}{0.8*RV}$

19900000*(0.8*RV) = 50000000*(20000000-100000)
15920000*RV=50000000*(20000000-100000)
RV=62500000 TMT

XV. Yes, insurance company will reimburse for 25 MLN TMT because yacht's real reimburse value is 62.5 MLN TMT. The less the better for insurance companies, because the rest of 25 MLN TMT they can invest and make profit on it.

XVI. No, you are not able to insure a Mickey Mouse because Mickey Mouse is not your property, thus, you distort the principle of insurable interest of Insurance companies.

XVII. According to one of the main principles of insurance companies, "Principle of insurable interest" property you want to insure must belong to insurer, thus, my aunt can purchase property insurance on my house only in case, if she owns share of my house.

XVIII. No, even if damaged property is fully insured, you can't apply for insurance damage compensation because if the reimbursement is less than deductible value, insurance company will not pay any reimbursements.

XIX. If damage was for 8000 TMT, which exceeds deductible value, the insurance company will reimburse, but after insurer has a full right to sue third party (the one who damaged the car).

XX. (1000+2000) *19+7000=64000 TMT (total expense for damage)
Deductibles= 3000 TMT
64000-3000=61000 TMT
If Co-insurance 10% to 90%:
61000*0.9=54900 TMT will be covered by insurance
The rest 61000*0.1%+3000(deductibles) =9100 TMT covers Jemile.

XXI. Mostly in developed countries insurance businesses are developed because income level affords to insure your property, life, house etc., and services are expensive, thus insurance companies are vital for population to incur less cost for living. While in developing countries services are affordable (not expensive), according to income level which creates unnecessity in insuring their property.

Chapter 9: Basic Finance 3

Main source of capital in economy are banks. When we need funds we go to banks for a loan. We are able to borrow from bank with one condition: collateral. Banks will not take a risk and lend without any collateral. First "headache" for any borrower from banks will be collateral. Second is going to be "fixed" interest rates for borrowed capital. "Fixed" because banks will not care if you make profit or loss but you have to pay your interest on time. Interest will not lower when you are doing "bad" and will not increase when you are doing "good". If you cannot pay your dues on time you might be put on a "black list" which will make borrowing even harder and pricier for you. Banks do not like a joke that is for sure. In case of failure of interest payment by borrower, bank will have a right to arrest the collateral and do with it whatever is needed to return the loss. Banks are also one of the major targets of investments because they pay not bad dividends. Deposit rates, long terms deposit rates and so on, to provide "guaranteed" returns. "Guaranteed" because, lots of countries have laws guaranteeing safety and security of all deposits. Government obliges banks to return deposits whenever the client demands it. Deposit rates are not high and generally banks always pay less return than market rate but the "guaranteed" return makes it still one of the most attractive investment opportunities. We have to admit that not everybody likes to risk after all. Stock exchange provides alternative source of both funds and investment opportunity. Companies can find capital through sales of shares and investors can buy those shares for earning dividends or reselling it. Stock exchanges require no collateral neither they require "fixed" dividends for investors. Investors might earn a lot more than they expected or they might go to "bottom" and lose all investments, that all will depend on how the company performs. Banks were not asking to be part of the business when companies asked them a loan, they only asked for collateral. The only way companies can raise capital in stock exchanges is selling company shares, thus, investors will be part of the company. That is why you will see how legal status of the "family company" changes to "public company" after sales of shares of company in stock exchanges because this company is no more under sole ownership, it is now owned by public, and profit is shared with shareholders from now on. If more than certain portion of shares have been bought by investors than investors actually can directly intervene into management process of the company. So, both banks and stock exchanges do the same thing: provide funds and opportunity for investment, but they do it in "own way". About how banks attract clients we have talked in previous chapters, they do that with interest rates. If they wanted more investment they increased they deposit rates, wanted to

increase borrowers then they decreased borrowing rates. All you needed when dealing with banks is your identification card and your collateral. Those two things will be enough to open an account and doing all banking operations like borrowing and depositing. This is called a banking procedure. Now, how things work in stock exchanges? Can any company come and start selling shares? No, in reality, stock exchanges have tougher procedures compared to banks.

Every stock exchange has its own conditions, both for investors and companies listed there. Let's start from conditions for companies which want to sell share to investors:

A. **Stock exchange requires companies to have years of experience:** Newly created companies will not be allowed to sell shares because simply nobody will buy their shares. Nobody wants to invest to the company that came out of nowhere and nobody has information about it: about company leadership, management culture, market strategy, labor policy, product quality, previous reports, finances, and so on. Company must have been already functioning and proven its capacity to survive harsh competition in the market and must have built at least satisfactory reputation over the years to earn investors‟ confidence.

B. **Stock exchanges require independent audit:** Before being enlisted in stock exchange, stock exchange management must make sure that you are doing business the way that it must be legally done. That is why each stock exchange require companies to be audited by one of their "chosen" audit companies and according to results of that audit report they will decide to enlist this company or not. Auditing is vital part of the stock exchange operations of companies and even after being enlisted, stock exchange will require companies to publish annual audit reports and punish harshly those companies that do it late or not properly to the level that closing down the company operations. Each stock exchange will have list of trusted audit companies and every company enlisted in that stock exchange will have to be audited only by those trusted audit companies. Audit companies play vital role in building trust between company and investors, and also in finding all kind of economic and financial irregularities which are punishable by law.

C. **Stock exchange require companies to be in certain value to be listed:** Each stock exchange has its own rule regarding "market capitalization", "size" or "value" of company which wants to be enlisted in that stock exchange but there are "minimum size" measurement that is for sure. Some stock exchange may require that companies must be at least 100

MLN USD worth, some lesser or some more **to be** enlisted in that stock exchange. It is not in market interest to put size "growth limits" for companies and actually nobody has a right to limit company growth, as long as company is legally doing everything right (without breaking any anti-monopoly rules, or competition ethics, tax evasion, etc.). But when it comes to being enlisted in stock exchange and dealing with public confidence issue, companies better be of "minimum size" required. Would you trust a company whose value is 1 MLN TMT or 1 TRLN TMT? Yep, when it comes to gaining investors‟ confidence "size" does really matters!

Obligations of investors

There are very few rules for all investors and "middle man" (brokers, agents, etc.) doing operations is stock exchanges. Few but very strict rules enforced by law in most of the countries. Here are:

1) All investors and their representatives must be licensed, must be of legal age and there must not be any kind of restriction against him/her on doing operations in stock exchange. These restrictions might be of ethical form like conflict of interest issue.

2) Insider information: Outflow of any kind of "good" or "bad" information out of management of company may deeply hurt the company finances and investors. If huge amount of sales or purchase was done in a very short period of time, just before the bad news or good news, this will catch interest of supervising agencies. "Insider trading" is illegal in all countries.

Above are the most important obligations of investors and middle man which **must be** followed by anybody who wants to deal with commercial papers and in all countries these obligations are enforced by law. In some countries speculative operations are also limited. Speculative operations are those that are done to influence the market price of stocks by purchasing or selling huge amount of shares in a very short period of time. This has damaging effect to investors who see the prices fall (or rise) deeply in blink of an eye. In panic, others also join. This even further accelerates fall (rise). When prices hit the bottom, speculators start to buy them (undervalued assets). Target of the speculative attack is try to artificially lower down (or increase) the prices of stock to gain profit. In some countries investors‟ daily operation/transaction amount is limited to protect markets from speculative attacks.

Top stock exchanges by their market capitalization (in BLN USD)

1	New York Stock Exchange	New York City	22,923
2	NASDAQ	New York City	10,857
3	Japan Exchange Group	Tokyo	5,679
4	London Stock Exchange	London Milan	4,590
5	Shanghai Stock Exchange	Shanghai	4,026
6	Hong Kong Stock Exchange	Hong Kong	3,936
7	Euronext	Amsterdam Brussels Dublin Lisbon Oslo Paris	3,927
8	Shenzhen Stock Exchange	Shenzhen	2,504
9	Toronto Stock Exchange	Toronto	2,095
10	Bombay Stock Exchange	Mumbai	2,056
11	National Stock Exchange	Mumbai	2,030
12	Deutsche Börse	Frankfurt	1,864
13	SIX Swiss Exchange	Zurich	1,523
14	Korea Exchange	Seoul, Busan	1,463
15	NASAQ Nordic Exchanges Copenhagen Stock Exchange Stockholm Stock Exchange Helsinki Stock Exchange Tallinn Stock Exchange Riga Stock Exchange Vilnius Stock Exchange Iceland Stock Exchange Armenia Securities Exchange	 Copenhagen Stockholm Helsinki Tallinn Riga Vilnius Iceland Yerevan	1,372
16	Australian Securities Exchange	Sydney	1,328
17	Taiwan Stock Exchange	Taipei	966
18	B3	São Paulo	938
19	JSE	Johannesburg	894
20	Bolsas y Mercados Españoles	Madrid	764
21	Singapore Exchange	Singapore	787
22	Moscow Exchange	Moscow	619
23	Stock Exchange of Thailand	Bangkok	549
24	Indonesia Stock Exchange	Jakarta	521
25	Bursa Malaysia	Kuala Lumpur	456

Stock exchanges are simple markets where financial goods such as stock and bonds are sold. They do not produce stocks; they do not find investors; they do not give consultations to companies regarding management or sales tactics; they do not recommend investors where to put money and so on. Stock exchanges just provide companies and investors a space where they can come together and do their business: find capital for one and find

investment opportunity for another one. Stock exchanges' main income are from renting the area (for brokers or agents), charging service fee for transactions (sales and purchases of stocks), and providing any other services easing your business in the stock market. The more companies decide to use area for selling their stocks the richer will get the stock exchange. Big investors with big money will try to rent a corner for making business in that stock exchange. In above table you can see top stock exchange markets in the world by their market capitalization. Market capitalization means total value of companies enlisted in the stock exchange. How it is found? It is found by multiplying number of total shares of companies enlisted in there to the current price of those shares. Let's take as an example the biggest stock exchange market in the world: New York Stock Exchange. Its market capitalization is around 23 TRNL USD, which is found by:

Quantity $_{\text{total shares}}$*Price $_{\text{price of each share}}$ = Market capitalization

Exercise 1: *Assume tiny stock exchange in an island where only 10 companies are listed. Below is information from stock exchange of an island. Find market capitalization.*

Company name	Total shares	Price per share in TMT
A	2000	10
AA	5000	9
AAA	5000	20
B	3000	30
BB	6000	50
BBB	5000	12
BBBB	9000	7
C	5000	40
CC	7000	50
CCC	8000	25

Market capitalization is equal to total shares of company multiplied by current price of that share. Let's find for each company:

Company name	Total shares	Price per share in TMT	Total price
A	2000	10	20000
AA	5000	9	45000
AAA	5000	20	100000
B	3000	30	90000
BB	6000	50	300000
BBB	5000	12	60000
BBBB	9000	7	63000
C	5000	40	200000
CC	7000	50	350000
CCC	8000	25	200000

Island's stock exchange market capitalization is equal to 1428000 TMT.

Regulations prohibit companies to register in different stock exchanges and selling shares, company has right to sell shares only in one stock exchange (where it is registered) in order to omit confusion and fraud. All stock exchanges have opening time and closing time as a regular market. Sales start with yesterday's closing prices and end with today's closing prices. Tomorrow sales opens with today's closing prices. Selling and purchasing of stocks and bonds are not allowed in other hours except working hours of exchange markets. Even if such transaction occurred, this transaction will not be registered by stock exchange which will technically annul transaction. Penalty will follow both seller and buyer. Thus:

$$\text{Today opens with prices} = P_{t-1}$$

$$\text{Today closes with prices} = P_t$$

$$\text{Tomorrow opens with prices} = P_t$$

$$\text{Tomorrow closes with prices} = P_{t+1}$$

While companies are obliged to sell shares only in one stock exchange by regulation, investors are not tied with that regulation that is why, a lot of investors use time differences to switch operations to another market when one is closed that day. As an example New York and London has 8 hours differences, so when London markets are closing investors can switch to New York markets easily.

Exercise 2: *Prices of shares of steel production company Steel Co. opened with 460 TMT per share this morning and closed with 458.9 TMT. What will be the share price of Steel Co. tomorrow at opening minutes?*

Price at opening is going to be 458.9 TMT per share.

What will be the prices in the middle of the trading day tomorrow? Nobody knows.

Company's legal status changes after selling shares to public in stock exchanges, becoming a **public company**. After being enlisted in stock exchanges, legally becoming public company has many obligations in front of investors, thus owners. **These obligations are to publish financial reports, quarterly, semi-annually and annually**; **publish annual (periodic) audit reports; official inspection results of various government agencies such as tax services, immigration agencies; legal law suits against company and results of legal battle; and other various things like penalties, fines and arrests if such a thing occurred**. Since company is a public company, public (investors) have absolute right to know the financial situation of company

because public financial interest is at stake (investors put money and they have taken risks). Moreover, if majority of stakes of company is owned (bought) by certain people or groups, they will have a right directly intervene in to the management of the company, influencing labor policy, product and service quality, sales tactics, production strategy, etc.

Now, let's get to the technical side of shares. Why companies sell their shares? Selling shares is selling ownership of the company, why would somebody want to share company with others? There are many reasons why companies sell their shares but the main reason is to **raise capital**. See, when you go to the bank and take a loan, first of all you need to find collateral and secondly is that you will have to pay fixed interest rate on loan. When you sell shares, you do not need collateral and you will have to pay shareholders profit (dividend) only if you make it. Thus, when you go to banks for capital, you take all risk on yourself, but when you sell shares, you share future risks with shareholders. How this thing works then? Assume you have a chicken farm. It is a family business and was fully built on family savings for 2 MLN TMT. Business is literally blossoming, bringing good profits, businessman feels like there is potential for growth and expansion. Capital is needed for growth and expansion. Goes to the bank for 2 MLN TMT but bank requires collateral of 2.3 MLN TMT. In order to satisfy bank's demands businessman needs to put his own house under collateral with farm. He does not want to do that. There is a second option: finding a partner! To find another businessman who will be interested in this project and invest 2 MLN TMT. This might look easy but after knocking on couple of doors he understands that investors are scared to put whole 2 MLN TMT at once for farming business. It is getting messy. A friend recommends him going to the stock exchange for consultation. He applies and stock exchange management after evaluating application form informs businessman that by years and experience (he was in farming business for 10 years) and a minimum capital requirement (1 MLN TMT) his business is fully satisfies. Stock exchange management gives list of independent audit companies and asks businessman to submit original audit report as soon as audit finished. Businessman immediately goes to one of those companies on the list and after paying audit fees got audited. Report comes with minor irregularities but overall with good conclusion. Businessman submits report to stock exchange management and stock exchange after evaluation of the report and comparison of businessman's farm business with other farming businesses submits report to independent rating agency for ratemaking. Agency gives a credit rating of AA, which is pretty good. Ratemaking is needed because it helps investors to make quick decision regarding purchasing or not. Credit rating is a measure of capability of company to face its debt obligations.

Higher ratings mean company is creditworthy, lower ratings mean the opposite. Rates are like school grades for companies, better grades show how successful student is and everybody likes that. Same is true for companies; better grades show sound financial stability, strong business ethics and so on. There are hundreds and thousands of companies listed in stock exchanges, do I have time to read all their published financial reports and play around with their audit reports one by one? Impossible! Nobody wants to waste time reading reports and evaluating companies, it is hard and time consuming. That is why ratemaking companies do that for investors (not for free of course). They evaluate company according to national standards, financial soundness and business ethics and give rates to them. This makes investors life easy. Now, all investors need is just to look at company ratings and make a decision to put money or not. There are three well-known ratemaking "giants": Moody's, Fitch and Standard and Poor's. They do not only give credit ratings to companies but also to economies. Below is a sample table.

S&P credit ratings of some countries

1	Croatia	BBB-
2	Indonesia	BBB
3	Ireland	AA-
4	Latvia	A+
5	Lithuania	A+
6	Pakistan	B-
7	Philippines	BBB+
8	Portugal	BBB
9	Spain	A
10	Serbia	BB+
11	Ukraine	B
12	United States	AA+
13	Vietnam	BB

After receiving rates stock exchange management decides that company is "good enough" to be put on sales. Invites businessman and they make a contract. Businessman needs 2 MLN TMT and with his own investment of 2 MLN TMT the value of future company will be 4 MLN TMT. Now, 4 MLN TMT worth company is divided to 40000 pieces by businessman (40000 shares) and businessman has right to keep 20000 shares because that is exact amount he had invested from his own pocket. The rest 20000 shares were put for sales for 100 TMT per share.

Total value of company before	2 MLN TMT
The legal status of company: family business, sole proprietorship	2 MLN TMT
Total capital needed for	2 MLN TMT

expansion	
Future value of company	2+2=4 MLN TMT
The legal status of company after selling shares: public company. The owners:	2 MLN TMT (50%) of company owned by businessman. 2 MLN TMT (50%) of company owned by investors, public
Total number of shares	40000
Ownership	20000 shares owned by businessman, 20000 owned by investors
Initial price per share	$\frac{\text{Total value of company}}{\text{Total shares}} = \frac{4000000}{40000}$ = 100 TMT

Stock exchange puts 20000 shares on sale and by contract charges 0.1% of fee for each share sold. Since ratemaking company gave good grades to company there should not be any problems with selling all shares and collecting needed capital for expansion of farm. Assume that all shares were sold. Each share sold is registered in stock exchange with owner of the share. How much did stock exchange earned? 0.1% of each share sold for 100 TMT per share. If all shares were sold then;

Stock exchange brokerage fee=0.1%*100TMT*20000=2000 TMT

Stock exchange takes 2000 TMT and other service fees required by contract from the deal and transfers the rest to the account of newly created chicken farm company. Owner of the company immediately starts expansion project and as per contract publishes all financial statement and other legal documentations on agreed term. A year later farmer reports 1 MLN TMT profit and it is time to pay dividends. Dividends are profits per share and generally paid once in a year in all public companies. How dividends are shared? Total profit is divided to total shares;

Dividends = $\frac{Total\ profit}{Total\ number\ of\ shares} = \frac{1000000\ TMT}{40000\ shares}$ = 25 TMT per share

Each shareholder will be paid 25 TMT. The owner had 20000 shares and his share of profits will be 20000*25=500000 TMT (half of the profits because he owns half of the company). Others will share 500000 TMT according to shares they own.

Exercise 3: *Stock exchange charges 0.01% of price of share sold and 10 TMT for each deal. How much stock exchange earned today*

if total deals were 1400 and price of total shares sold is 34 MLN TMT?

*1400*10=14000 TMT from deals and 34 MLN TMT*0.01%=3400 TMT from total prices of shares sold. In total, stock exchange made 14000+3400=17400 TMT today.*

Now, since it is no more a family company, how will the management of the company change? It will not be led by one person anymore; now, main management decision will be taken by Board of Directors. Board of Directors will be consisted of majority stake owners (normally an investor who owns 10% of shares of a company will automatically be in Board of Directors), and each country has its own legal frame regarding majority stockholder's management rights. Main tactical plans and other financial decisions will be approved in Board of Director's meetings which will be held once in a month or quarterly. It depends on company policy. In those meetings, main problems of company will be discussed like electing Chief Executive Officer, Chief Financial Officer or Main legal consultant of company, business strategy, partnership, financing projects and other important issues. Each meeting will be officially registered and decisions generally approved by voting among the members of Board of Directors. Again, Board of Directors will decide who is going to be Chief Executive Officer (CEO); it might be one of members of Board of Directors or any other suitable person for that position from outside. CEO will lead company, implement corporate policy put forward by Board of Directors, hire and fire people (except those who were appointed by Board of Directors), long word short, CEO will be a person who will be dealing with daily operations of the company. So, will shares stay at same price?

No, of course! Shares are now sold and purchased in secondary market. First of all we have to understand that each share is an investment that brings returns. What is the return rate of our shares in example? It is 25%. Each share earned 25 TMT profit, cost of each share is 100 TMT, and then return rate is:

$$\text{Return on investment} = \frac{Dividend\ per\ share}{Price\ of\ one\ share} * 100 = \frac{25}{100} * 100 = 25\%$$

This is much better investment than simply depositing money in bank, because annual deposit rates in banks are 7% (presumably!).

Exercise 4: *Rate of return of Garagum Co. company shares is around 12% and I have 1000 shares of company for 50 TMT each. What are my total dividends earned?*

$$Return\ on\ investment = \frac{Dividend\ per\ share}{Price\ of\ one\ share} * 100$$

$$12 = \frac{Dividend\ per\ share}{50} * 100$$

$$\text{Dividend per share} = 6\ \text{TMT}$$

$6*1000=6000$ TMT *my total dividends earned*

Share prices will fall and rise according to companies earnings and success. As company makes profits share prices will raise, as company tumbles, share prices start to fall. Gossips about lying off, rumors about management disarray, or some heavy fee, even filed lawsuit can hugely impact share prices. As any other good, price of the shares also depends on demand and supply. Good news will boost investors' confidence about company's bright future and they will rush to get piece of that company. This will increase demand for shares and prices will climb up. Instead, bad news will weaken confidence and investors will try to get rid of potentially bad investment and supply will increase. This will decrease prices of shares. Do price fluctuations of shares in secondary market have any impact on company? Yes, there will be huge impact. Increasing share prices means investors' confidence. Let's say our farm has been doing well for couple of years and wants to expand. Now, target is to be 10 MLN TMT company. Company decides to issue 60000 shares for 100 TMT per share price. It will be total 6000000 TMT which will increase the value of 4 MLN TMT valued company to 10 MLN TMT company as targeted by Board of Directors. Almost in all public companies, when new shares are issued, or if old shares are resold, there will be strict condition that shares must be offered first to the stockholders, if they do not want to buy then, company has a right to offer it to public. New shares will be offered first to members of Board of Directors and let's say that first owner decided to purchase 10000 shares (the first owner buys 10000 shares for 100TMT each and 10000*100=1000000 TMT deposits money to company account! What I want to say is that he cannot just take shares; he must buy it as a regular investor paying money. The only difference is that he would be offered as much shares as he wishes as long as he can pay for it **first**!) And other members also decided to purchase 10000 shares in total (all Board members will have a right to purchase shares **first**, and then it will be offered to other investors through stock exchange!). Thus, out of 60000 newly issued shares for 100 TMT value 20000 shares were purchased by members, the rest is brought to stock exchange for sales. If company was doing well before 40000 shares will have no problem in finding buyers, if company was not doing well, then company will have problems in raising capital. At the end of the day, nobody wants to deal with "falling" company right? If company were doing badly, ratings will fall, investors will run away, shares will drop, and even banks will increase interest rates. Let's assume that all

40000 shares were sold successfully and year later company reports profit of 4 MLN TMT. How profits are shared?

Total value of company	10 MLN TMT
Total shares	100000
Owners of shares	30000 shares owned by creator of company, 30000 shares are owned by board members (20000 previous shares and 10000 new shares), the rest 40000 shares are owned by public
Initial price per share	100 TMT per share
Annual profit reported	4 MLN TMT
Dividend per share	$\frac{Total\ annual\ profit}{Total\ shares} = \frac{4000000}{100000} = 40$ TMT
Rate of return on investment of company shares for investors	$\frac{Dividend\ per\ share}{Price\ of\ one\ share} * 100 = \frac{40}{100} * 100 = 40\%$

Exercise 5: *Small iron mine was started for 10 MLN TMT. Later on with increasing market demand, owners decided to expand the business but they needed 10 MLN TMT. They have applied to stock exchange and been audited and results were satisfactory, they have got good credit ratings and finally they decided to divide the future company to 1 MLN pieces (shares). What will be initial price of shares?*

Initial price will be 20 MLN TMT divided by 1 MLN shares= 20 TMT per share.

A year later company reported a profit of 4 MLN TMT. What is return rate of investments to company shares?

Dividend per share = $\frac{Total\ annual\ profit}{Total\ shares} = \frac{4000000}{1000000} = 4\ TMT$

Return rate = $\frac{Dividend\ per\ share}{Price\ of\ one\ share} * 100 = \frac{4}{20} * 100 = 20\%$

Company is doing extremely well and there are plenty of opportunities for growth and company decides to "pour in" 20 MLN TMT. They decide to issue 2 MLN new shares. What will be initial offer price of shares?

Initial offer prices of shares will be equal to 20 MLN TMT/2MLN TMT=10TMT per share. New shares will cost 10TMT per share. A year later company reports profit of 8 MLN TMT. How profits will be divided? There are two types of shares: 1 MLN shares for 20

TMT each and 2 MLN shares 10 TMT each. Easiest way to solve this question is just find percentage of profit from overall value of company. Thus:

$$\frac{Total\ profit}{Value\ of\ company} * 100 = \frac{8\ MLN\ TMT}{40\ MLN\ TMT} * 100 = 20\%$$

Investors will earn 20% profit from each 1 TMT invested. Those who hold 20 TMT shares will earn 4 TMT per share, and those who hold 10 TMT shares will earn 2 TMT per share.

Since its creation, stock exchanges evolved into important financial institution impacting hugely economic and financial environment. These impacts are:

a) **Corporate governance:** Stock exchanges almost "force" companies to improve their corporate governance culture because they make companies transparent. After becoming public company, companies are obliged to publish all required reports. Companies' behavior changes: they become more law obedient, more business oriented, corporate ethics improves, more practical, aggressive, more competitive, ecofriendly and more humanitarian, because all these things will hugely impact on public relations and stock prices of company. Stock exchange helps companies to "level up" in their corporate governance culture.
b) **Stock exchanges develop indexes for market evaluation:** You may have already heard things like Dow Jones, Nikkei, S&P Global 100, etc. these are all stock market indexes. Each stock exchange creates its own stock market index from top companies listed in that stock exchange. They try to pick companies from various industries (banking, IT, construction, etc.) and only choose the most valuable ones (with high market capitalization). How this index is created? First of all as I told above stock exchange chooses the most valuable companies listed in that stock exchange. Secondly, total values of those companies are calculated. Thirdly, total value of company is divided to total number of shares. Voila! In professional financial slang, shares of those companies are called "blue chip" which has similar meaning to the idiom "blue blood" (old idiom which means "from ruling family"). By fluctuation of index up and down economists can determine "the mood" of investors in the market. When investors believe that prices of shares will increase, optimistic mood, they will start buying shares and this will increase prices due to increased demand for shares and index will rise. Those investors with optimistic mood are called "bulls" in professional financial slang. When indexes rise we say "bulls are winning today". If instead investors believe that prices of shares will fall in future, they

will start selling off shares, this will increase supply of shares and prices will fall. Falling prices will drag down index and in professional financial slang those investors with pessimistic mood are called “bears”. So, when indexes fall, we say “bears are winning today”. Investors’ belief may be based upon many things like news about company, rumors, economic expectations, legal lawsuits and so on. That is the reason why public companies always worry about their “public image” and always try to keep good “public relationship” level. Below are some examples of well-known indexes and their components (blue chip companies).

S & P Global 100 components

1	3M Company
2	ABB Ltd
3	Aegon
4	Allianz AG
5	Anglo American plc
6	Apple Inc.
7	AstraZeneca
8	Aviva
9	AXA
10	Banco Bilbao Vizcaya Argentaria, S.A.
11	Banco Santander, S.A.
12	Barclays
13	BASF AG
14	Bayer AG
15	BHP Billiton Limited
16	BP plc
17	Bridgestone Corp.
18	Bristol-Myers Squibb
19	Canon Inc.
20	Carrefour SA
21	Caterpillar Inc.
22	Chevron Corp.
23	Citigroup Inc.
24	Coca-Cola Co.
25	Colgate-Palmolive
26	Compagnie de Saint-Gobain
27	Credit Suisse Group
28	Daimler AG
29	Dell Inc.
30	Deutsche Bank AG
31	Deutsche Telekom AG
32	Diageo
33	Dow Chemical
34	DuPont
35	E.ON AG
36	EMC Corporation

37	LM Ericsson Telephone Co.
38	Exxon Mobil Corp.
39	Ford Motor Company
40	GDF Suez
41	General Electric
42	GlaxoSmithKline plc
43	Goldman Sachs Group Inc
44	Google
45	Hewlett-Packard
46	Honda Motor Corp.
47	HSBC Holdings plc
48	ING Groep NV
49	Intel Corp.
50	International Business Machines
51	Johnson & Johnson
52	JPMorgan Chase & Co.
53	Kimberly-Clark Corp.
54	L'Oréal SA
55	LVMH Moet Hennessy Louis Vuitton
56	McDonald's Corp.
57	Merck & Co.
58	Microsoft Corp.
59	Morgan Stanley
60	Munich Re AG
61	National Grid plc
62	Nestlé SA
63	Nike, Inc.
64	Nissan Motor Co.
65	Nokia
66	Novartis AG
67	Orange S.A.
68	Panasonic
69	PepsiCo Inc.
70	Pfizer, Inc.
71	Philip Morris International
72	Koninklijke Philips Electronics NV
73	Procter & Gamble
74	Prudential Plc
75	Repsol YPF, S.A.
76	Rio Tinto
77	Royal Dutch Shell
78	RWE AG
79	Samsung Electronics Company Limited
80	Sanofi-Aventis
81	Schneider Electric
82	Seven & I Holdings Co., Ltd.
83	Siemens AG
84	Société Générale
85	Sony Corp.

86	Standard Chartered Bank
87	Swiss Re
88	Telefónica, S.A.
89	Texas Instruments
90	Toshiba Corp.
91	Total S.A.
92	Toyota Motor Corporation
93	Twenty-First Century Fox, Inc.
94	UBS AG
95	Unilever NV
96	United Technologies Corporation
97	Vivendi Universal SA
98	Vodafone Group PLC
99	Volkswagen AG
100	Wal-Mart Stores
101	Westfield Group

Dow Jones Global Titans 50 Components

1	United States	3M Co.	Industrial Goods
2	United States	AbbVie Inc.	Pharmaceutical
3	Germany	Allianz	Insurance
4	United States	Alphabet Inc.	Technology
5	United States	Amazon	Technology
6	United States	Amgen	Pharmaceutical
7	Belgium/United States	Anheuser-Busch InBev	Beverage
8	United States	Apple Inc.	Technology
9	Australia/United Kingdom	BHP	Metals
10	United States	The Boeing Company	Aerospace
11	United Kingdom	BP	Oil & Gas
12	United Kingdom	British American Tobacco	Tobacco
13	United States	Chevron Corporation	Energy
14	United States	Cisco Systems	Technology
15	United States	Citigroup	Banking
16	United States	Coca-Cola	Food & Beverage
17	United States	DuPont	Chemicals
18	United States	ExxonMobil	Energy
19	United States	Facebook	Technology
20	United States	General Electric	Conglomerate
21	United Kingdom	GlaxoSmithKline	Pharmaceutical
22	United Kingdom	HSBC Holdings PLC	Banking
23	United States	Intel	Technology
24	United States	IBM	Technology
25	United States	Johnson & Johnson	Pharmaceutical
26	United States	JPMorgan Chase	Banking
27	United States	Mastercard	Banking
28	United States	McDonald's	Restaurant

29	United States	Merck & Co.	Pharmaceutical
30	United States	Microsoft	Technology
31	Switzerland	Nestlé	Food & Beverage
32	Switzerland	Novartis	Pharmaceutical
33	United States	Nvidia	Technology
34	United States	Oracle Corporation	Technology
35	United States	PepsiCo	Food & Beverage
36	United States	Pfizer	Health Care
37	United States	Philip Morris International	Tobacco
38	United States	Procter & Gamble	Consumer Goods
39	Switzerland	Roche	Health Care
40	Canada	Royal Bank of Canada	Banking
41	Netherlands	Royal Dutch Shell	Oil & Gas
42	South Korea	Samsung Electronics	Conglomerate
43	France	Sanofi	Pharmaceutical
44	Germany	Siemens	Conglomerate
45	Taiwan	Taiwan Semiconductor	Technology
46	France	Total S.A.	Energy
47	United States	Visa Inc.	Banking
48	Japan	Toyota	Automobiles & Parts
49	United States	Walmart	Retail
50	United States	The Walt Disney Company	Entertainment

Dow Jones Industrial average Components

	Company	Industry	Index Weighting
1	3M	Conglomerate	4.35%
2	American Express	Financial services	2.68%
3	Apple Inc.	Information technology	8.01%
4	Boeing	Aerospace and defense	3.87%
5	Caterpillar Inc.	Construction and Mining	3.34%
6	Chevron Corporation	Petroleum industry	2.63%
7	Cisco Systems	Information technology	1.21%
8	The Coca-Cola Company	Food industry	1.31%
9	Dow Inc.	Chemical industry	1.04%
10	ExxonMobil	Petroleum industry	1.32%
11	Goldman Sachs	Financial services	5.29%
12	The Home Depot	Retailing	6.17%
13	IBM	Information technology	3.58%
14	Intel	Information technology	1.72%
15	Johnson & Johnson	Pharmaceutical industry	4.18%
16	JPMorgan Chase	Financial services	2.72%
17	McDonald's	Food industry	5.23%
18	Merck & Co.	Pharmaceutical industry	2.25%
19	Microsoft	Information technology	4.94%

20	Nike	Apparel	2.45%
21	Pfizer	Pharmaceutical industry	1.06%
22	Procter & Gamble	Fast-moving consumer goods	3.26%
23	Raytheon Technologies	Aerospace and defense	1.89%
24	The Travelers Companies	Financial services	2.95%
25	UnitedHealth Group	Managed health care	8.01%
26	Verizon	Telecommunication	1.62%
27	Visa Inc.	Financial services	5.06%
28	Walmart	Retailing	3.44%
29	Walgreens Boots Alliance	Retailing	1.26%
30	The Walt Disney Company	Broadcasting and entertainment	3.12%

Mexican BMV/IPC Index components

1	Walmart de México
2	América Móvil
3	Fomento Económico Mexicano
4	Grupo México
5	Grupo Elektra
6	Grupo Financiero Banorte
7	Coca-Cola Femsa
8	Arca Continental
9	Banco Santander México
10	Grupo Bimbo
11	Grupo Financiero Inbursa
12	El Puerto de Liverpool
13	Kimberly Clark de Mexico
14	Grupo Carso
15	Infraestructura Energética Nova
16	Becle
17	Grupo Aeroportuario del Pacífico
18	Cemex
19	Grupo Televisa
20	Industrias Peñoles
21	Grupo Aeroportuario del Sureste
22	Alfa
23	Gruma
24	Mexichem
25	Promotora y Operadora de Infraestructura
26	Megacable Holdings
27	Alpek
28	Grupo Aeroportuario del Centro Norte
29	Banco del Bajío
30	Alsea
31	Grupo Cementos de Chihuahua
32	Regional
33	Gentera
34	Bolsa Mexicana de Valores

35	Genomma Lab Internacional

Swiss SMI index components

1	Nestlé	Food
2	Novartis	Pharmacy
3	Hoffmann-La Roche	Pharmacy
4	Zurich Insurance Group	Insurance
5	UBS	Banks
6	ABB	Electrical equipment
7	Richemont	Luxury Goods
8	Lonza	Chemistry
9	Givaudan	Chemistry
10	Alcon	Pharmacy
11	Sika	Chemistry
12	Swiss Re	Insurance
13	Credit Suisse	Banks
14	LafargeHolcim	Building materials
15	Geberit	Sanitary engineering
16	SGS	Services
17	Swisscom	Telecommunications
18	Swiss Life Holding	Insurance
19	Swatch Group	Watches
20	Adecco	Services

Chinese SSE 50 Index components

1	Shanghai Pudong Development Bank	Banking
2	China Minsheng Bank	Banking
3	Sinopec	Oil & gas
4	China Southern Airlines	Airline
5	CITIC Securities	Financial services
6	China Merchants Bank	Banking
7	Poly Real Estate	Real estate
8	China United Network Communications	Telecommunication
9	Tsinghua Tongfang	
10	SAIC Motor	Automotive
11	China Northern Rare Earth	Mining
12	China Fortune Land Development	Real estate
13	Xinwei Group	
14	Kangmei Pharmaceutical	Pharmaceutical
15	Kweichow Moutai	Beverages
16	Shandong Gold Mining	Mining
17	Greenland Holdings	Real estate
18	Haitong Securities	Financial services
19	Yili Group	Dairy
20	Bank of Jiangsu	Banking
21	Orient Securities	Financial services
22	China Merchants Securities	Financial services

23	Daqin Railway	Rail transport
24	China Shenhua Energy	Energy
25	Industrial Bank	Banking
26	Bank of Beijing	Banking
27	China Railway Construction	Construction
28	Dongxing Securities	Financial services
29	Guotai Junan Securities	Financial services
30	Bank of Shanghai	Banking
31	Agricultural Bank of China	Banking
32	Ping An Insurance	Insurance
33	Bank of Communications	Banking
34	New China Life Insurance	Insurance
35	China Railway	Construction
36	Industrial and Commercial Bank of China	Banking
37	China Pacific Insurance	Insurance
38	China Life Insurance	Insurance
39	China State Construction Engineering	Construction
40	Huatai Securities	Financial services
41	CRRC	Rolling stock
42	Everbright Securities	Financial services
43	China Communications Construction	Construction
44	China Everbright Bank	Banking
45	PetroChina	Oil & gas
46	China Galaxy Securities	Financial services
47	Founder Securities	Financial services
48	China National Nuclear Power	Energy
49	Bank of China	Banking
50	China Shipbuilding Industry	Shipbuilding

Dutch AEX Index components

1	ABN AMRO	banks
2	Adyen	support services, financial administration
3	Aegon	life insurance
4	Ahold Delhaize	food retailers & wholesalers
5	AkzoNobel	specialty chemicals
6	ArcelorMittal	iron & steel
7	ASM International	semiconductors
8	ASML Holding	semiconductors
9	ASR Nederland	full line insurance
10	DSM	specialty chemicals
11	Galapagos	biotechnology
12	Heineken	brewers
13	IMCD	specialty chemicals
14	ING Group	banks
15	Just Eat Takeaway	specialized consumer services
16	KPN	fixed line telecommunications
17	NN Group	life insurance

18	Philips	medical equipment
19	Prosus	Consumer Digital Services
20	Randstad	business training & employment agencies
21	RELX	publishing
22	Royal Dutch Shell	integrated oil & gas
23	Unibail-Rodamco-Westfield	retail REITs
24	Unilever	personal products
25	Wolters Kluwer	publishing

India's NIFTY 50 Index components

1	Adani Ports	Infrastructure
2	Asian Paints	Consumer Goods
3	Axis Bank	Banking
4	Bajaj Auto	Automobile
5	Bajaj Finance	Financial Services
6	Bajaj Finserv	Financial Services
7	Bharti Airtel	Telecommunication
8	Bharti Infratel	Telecommunication
9	Bharat Petroleum	Energy - Oil & Gas
10	Britannia Industries	Consumer Goods
11	Cipla	Pharmaceuticals
12	Coal India	Energy & Mining
13	Dr. Reddy's Laboratories	Pharmaceuticals
14	Eicher Motors	Automobile
15	GAIL	Energy - Oil & Gas
16	Grasim Industries	Cement
17	HCL Technologies	Information Technology
18	HDFC	Financial Services
19	HDFC Bank	Banking
20	Hero MotoCorp	Automobile
21	Hindalco Industries	Metals
22	Hindustan Unilever	Consumer Goods
23	ICICI Bank	Banking
24	IndusInd Bank	Banking
25	Infosys	Information Technology
26	Indian Oil Corporation	Energy - Oil & Gas
27	ITC Limited	Consumer Goods
28	JSW Steel	Metals
29	Kotak Mahindra Bank	Banking
30	Larsen & Toubro	Construction
31	Mahindra & Mahindra	Automobile
32	Maruti Suzuki	Automobile
33	Nestle India	Consumer Goods
34	NTPC	Energy - Power
35	Oil and Natural Gas Corporation	Energy - Oil & Gas
36	Power Grid Corporation of	Energy - Power

	India	
37	Reliance Industries	Energy - Oil & Gas
38	State Bank of India	Banking
39	Shree Cements	Cement
40	Sun Pharmaceutical	Pharmaceuticals
41	Tata Motors	Automobile
42	Tata Steel	Metals
43	Tata Consultancy Services	Information Technology
44	Tech Mahindra	Information Technology
45	Titan Company	Consumer Goods
46	UltraTech Cement	Cement
47	United Phosphorus Limited	Chemicals
48	Vedanta	Metals
49	Wipro	Information Technology
50	Zee Entertainment Enterprises	Media & Entertainment

Exercise 7: *Assume that Swiss SMI Index components and their share prices are given below (numbers are not real!). Please find market capitalization and SMI Index.*

	Swiss SMI index components	**Total shares**	**Current price per share in FRANKS**	**Market capitalization**
1	Nestlé	10000000	300	
2	Novartis	2000000	200	
3	Hoffmann-La Roche	300000	250	
4	Zurich Insurance Group	60000	600	
5	UBS	1000000	800	
6	ABB	250000	1000	
7	Richemont	800000	650	
8	Lonza	90000	400	
9	Givaudan	40000	100	
10	Alcon	300000	200	
11	Sika	600000	350	
12	Swiss Re	70000	580	
13	Credit Suisse	5000000	700	
14	LafargeHolcim	1000000	320	
15	Geberit	2000000	740	
16	SGS	3000000	850	
17	Swisscom	600000	950	
18	Swiss Life Holding	2000000	750	
19	Swatch Group	500000	450	
20	Adecco	400000	400	
	Swiss SMI index			

Solution:

	Swiss SMI index components	Total shares	Current price per share in FRANKS	Market capitalization
1	Nestlé	10000000	300	3000000000
2	Novartis	2000000	200	400000000
3	Hoffmann-La Roche	300000	250	75000000
4	Zurich Insurance Group	60000	600	36000000
5	UBS	1000000	800	800000000
6	ABB	250000	1000	250000000
7	Richemont	800000	650	520000000
8	Lonza	90000	400	36000000
9	Givaudan	40000	100	4000000
10	Alcon	300000	200	60000000
11	Sika	600000	350	210000000
12	Swiss Re	70000	580	40600000
13	Credit Suisse	5000000	700	3500000000
14	LafargeHolcim	1000000	320	320000000
15	Geberit	2000000	740	1480000000
16	SGS	3000000	850	2550000000
17	Swisscom	600000	950	570000000
18	Swiss Life Holding	2000000	750	1500000000
19	Swatch Group	500000	450	225000000
20	Adecco	400000	400	160000000
	Total number of shares	**30010000**	Total market capitalization (values of company)	**15736600000 FRANKS**
	Swiss SMI index $(\frac{Total\ market\ cap.}{Total\ number\ of\ shares})$	**524.37854**		

Assume next day prices of shares fluctuate and our SMI Index will reflect how Swiss companies are doing in market.

	Swiss SMI index components	Total shares	Current price per share in FRANKS	Market capitalization
1	Nestlé	10000000	297	2970000000
2	Novartis	2000000	201	402000000
3	Hoffmann-La Roche	300000	240	72000000
4	Zurich Insurance Group	60000	594	35640000
5	UBS	1000000	810	810000000
6	ABB	250000	980	245000000
7	Richemont	800000	651	520800000
8	Lonza	90000	409	36810000
9	Givaudan	40000	97	3880000
10	Alcon	300000	188	56400000
11	Sika	600000	354	212400000

12	Swiss Re	70000	600	42000000
13	Credit Suisse	5000000	680	3400000000
14	LafargeHolcim	1000000	300	300000000
15	Geberit	2000000	728	1456000000
16	SGS	3000000	839	2517000000
17	Swisscom	600000	940	564000000
18	Swiss Life Holding	2000000	710	1420000000
19	Swatch Group	500000	488	244000000
20	Adecco	400000	410	164000000
	Total number of shares	**30010000**	Total market capitalization (values of company)	**15471930000**
	Swiss SMI index	**515.55915**		

As you can see from above calculations, SMI Index is down by $\frac{524.37854-515.55915}{524.37854} * 100 = 1.68\%$ *which means Swiss companies did "bad" today.*

In secondary market, shares of the companies are purchased and sold. Majority of traders in stock exchange are speculators: those who want to make profit in differences of prices. Speculators purchase shares and then they sell when prices are high enough. Prices might go either way depending on company situation that is why there are no guarantees that there will be any profit at all. Assume that you have bought K amount of shares with price of P_1 *per each share. During a day price changed and market closed with price* P_2*, difference will be either your profit or loss:*

$$(P_1 - P_2)*K$$

Next day markets open with price P_2 *and closes with* P_3*. Assume you kept shares for 50 days, what will be your gain or loss from this investment? Answer:*

$$(P_1 - P_2)*K + (P_3 - P_4)*K + (P_5 - P_6)*K \ldots (P_{50} - P_{51})*K$$

We can take K's out:

$$[(P_1 - P_2) + (P_3 - P_4) + (P_5 - P_6 \ldots (P_{50} - P_{51})]*K$$

$$[P_1 - P_2 + P_3 - P_4 + P_5 - P_6 \ldots P_{50} - P_{51}]*K$$

After cancellations we will have only:

$$(P_1 - P_{51})*K$$

Thus, your gain or loss from investment will depend only on your purchase price and sales day price, other fluctuations does not matter!

Not only private corporate shares are traded in stock exchanges but also government bonds (debt obligations) and other financial goods. We have solved some exercises with shares and now let's explore bonds a little bit. A bond is a debt obligation generally issued by governments; it is a piece of paper (promissory note) with face value (or par value) on it and sometimes with coupon payment paid in terms, which could be redeemed when maturity date comes up (or redemption date).

***Exercise 8**: A bond with face value of 1000 TMT with maturity date of 2 years is traded in stock exchange for 900 TMT. Buyer pays 900 TMT and purchases bond. After two years trader redeems bond back to government and receives 1000 TMT. What is bond's yield to maturity (YTM) (current rate, or rate of return)?*

*Since this bond pays no coupons, this bond is called **zero coupon bonds.** Then:*

$$900(1+r)^2=1000$$

$$(1+r)^2=1.11$$

$$\ln[(1+r)^2]=\ln(1.11)$$

$$2\ln(1+r)=0.104$$

$$\ln(1+r)=0.052$$

$$e^{0.052}=1+r$$

$$1.053=1+r$$

$$r=5.3\%$$

Yield to maturity rate of these bonds are 5.3%.

***Exercise 9:** 10000 TMT bond is maturing in 10 years. It has a coupon of 5% of the face value paid semi-annually. What is the price of this bond if yield to maturity rate is 8%?*

First let's understand this question: We must purchase this bond for price of P today and 10 years later we will be paid 10000 TMT (redemption value). This purchase yields us two things: coupon of 5% of face value (500 TMT) once in a six month (total of 20

coupon payments) and annual interest rate of 8%. Let's write it down in equation:

$$P=\frac{500}{(1+0.04)^1}+\frac{500}{(1+0.04)^2}+\frac{500}{(1+0.04)^3}+\cdots+\frac{500}{(1+0.04)^{20}}+\frac{10000}{(1+0.04)^{20}}$$

We know how to solve this kind of equations from previous chapters.

$$P=500\left(\frac{1}{1+0.04}\right)\left(\frac{1-\left(\frac{1}{1+0.04}\right)^{20}}{1-\left(\frac{1}{1+0.04}\right)}\right)+\frac{10000}{(1+0.04)^{20}}=6788+4563=$$

11351 TMT

Bond's initial price must be around 11351 TMT. What if initial price is lower? Let's say bond is priced at 11000 TMT. If that is true, then this bond will be yielding more than 8% annually. If bond's initial price is more than 11351 TMT, then this bond will be yielding less than 8% annual income.

***Exercise 10**: 5000 TMT bond is maturing in 5 years. It has a coupon 10% of face value paid semi-annually and yield is 5%. What is fair price for this bond?*

$$P=500\left(\frac{1}{1+0.025}\right)\left(\frac{1-\left(\frac{1}{1+0.025}\right)^{10}}{1-\left(\frac{1}{1+0.025}\right)}\right)+\frac{5000}{(1+0.025)^{10}}=4376+3906=$$

8282 TMT

Almost all textbooks never explain what this 8282 TMT price of bond means. Let's solve this once and for all. So, this bond will pay us 500 TMT once in a six month and in the end there will be last 500 TMT and 5000 TMT redemption. If we deposit to the bank each 500 TMT we receive from coupon payment with annual interest rate of 5% compounded semi-annually we will have in the end:

$$500(1+0.025)^9+500(1+0.025)^8+\ldots+500(1+0.025)^1+500+5000=$$

Take 500's out of parenthesis:

$$500[(1+0.025)^9+(1+0.025)^8+\ldots+(1+0.025)^1+1]+5000=$$

Same mathematical series principle applied in here:

$$500\left(\frac{1-(1+0.025)^{10}}{1-(1+0.025)}\right)+5000=10601 \text{ TMT}$$

So, in total over 5 years we will earn 10601 TMT. It is same amount as:

$$8282(1+0.025)^{10}=10601 \text{ TMT}$$

Homework:

I. "Angry Ram" mining company has 1 MLN shares and each share's current market price is 11.3 TMT. Find market capitalization of the company? What will happen to the value of company if share prices drop to 11 TMT?

II. Which stock exchange, do you think, has more demanding regulations (strict) Bombay Stock Exchange or New York Stock Exchange? Rationalize your view.

III. In which stock exchange do you think, brokerage fees and other transaction payments are higher, in London Stock Exchange or in B3 (Brazil)? Rationalize your view.

IV. Assume two country representatives approach you with loan requests: Indonesia and Spain. Which country you will prefer to give loan and why? If your advisor recommends to give to both countries, will the rates be the same or different?

V. "Angry Ram" issued 300000 shares for 8 TMT each and offered for sales in stock exchange. Stock exchange charges 0.03% fee for each share sold. If only 80% of "Angry Ram" offered stocks were sold yesterday, what was stock exchange's earning from this sales yesterday?

VI. JST (Johannesburg Stock Exchange) charges 20 RANDS (South Africa's national currency) for each deal plus 0.02% of brokerage fee for total trade made. There were 456 trade deals registered yesterday and total trade amount were for 2034585800 RANDS. How much JST earned yesterday?

VII. Shortly, what do you know about **Galapagos** from Dutch AEX Index? (What this company does? Market capitalization of company (value)? Managed by whom? When was it created? Employee numbers? What is the main market for its products? Etc.)

VIII. Shortly, what do you know about **Westfield Group** from S&P Global 100 Index?

IX. Shortly, what do you know about **NVidia** from Dow Jones Global Titans 50 Index?

X. Shortly, what do you know about B3 (Stock exchange of Brazil)?

XI. Shortly, what do you know about Shanghai Stock Exchange?

XII. Shortly, what do you know about **3M** from Dow Jones Industrial average Index?

XIII. Shortly, what do you know about **Cisco Systems** from Dow Jones Industrial average Index?

XIV. Shortly, what do you know about **CEMEX** from Mexican BMV/ IPC Index?

XV. Shortly, what do you know about **Mexichem** from Mexican BMV/ IPC Index?

XVI. Shortly, what do you know about **Geberit** from Swiss SMI Index?

XVII. Shortly, what do you know about **Richemont** from Swiss SMI Index?

XVIII. Shortly, what do you know about **Kweichow Moutai** from Chinese SSE 50 Index?

XIX. Shortly, what do you know about **Yili Group** from Chinese SSE 50 Index?

XX. Shortly, what do you know about **China Fortune Land Development** from Chinese SSE 50 Index?

XXI. Shortly, what do you know about **Adani Ports** from India's NIFTY 50 Index?

XXII. Shortly, what do you know about **Zee Entertainment Enterprises** from India's NIFTY 50 Index?

XXIII. "Funny Giraffe" company earned a profit of 5 MLN TMT which made a return rate of a company equal to 10% and with 400000 shares. What is total value of company and what is initial price of those shares?

XXIV. 10 Zero Coupon Bonds with face value of 10000 TMT and maturity of 8 years were bought for 6000 TMT each. What is a YTM of these bonds?

XXV. An investor bought 10000 shares of company for 52 TMT each. He kept them for 9 days and sold them. Here are prices of shares per each day: (52.2), (52.6), (53.7), (52), (51.2), (50), (53.2), (53.4), and (53.5). What is trader's total loss or profit if stock exchange charges 200 TMT per deal and 0.01% of total traded amount in market?

XXVI. 7000 TMT bond is maturing in 7 years. It has a coupon 7% of face value paid semi-annually and yield is 7%. What is fair price for this bond?

XXVII. GDP of USA is around 22 TRLN USD (2019) and total market capitalization of companies listed in NASDAQ and New York Stock Exchange is above 32 TRLN USD. Can you explain this?

XXVIII. "Space Bull" company is a family farm with 10 MLN TMT initial capitals. They want to expand business and they need 10 MLN TMT. How they can find this money using stock exchange mechanism?

XXIX. 9000 TMT bond is maturing in 10 years. It has a coupon 3% of face value paid semi-annually and yield is 5%. Bank offers saving services for 6% annual savings rate compounded semi-annually for 10 years. Which investment is preferable?

XXX. Below is Dutch AEX Index components and information on current prices (numbers are not real).

	Dutch AEX Index components	**Shares**	**Price**	**Market Cap.**
1	ABN AMRO	300000	36	
2	Adyen	2000000	50	
3	Aegon	50000	200	

4	Ahold Delhaize	250000	90	
5	AkzoNobel	500000	110	
6	ArcelorMittal	700000	300	
7	ASM International	3000000	400	
8	ASML Holding	4000000	80	
9	ASR Nederland	5000000	75	
10	DSM	1000000	60	
11	Galapagos	6000000	180	
12	Heineken	250000	55	
13	IMCD	400000	80	
14	ING Group	800000	300	
15	Just Eat Takeaway	360000	500	
16	KPN	60000	600	
17	NN Group	95000	700	
18	Philips	1400000	800	
19	Prosus	1800000	900	
20	Randstad	80000	1000	
21	RELX	100000	2000	
22	Royal Dutch Shell	200000	400	
23	Unibail-Rodamco-Westfield	1000000	500	
24	Unilever	7000000	600	
25	Wolters Kluwer	8000000	800	

Find Market Capitalization and Dutch AEX Index.

XXXI. Prices changed next day:

	Dutch AEX Index components	**Shares**	**Price.**	**Market Cap**
1	ABN AMRO	300000	38	
2	Adyen	2000000	48	
3	Aegon	50000	210	
4	Ahold Delhaize	250000	100	
5	AkzoNobel	500000	100	
6	ArcelorMittal	700000	295	
7	ASM International	3000000	390	
8	ASML Holding	4000000	78	
9	ASR Nederland	5000000	70	
10	DSM	1000000	55	
11	Galapagos	6000000	170	
12	Heineken	250000	50	
13	IMCD	400000	88	
14	ING Group	800000	330	
15	Just Eat Takeaway	360000	525	
16	KPN	60000	640	
17	NN Group	95000	700	
18	Philips	1400000	820	
19	Prosus	1800000	950	
20	Randstad	80000	990	
21	RELX	100000	1880	
22	Royal Dutch Shell	200000	390	
23	Unibail-Rodamco-Westfield	1000000	470	

24	Unilever	7000000	570	
25	Wolters Kluwer	8000000	770	

Find new market capitalization and Dutch AEX Index and tell who is winning over the market, bulls or bears?

XXXII. Assume NASDAQ Index is 548004. Find total number of shares traded in NASDAQ.

XXXIII. What will be B3 Index if we knew that total number of shares traded is 3 BLN?

XXXIV. Dividends are 20 TMT per share, and I have 3000 shares. What is price of shares if my return rate is 12%? What is my initial investment amount? How much will I earn if I sell shares for 190 TMT per share?

Solutions:

I. **Market capitalization=Quantity$_{\text{total shares}}$*Price$_{\text{price of each share}}$**
Market capitalization=1000000*11.3=11300000 TMT
If price drops to 11 TMT:
Market capitalization=1000000*11=11000000 TMT

II. More demanding (strict) regulations give us well done job result, and well done job leads us to higher rating. According to the rating New York Stock Exchange on the 1st place worldwide shows us that it has more demanding regulations than Bombay Stock Exchange.

III. In London's Stock Exchange brokerage fees and other payments are higher than in B3. Because, all companies in order to raise funds want to sell their shares in a profitable place where they are sure that they will find investors, but also they know that to find good place they must pay good fees.

IV. I would give loan to Spain, because Spain is more creditworthy (according to statistics of S&P, table above in the chapter). If my advisor will recommend me to give loan to both countries, loan rate would be different for each of them. For Spain, loan rate would be lower due to higher creditworthiness (A rating), and Indonesia's would be higher due to lower creditworthiness (rating BBB).

V. 8*0.0003=0.0024
0.0024*300000*0.8=576 TMT earned from sales.

VI. 456*20=9120 TMT earnings from trade deals
2034585800*0.0002=406917.16 TMT earnings from fees
9120+406917.16=416037.16 TMT was earned by stock exchange yesterdays.

VII. Galapagos is pharmaceutical company which was based by Onno Van de Stolpe in 1999 and still he is CEO (Chief Executive Officer) and Raj Parekh is a chairman, with 725 employees. Market capitalization is 11.9 Billion USD. Main markets for its products are: Galapagos Biotech Ltd., BioFocus DPPI AG, and Galapagos Inc etc...

VIII. Westfield group provides asset/funds management, leasing services, property management, construction, design and real estate investment since 1960 by John Sounders. Its market capitalization is 7 BLN USD. Current (2020) CEO of the company is Peter S. Lowy.

IX. NVidia is an Aericam multinational technology company that produces graphic processing units, drivers, shipsets and so on. It was founed in 1993 by Jensen Huang, Curtis Priem and Chris Malachowsky. Its market capitalization is 266.9 BLN USD with over 13000 employees.

X. B3 is Brazil Stock Exchange and OTC (over-the-counter) market with 2.37 TRLN USD market capitalizations. It was based in 1890 by Emilio Rangel Pestana. Current chairman is Antonio Carlos Quintella and CEO I Gilson Finkelsztain with Edemir Pinto.

XI. The Shanghai stock Exchange is Chinese stock exchange market which was based in 1990 in Shanghai, China. Its market capitalization is 5 TRLN USD. President of the stock exchange is Zhang Yujun with a chairman Geng Liang.

XII. 3M (Minnesota Mining and Manufacturing Company) is an American technology corporation which was founded in 1902 by John Dwain, Henry Brian and William McGonagall. It has over 93500 employees. CEO is Mike Roman and its market capitalization is 96 BLN USD.

XIII. Cisco System is an American multinational technology conglomerate which was based in 198 by Leonard Bosack and Sandy Lerner. CEO and a chairman is Chuck Robbins. Its market capitalization is 180 BLN USD with 75900 employees.

XIV. CEMEX is Mexican multinational company that produces building materials which was based in 1906 by Lorenzo Zambrano Guttierez. CEO is Fernando A. Gonzalez and chairman is Rogelio Zambrano Lozano. This company has over 42000 employees and its market capitalization is 30 BLB USD.

XV. Mexichem is a Mexican company that produces chemical and petrochemical goods. It was founded in 1953. Chairman of the company is Juan Pablo Del Valle Perochena and CEO of the company is Daniel Martinez-Valle. This company has more than 22000 employees and its market capitalization is 78 BLN USD.

XVI. Geberit is a Swiss multinational group which manufactures and supplies plumbing and sanitary relate goods. It was founded in 182 by Caspar Melchior Gebert. CEO is

Christian Buhl and chairman is Albert M. Baehny. Market capitalization of the company is about 18 BLN USD and the company has 12000 employees.

XVII. Richemont luxury goods holding company which was based in Switzerland in 1988 by African businessman Johann Rupert. CEO of the company is Jerome Lambert with Chairman Johann Rupert. Its market capitalization is about 31 BLN USD and it has over 3000 employees.

XVIII. Kweichow Moutai Co. Ltd is partially state owned and partially publicly owned enterprise which produces packaging material, food, research and development relevant information technology and Maotai baijiu (alcohol) in China. It was founded in 1999 by Mao Zedong. Its market capitalization is 2.043 BLN USD. Current chairman is Yuan Renguo.

XIX. Yili Group is Chinese company specialized in processing and manufacturing of milk products as well as ice cream, milk tea powder and organic milk. It was founded in 1993. Its chairman is Mr. Pang Gang and market capitalization of 24.5 BLN USD.

XX. China Fortune Land Development is company that operates and develops real estate in China which was founded by Wang Wenxue in 1993. Its market capitalization is 10 BLN USD and has over 1000 employees. CEO of the company is Wu Xiangdong.

XXI. Adani Ports is India's largest multiport operator which specializes in maintenance of port and port based related facilities. It was founded in 1998 by Gauam Adani, CEO is Karan Adani. Its market capitalization is 694 BLN USD with 420 employees.

XXII. Zee entertainment Enterprises is a media and entertainment broadcasting company of Essel group. It was founded in 1926 by Jagannath Goenk in India. This company has over 10000 employees. Its market capitalization is 154 BLN. Chairman of the company is Subhash Chandra.

XXIII. Dividends $= \frac{Total\ profit}{Total\ number\ of\ shares} = \frac{5000000\ TMT}{400000\ shares} =$ 12.5 TMT

Return on investment $= \frac{Dividend\ per\ share}{Price\ of\ one\ share} * 100$

Return on investment = $\frac{12.5\ TMT}{P}$*100 = 10%

$$\frac{10}{100} = \frac{12.5}{P}$$

0.1*P=12.5

P=125 TMT per share

Total value of the company = 125*400000=50000000 TMT

Initial price of the shares is=125

XXIV. $6000(1+r)^8=10000$

$(1+r)^8=1.67$

$\ln [(1+r)^8] = \ln (1.67)$

$8\ln (1+r) =0.51$

$\ln (1+r) =0.0638$

$e^{0.0638}=1+r$

1.066=1+r

r=6.6%

Yield to maturity rate of these bonds are 6.6%.

XXV. Final price of shares was 53.5 TMT:

53.5-52=1.5

1.5*10000=15000 TMT

15000-200-53.500*0.0001= 14746.5 TMT total profit

XXVI. $P=490\left(\frac{1}{1+0.035}\right)\left(\frac{1-\left(\frac{1}{1+0.035}\right)^{7*2}}{1-\left(\frac{1}{1+0.035}\right)}\right)+\frac{7000}{(1+0.035)^{7*2}} = 5351 + 4324.47 = 9675.47TMT$

XXVII. GDP and market capitalization are different indicators of the country. GDP of USA shows aggregate output in a given year while market capitalization shows total value of shares that are traded in the stock exchanges.

XXVIII. They (family that owns "Space Bull") go to the stock exchange to raise funds and stock exchange and face with requirements of stock exchange: firstly, they will require certain years of experience, if they are experienced enough then, stock exchange will require independent audit, to be checked from their own audit companies. And finally, stock exchange will require the company to fulfill "minimum size" measurements. After passing all these requirements the family will be allowed to raise funds by selling shares of the company.

XXIX. If we invest to the bond we will get:

$$P=270\left(\frac{1}{(1+0.025)}\right)\left(\frac{1-\left(\frac{1}{1+0.025}\right)^{2*10}}{1-\left(\frac{1}{1+0.025}\right)}\right)+\frac{9000}{(1+0.025)^{2*10}}=$$

$4209.07+5492.4=9701.47$ is fair price of the bond.

If we invest each coupon to the bank we will get:

$$270\left(\frac{1-\left(1+\frac{0.06}{2}\right)^{2*10}}{1-\left(1+\frac{0.06}{2}\right)}\right)+9000=16255 \text{ TMT}$$

$$9701.47*\left(1+\frac{0.06}{2}\right)^{2*10}=17521.9$$

Investment to the bank is more preferable.

XXX.

	Dutch AEX Index components	**Shares**	**Price**	**Market Cap.**
1	ABN AMRO	300000	36	10800000
2	Adyen	2000000	50	100000000
3	Aegon	50000	200	10000000
4	Ahold Delhaize	250000	90	22500000
5	AkzoNobel	500000	110	55000000
6	ArcelorMittal	700000	300	210000000
7	ASM International	3000000	400	1200000000
8	ASML Holding	4000000	80	320000000
9	ASR Nederland	5000000	75	375000000
10	DSM	1000000	60	60000000
11	Galapagos	6000000	180	1080000000
12	Heineken	250000	55	13750000
13	IMCD	400000	80	32000000
14	ING Group	800000	300	240000000
15	Just Eat Takeaway	360000	500	180000000
16	KPN	60000	600	36000000
17	NN Group	95000	700	66500000
18	Philips	1400000	800	1120000000
19	Prosus	1800000	900	1620000000
20	Randstad	80000	1000	80000000
21	RELX	100000	2000	200000000
22	Royal Dutch Shell	200000	400	80000000
23	Unibail-Rodamco-Westfield	1000000	500	500000000
24	Unilever	7000000	600	4200000000
25	Wolters Kluwer	8000000	800	6400000000

Market capitalization=18211550000 TMT

Total number of shares=44345000

Dutch AEX index = $\left(\frac{Total\ market\ cap.}{Total\ number\ of\ shares}\right)$ =

$\left(\frac{18211550000}{44345000}\right)$ = 410.6788

XXXI.

	Dutch AEX Index components	Shares	Price	Market Cap.
1	ABN AMRO	300000	38	11400000
2	Adyen	2000000	48	96000000
3	Aegon	50000	210	10500000
4	Ahold Delhaize	250000	100	25000000
5	AkzoNobel	500000	100	50000000
6	ArcelorMittal	700000	295	206500000
7	ASM International	3000000	390	1170000000
8	ASML Holding	4000000	78	312000000
9	ASR Nederland	5000000	70	350000000
10	DSM	1000000	55	55000000
11	Galapagos	6000000	170	1020000000
12	Heineken	250000	50	12500000
13	IMCD	400000	88	35200000
14	ING Group	800000	330	264000000
15	Just Eat Takeaway	360000	525	189000000
16	KPN	60000	640	38400000
17	NN Group	95000	700	66500000
18	Philips	1400000	820	1148000000
19	Prosus	1800000	950	1710000000
20	Randstad	80000	990	79200000
21	RELX	100000	1880	188000000
22	Royal Dutch Shell	200000	390	78000000
23	Unibail-Rodamco-Westfield	1000000	470	470000000
24	Unilever	7000000	570	3990000000
25	Wolters Kluwer	8000000	770	6160000000

Market capitalization= 17735200000 TMT
Total number of shares=44345000

Dutch AEX index = $\left(\frac{Total\ market\ cap.}{Total\ number\ of\ shares}\right)$

Dutch AEX index = $\left(\frac{17735200000}{44345000}\right)$ = 399.9369

$\frac{399.9369-410.6788}{410.6788}$*100 = - 2.61%

Bears are winning over the market.

XXXII. **NASDAQ index** = $\left(\frac{Total\ market\ cap.}{Total\ number\ of\ shares}\right)$ = $\frac{10857000000}{Total\ number\ of\ shares}$ = 548004

Total number of shares = $\frac{10857000000}{548004}$ = 19812

XXXIII. B3 index = $\left(\frac{Total\ market\ cap.}{Total\ number\ of\ shares}\right)$ = $\frac{938000000000}{3000000000}$ = 312.67

XXXIV. Return on investment = $\frac{Dividend\ per\ share}{Price\ of\ one\ share}$*100

$12 = \frac{20\ TMT}{P}$*100

$\frac{12}{100} = \frac{20}{P}$

0.12P=20

P=166.67 TMT per share

Initial investment amount = 166.67*3000=500010 TMT

If I sell shares for 190 TMT each, my earnings will be:

(190-166.67)*3000=69990 TMT

Chapter 10: Basic Finance 4

Our world has evolved socially and economically. Global production (GDP of all countries) is highest and keeps on growing every year. New products, services and technology are just mind blowing. Honestly, I am a bit scared to think about products and services that we will have 50 years later: surgeries done by robots with artificial intelligence, pills that cure immediately, liquid food, humanoid real like robots, borderless global world, fast education, eco-cities with zero emission, genetically modified animals, no cash at all, zero bureaucracy, etc. these things mind sound fantastic but do not forget that technologies that we are having today is "unbelievable and fantastic" for people lived just 100 years ago from now. Technological and scientific advancement is growing exponentially in our times. This growth has huge impact on society both sociologically and economically. Sociologically, human beings are becoming more and more liberal. While life expectancy is constantly increasing, our mental health conditions are a bit worsening. We became more isolated, this leads to stress and depression. Economically, countries are becoming more and more dependent to each other. Trade increased and we started producing and consuming a lot more than our predecessors. This volume of consumption and production has a price: global pollution. Our planet is polluted more than ever and if this production and consumption level goes on, nothing good is awaiting us in future that are for sure. These rapid advancements in technology and science cause society to change its consumption and saving preferences. Innovations does not touch only certain parts of human lives, it actually touches whole our life, our economy. New financial instruments, new institutions, new ideas, even potential new currency is being intensively developed to meet our demands.

Derivatives and Securities

Derivatives are financial instruments that derive their value from underlying asset. This instrument became extremely popular not long 30-40 years ago, (in financial world that is very "young" innovation) and ever since market share increased rapidly. Derivatives are traded in two types of markets generally: **future markets** and **over-the counter market**.

Top Futures exchange markets	
1	CME Group (US)
2	National Stock Exchange of India (India)
3	Intercontinental Exchange (US)
4	CBOE Holdings (US)
5	Eurex (Europe)

6	NASDAQ (US)
7	Moscow Exchange (Russia)
8	Korea Exchange (South Korea)
9	Shanghai Futures Exchange (China)

Future markets are a place where derivatives are traded and exchange clearing house (Exchange market) acts as a guarantor for both traders on compliance of rules of trade. Thus, in futures exchanges, traders do not worry about other party's creditworthiness (same as in stock exchanges!) and all contract details are open to public (everybody can see the prices and delivery date of contract). Exchange clearing house require both parties to open a trader account, it is called **margin account**, and require traders to deposit certain percentage of contract deal to the **margin account** to ensure that both traders will comply with their obligations. Daily earnings (or losses) will be settled through **daily settling** mechanism of futures exchanges. According to price fluctuations of assets underlying derivatives, daily earning or loss margins will be transferred from one party's margin account to another's. Futures exchanges earn by providing clearing services (guarantor) and from brokerage deals (% fee from total trade). In **Over-the-Counter markets** there is no clearing house, thus traders bilaterally agree to cover all expenses or they just present the role of judge to the third party, who is called Central Counterpart, for certain fee. There will be no such a thing as **margin account** or **daily settling mechanism** like in futures exchanges and all transactions (payment and transfer of goods) will occur at delivery date of contract. Trade deals, prices and deliveries will not be open to public and only trading parties will know all details of contract, providing parties needed privacy. There are four types of derivatives contracts in general: **Forwards, Futures, Options** and **Swaps.** Three of them: **Futures, Options** and **Swaps** are traded in *Futures exchanges* and three of them: **Forwards**, **Options** and **Swaps** are traded in *Over-the-Counter markets*. In derivatives exchange markets having **long position** means *agreeing to buy* (certain asset for certain price), and having **short position** means agreeing to sell (certain asset at certain price). We will discuss only fundamentals of derivatives and its impacts on economy because of two reasons. First of all because it is huge topic to be explained in one chapter whole trade strategies and other detailed topics. Secondly, this is advanced finance topic which is understood better when application is used (programs and games). Simply explaining it on paper will be same as explaining brain-surgery without any surgery. This is the case where students must see it, rather than read about it. (Actually, all exchanges; stock and futures are better learned in practice!).

Let's start from technical side first: technicality of contract types and mathematics, later on we will discuss impact on economy.

Futures contract

As name says it all, **futures contract** is an agreement between traders to buy or sell an asset on agreed price at a certain time in future.

Exercise 1: *Assume a farmer specializing in beef meat is expecting to produce 1000 tons of beef meat in 9 month (it is February now and meat will be ready to deliver on December) and current beef meat prices are good for farmer, it is 10000 TMT per ton. So, total contract size is 10 TML TMT (10000 TMT*1000 Tons=10000000 TMT). Farmer makes a contract where he obliges to deliver 1000 Tons of beef meat to buyers place on 1 December for price of 10000 TMT/ton and brings this contract for sales to local futures exchange. Futures exchange clearing house requires farmer to open a margin account and deposit 0.5% of total contract size to margin account as a proof that product will be delivered. After submitting all legal documents to exchange clearing house where farmer proves that the he can deliver the product in contract and depositing 500000 TMT (0.5% of size of contract) to margin account, clearing house puts this futures contract on sale. A buyer, who is interested in this contract (fast food restaurant chain owner for example) which needs beef meat 9 month later after looking into contract and learning about meat quality decides to buy this futures contract. He applies to exchange clearing house for purchasing and clearing house requires buyer to open a margin account and deposit 0.5% of size of contract that will ensure his creditworthiness. Buyer will immediately open a margin account and after depositing 500000 TMT (0.5% of contract size) to his margin account trade deal will be sealed. Seller will be obliged to deliver 1000 Tons of beef meat on 1 December and buyer will be obliged to buy 1000 Tons of beef meat on 1 December for 10000 TMT/ton. Exchange clearing house clearing fees are 1000 TMT per deal which is charged to both parties and 0.01% fees of contract size (sales fees) which is charged only to seller (these fees change depending on futures exchange markets) on delivery date when seller receives money from buyer. Daily settlement mechanism of futures exchange markets work this way: due to daily price fluctuations in market either one side will win from this trade. If prices go up, (for example 10080 TMT per ton) then buyer wins because he purchased beef meat cheaper (he purchased it for 10000 TMT per ton). In this case, amount equal to total loss amount of seller (80 TMT per ton, 1000 Tons then total loss of seller is 80*1000=80000 TMT) will be transferred from seller's*

*margin account to buyers margin account (Now, sellers margin account balance is 420000 TMT and buyer's margin account balance is 580000 TMT). If prices fall next day (for example to 10020 TMT per ton), then seller wins (20*1000=20000 TMT will be transferred from buyer's margin account to sellers) because price was fixed and any fall from this price is a win (Now, margin account balance of seller will be 440000 TMT and buyer's margin account will be 560000 TMT) for seller (or spot rate). Below is table of margin account changes for next 20 days:*

	Trade price	**Initial Margin account of buyer**	**Initial Margin account of seller**
	10000	500000	500000
days	**Settlement price**	**Margin account balance of buyer**	**Margin account balance of seller**
1	9970	470000	530000
2	9978	478000	522000
3	9983	483000	517000
4	9994	494000	506000
5	10041	541000	459000
6	10050	550000	450000
7	9990	490000	510000
8	9987	487000	513000
9	10007	507000	493000
10	10010	510000	490000
11	10057	557000	443000
12	10043	543000	457000
13	10040	540000	460000
14	10037	537000	463000
15	10040	540000	460000
16	10018	518000	482000
17	10009	509000	491000
18	9991	491000	509000
19	9990	490000	510000
20	9986	486000	514000

*As you can see from above, margin account movements changed daily according to spot price (current price) of beef meat in market. At the end of 20th day, as you can see seller is winning (gaining profit) because he sold beef meat for 10000 TMT per ton, and prices dropped to 9986 TMT per ton now, which means seller was able to sell his good for higher than expected market price. Seller is winning 14 TMT per ton, and 14*1000=14000 TMT on green (which means making profit!). Buyer is on red (making loss), 14 TMT per ton, in total 14000 TMT. Now, assume that buyer, the restaurant chain owner wants to sell this contract. Can he do that? Yes, he can sell this contract until the couple of weeks to delivery*

date. Couple of weeks before delivery date of contract, sales will be not allowed in exchange due to date of contract is coming. Due date of each contract will be mentioned in contract and it might change from contract to contract. The reason why contract sales is over couple of weeks before the contract due date is that seller must be able to deliver it on time. Delivery might take from couple of days to couple of weeks depending on location differences of seller and buyer. Assume restaurant chain owner wants to sell this contract to potential buyer, transferring his own obligation to another one. He has a right to do that, and he does it. What will be price then? Price will be the market price, which is settlement price in our table. You remember that nobody can sell any good in market economy above the market price right? I mean, even if restaurant chain owner would have wished to sell this contract higher than 10000 TMT per ton he would not be able to do that, because on 20th day market price per ton of beef is at 9986 TMT per ton. With pushing couple of buttons he puts this futures contract on sale. All exchange markets are digital nowadays (like stock exchanges) and trades take place in couple of seconds, no more. Assume that another buyer is interested in this futures contract and after thinking she decides to buy it. Let's say she is major supplier of supermarkets. Supermarket supplier before purchasing this futures contract must have to create margin account and deposit 0.5% of initial contract size, thus 500000 TMT. After depositing and proving her creditworthiness she will have a right to make a purchase. She buys the future contract (purchase obligation, a promise to buy) from restaurant chain owner for 9986 TMT per ton (trade price) but she does not pay anything to anybody. Restaurant owner also did not receive any payments because he sold the futures contract (purchase obligation, a promise to buy), that is it. No real trade is occurred, no commodity was exchanged only the "purchase obligators changed". For initial seller nothing changed, he promised to deliver 1000 Ton of beef meet on 1 December for 10000 TMT per ton and he is still keeping his promise, even if other party changed, it does annul his promise. At the end of the day farmer does not care who pays him, as long as he gets paid. Only buyers change. One buyer changed to another buyer. What restaurant chain owner gained from this trade? He gained nothing; actually he made a loss of 14000 TMT and 1000 TMT (total loss of 15000 TMT). 14000 TMT from price fluctuations, which is transferred from his margin account on daily basis (his margin account on 20th day is 486000 TMT) and plus 1000 TMT, he paid to clearing house for the deal. After re-selling the futures contract to supermarket supplier, restaurant chain owner will have no obligations left, so if he wishes he can close his margin account opened in exchange clearing house transferring the balance 486000 TMT to anywhere he wants or he can actually do any other trade at the same exchange if he wishes to stay. Now, only two parties go on from now on: the seller

(farmer) and buyer (supermarket supplier) and daily price fluctuations on market go on like shown below:

	Trade price	**Initial Margin account of buyer**	**Initial Margin account of seller**
	9986	500000	514000
days	**Settlement price**	**Margin account of buyer**	**Margin account of seller**
21	9983	497000	517000
22	9980	494000	520000
23	9977	491000	523000
24	9972	486000	528000
25	9971	485000	529000
26	9966	480000	534000
27	9951	465000	549000
28	9942	456000	558000
29	9921	435000	579000
30	9905	419000	595000
31	9870	384000	630000
32	9851	365000	649000
33	9841	355000	659000
34	9834	348000	666000
35	9822	336000	678000
36	9814	328000	686000
37	9803	317000	697000
38	9788	302000	712000
39	9787	301000	713000
40	9781	295000	719000

On 40th day, prices fall to 9781 and something happens to margin accounts. As we know, futures exchange put initial margins for both traders ensuring their creditworthiness of 0.5% of contract size and both traders were obliged to deposit that sum to trade. Second condition that will exchange clearing house will put is minimum margin which is a minimum amount that margin accounts must hold. This is called maintenance margin. Both initial margins and maintenance margins will be put by exchange clearing house. Assume maintenance margin for this contract was put 300000 TMT. Thus, any trader whose margin account will fall below 300000 TMT will have 24 hours (this is fixed in all exchanges!) to raise balance to initial margin (to 500000 TMT). If it will not be raised to initial margin amount, clearing house will immediately annul trade and penalize the party whose margin fall below maintenance margin. Penalty amount is hefty and moreover, exchange clearing house may put trade restriction on that trader, which will put him on a "black list" and maybe even banned from trading at all. It is a nightmare for any trader that is why in almost

all times all traders will do whatever they can to raise capital to initial level. In our case, on 40th day, buyers initial margin fall below maintenance margin of 300000 TMT, that is why, exchange clearing house will give him 24 hours to raise capital to 500000 TMT. Let's assume buyer raised its margin account balance to initial margin. Trade will go on:

	Trade price	**Initial Margin account of buyer**	**Initial Margin account of seller**
	9986	500000	514000
days	**Settlement price**	**Margin account of buyer**	**Margin account of seller**
40	9781	295000+205000	719000
41	9771	490000	729000
42	9773	492000	727000
24	9770	489000	730000

When both parties agreed on price P, their profits and loss will depend on price fluctuation. When prices go up, sellers lose because they have sold asset for cheaper price. When prices fall, sellers win because they sold the asset for higher price. Easiest way to find out if seller lost or won from this trade will be:

$$(P_i - P_f)*Q_{\text{underlying asset}}$$

For example on 40th day beef prices were 9781 TMT per ton and if we put is on equation, then seller is on a green for;

$$(P_i - P_f)*Q_{\text{underlying asset}}$$

*(10000 - 9781)*1000=219000 TMT*

If seller would have chosen to sell beef meat not 40 days ago but today, market prices would have been 9781 TMT per ton. Because he sold meat at a price of 10000 TMT per ton, he is earning profits because the price that he sold the meat is above current price of beef meat. Since trade is zero sum game (there is one winner and one loser) we can also interpret this equation reverse by saying that buyer had lost 219000 TMT from this trade deal in 40 days. Let's assume the prices of last 20 days (on last 20th day deal closed and futures contract cannot be sold to anyone, assume that 9 month equals to 275 days).

	Trade price	**Initial Margin account of buyer**	**Initial Margin account of seller**
	9986	500000	514000
days	**Settlement price**	**Margin account of buyer**	**Margin account of seller**

256	9982	701000	518000
257	9984	703000	516000
258	9983	702000	517000
259	9980	699000	520000
260	9987	706000	513000
261	9988	707000	512000
262	9991	710000	509000
263	9990	709000	510000
264	9986	705000	514000
265	9983	702000	517000
266	9980	699000	520000
267	9984	703000	516000
268	9987	706000	513000
269	9986	705000	514000
270	9990	709000	510000
271	9991	710000	509000
272	9994	713000	506000
273	9991	710000	509000
274	9990	709000	510000
275	9992	711000	508000

*Let's make analysis what happened now. When beef meat was first sold for 10000 TMT per ton, buyer resold the contract on 20th day when prices fell to 9986 TMT per ton. Pay attention to margin account of seller, his balance on 20th day is 514000 TMT. Seller already made 14000 TMT profit. On 20th day same futures contract was resold to supermarket supplier for 9986 TMT per ton. On 40th day, buyer's margin account fell below maintenance margin of 300000 TMT and buyer had to raise balance to initial margin due to regulation of exchange clearing house, he deposited 205000TMT. At the end of 275 day meat was delivered and buyer pays 9986 TMT per ton for 1000 Ton of beef meat (9986*1000=9986000 TMT). He initially deposited 500000 TMT later on he had to deposit 205000 TMT more and at the end of the contract he has 711000 TMT in his account, thus 711000-500000-205000=6000 TMT, this is his profit amount, 6000 TMT. If he had not purchased this futures contract he would have purchased beef meat for 9992 TMT per ton today. Now, let's switch to seller. On 20th day in sellers margin account was 514000 TMT and he was already on 14000 TMT on green. On 20th day contract was resold for 9986 TMT per ton and on 275th day he had 508000 TMT in his margin account balance. Thus, seller first made 14000 TMT profit, then 6000 TMT loss, in total, he made 8000 TMT profit from this trade deal because he was able to sell a good at 10000 TMT per ton while prices fell to 9992 TMT per ton. He was able to sell good above the market price. Let's use our formula for seller:*

$(P_i - P_f)*Q_{underlying\ asset}$

(10000 - 9986)*1000=14000 TMT

Contract was resold and:

$(P_i - P_f)*Q_{underlying\ asset}$

(10000 - 9992)*1000=8000 TMT

Seller made 8000 TMT revenue from this deal. Now let's use this for buyers:

$(P_i - P_f)*Q_{underlying\ asset}$

(9986-10000)*1000= -14000 TMT

First buyer made 14000 TMT losses from price fluctuations plus 1000 TMT for clearing house service fee (total loss of Buyer 1 is 15000 TMT) and resold the contract afraid from continuous falling of price. He sold the contract for 9986 TMT per ton. Equation for second purchaser is:

$(P_i - P_f)*Q_{underlying\ asset}$

(9992-9986)*1000= 6000 TMT

Second buyer actually finished in green, with profit of (6000-1000 TMT which he must pay for clearing house services) 5000 TMT. Last thing left is the sales fee of 0.01% of initial contract size of exchange clearing house which is only paid by sellers, thus 0.01% of 10 MLN TMT is equal to 1000 TMT. Total profit of seller is 8000 TMT-1000 TMT (clearing house service fee) -1000 TMT (sales fee) =6000 TMT.

Exercise 2: *1000 ton aluminum futures contract with 50 day delivery was sold for 7000 TMT per ton. Initial margin was put by exchange clearing house as 1% of contract size and maintenance margin was put as 40000 TMT. On 12^{th} day contract was resold to buyer 2. On 30^{th} day contract was resold to buyer 3. On 45^{th} day (last day due to delivery) contract was resold to buyer 4.*

Price fluctuations are given in below table, please find how much profit or loss each buyer made and seller in the end and explain if where explanation is needed. (Clearing house fee is 1000 TMT per deal and 0.01% is commission is charged to initial seller).

	Trade price	**Initial Margin account of seller**	**Initial Margin account of buyer 1**	**Initial Margin account of buyer 2**	**Initial Margin account of buyer 3**	**Initial Margin account of buyer 4**
	7000	70000	70000	70000	70000	70000
days	**Settlement price**	**Margin account of seller**	**Margin account of buyer 1**	**Margin account of buyer 2**	**Margin account of buyer 3**	**Margin account of buyer 4**
1	7002					
2	7004					
3	7001					
4	7002					
5	6998					
6	6996					
7	6997					
8	6998					
9	6999					
10	7000					
11	7001					
12	7002					
13	7003					
14	7004					
15	7002					
16	7001					
17	6997					
18	6992					
19	6991					
20	6990					
21	6991					
22	6992					
23	6993					
24	6998					
25	7004					
26	7000					
27	6998					
28	6997					
29	6996					
30	6995					
31	6994					
32	6991					
33	6992					
34	6990					
35	6987					
36	6988					
37	6988					
38	6989					
39	6980					

40	6975					
41	6972					
42	6970					
43	6969					
44	6965					
45	6960					
46	6954					
47	6930					
48	6925					
49	6920					
50	6900					

Now, let's first see what happens in margin accounts of sellers and buyers:

	Trade price	**Initial Margin account of seller**	**Initial Margin account of buyer 1**	**Initial Margin account of buyer 2**	**Initial Margin account of buyer 3**	**Initial Margin account of buyer 4**
	7000	70000	70000	70000	70000	70000
days	**Settlement price**	**Margin account of seller**	**Margin account of buyer 1**	**Margin account of buyer 2**	**Margin account of buyer 3**	**Margin account of buyer 4**
1	7002	68000	72000			
2	7004	66000	74000			
3	7001	69000	71000			
4	7002	68000	72000			
5	6998	72000	68000			
6	6996	74000	66000			
7	6997	73000	67000			
8	6998	72000	68000			
9	6999	71000	69000			
10	7000	70000	70000			
11	7001	69000	71000			
12	7002	68000	72000			
13	7003	67000		71000		
14	7004	66000		72000		
15	7002	68000		70000		
16	7001	69000		69000		
17	6997	73000		65000		
18	6992	78000		60000		
19	6991	79000		59000		
20	6990	80000		58000		
21	6991	79000		59000		
22	6992	78000		60000		
23	6993	77000		61000		
24	6998	72000		66000		

25	7004	66000		72000		
26	7000	70000		68000		
27	6998	72000		66000		
28	6997	73000		65000		
29	6996	74000		64000		
30	6995	75000		63000		
31	6994	76000			69000	
32	6991	79000			66000	
33	6992	78000			67000	
34	6990	80000			65000	
35	6987	83000			62000	
36	6988	82000			63000	
37	6988	82000			63000	
38	6989	81000			64000	
39	6980	90000			55000	
40	6975	95000			50000	
41	6972	98000			47000	
42	6970	100000			45000	
43	6969	101000			44000	
44	6965	105000			40000	
45	6960	110000			35000	
46	6954	116000				64000
47	6930	140000				40000
48	6925	145000				35000
49	6920	150000				30000+40000
50	6900	170000				50000

Futures contract was sold for 7000000 TMT (size of contract), 1000 tons of aluminum for 7000 TMT per ton. On 12th day prices rose to 7002 TMT per ton Buyer 1 decided to resell the contract. Revenue of Buyer 1 is:

$$(P_f - P_i)*Q_{\text{underlying asset}}$$

(7002-7000)*1000= 2000 TMT

Revenue-Cost=Profit

2000-1000 (clearing house fee) =1000 TMT

Buyer 1 made 1000 TMT profit in 12 days, not bad I will say. Buyer 2 purchased aluminum for 7002 TMT per ton, then his revenue and profit in 18 days (he resold it on 30th day!):

$$(P_f - P_i)*Q_{\text{underlying asset}}$$

(6995-7002)*1000= -7000 TMT

Revenue-Cost=Profit

-7000-1000 (clearing house fee) =-8000 TMT

Buyer 2 made -8000 TMT of loss in 18 days. Buyer 2 sold futures contract to Buyer 3 on 30th day for 6995 TMT per ton. Buyer 3 made a revenue and profit of:

$$(P_f - P_i)*Q_{\text{underlying asset}}$$
(6960-6995)*1000= -35000 TMT
Revenue-Cost=Profit
-35000-1000 (clearing house fee) =-36000 TMT

Buyer 3 made -36000 TMT of loss in 15 days (It could be worse I would say). On 44th day Buyer 3 was right on the maintenance margin, at 40000 TMT. 45th day prices fell even more and probably Buyer 3 thought that selling will be better than raising capital to initial margin of 70000 TMT. Traders can do that, they can sell contract if they cannot find capital but they must do it within 24 hours! Exchange clearing house will give them 24 hours for decision: to raise capital, to sell or do something else (barter the contract, yep, it is allowed too). Buyer 3 sold futures contract to Buyer 4 on 45th day for 6960 TMT per ton. Buyer 4 made a revenue and profit of:

$$(P_f - P_i)*Q_{\text{underlying asset}}$$
(6900-6960)*1000= -60000 TMT
Revenue-Cost=Profit
-60000-1000 (clearing house fee) =-61000 TMT

Somebody must show this to Buyer 3! I told you, "It could be worse!" The worst 5 days I will say to Buyer 4! On 49th day he was even obliged to raise capital to 70000 TMT again from low dropping fall to 30000 TMT. The biggest loser among all buyers! Only Buyer 1 made a profit, other buyers all made losses. So how well did seller? Use the formula again (do not forget to reverse it!) for initial price putting trade price of 7000 TMT and final price as the price of last 50th day of 6900 TMT.

$$(P_f - P_i)*Q_{\text{underlying asset}}$$
(7000-6900)*1000= 100000 TMT
Revenue-Cost=Profit
100000-1000 (clearing house fee) -700 (0.01% commission fee)
=98300 TMT

Seller made 98300 TMT profit because he was able to sell aluminum to much expensive price than market price. Simple way of finding sellers revenue is just look at margin account balance in the end, seller had 170000 TMT. As we know trade started with 70000 TMT on margin account and it increased to 170000 TMT. Seller made 100000 TMT of revenue. All margin account balances actually show revenue amount. Buyer 1's margin account balance started with 70000 TMT ending balance was 72000 TMT, 2000

TMT of revenue. Buyer 2 started with 70000 TMT margin account balance and ended with 63000 TMT, -7000 TMT of revenue. Buyer 3 started with 70000 TMT account balance and ended with 35000 TMT, -35000 TMT of revenue. Buyer 4 started with 70000 TMT and ended with (50000-40000 he raised the 40000 extra, do not forget about it) 10000 TMT, -60000 TMT of revenue. All fits!

Exercise 3: *30 day titanium futures contract was put on sales for 12000 TMT per ton and 1000 ton in total. Below are 30 day prices and find winner and loser and also size of the contract. Exchange clearing house service fee is 1000 TMT and 0.01% of sales commission fee is charged to seller only when he delivers and receives payment. Initial margin is 1000000 TMT and maintenance margin is 500000 TMT was put by exchange clearing house.*

	Trade price	**Initial Margin account of seller**	**Initial Margin account of buyer**
	12000	1000000	1000000
days	**Settlement price**	**Margin account of seller**	**Margin account of buyer**
1	12001		
2	12003		
3	11998		
4	11997		
5	11995		
6	11990		
7	11992		
8	11986		
9	11982		
10	11981		
11	11973		
12	11965		
13	11952		
14	11930		
15	11921		
16	11906		
17	11885		
18	11880		
19	11900		
20	11925		
21	11935		
22	11968		
23	11990		
24	12020		
25	12047		

26	12125		
27	12158		
28	12204		
29	12210		
30	12190		

*Total contract size is 12000*1000=12000000 TMT. Below is table where margin account balances changes due to daily settlement.*

	Trade price	**Initial Margin account of seller**	**Initial Margin account of buyer**
	12000	1000000	1000000
days	**Settlement price**	**Margin account of seller**	**Margin account of buyer**
1	12001	999000	1001000
2	12003	997000	1003000
3	11998	1002000	998000
4	11997	1003000	997000
5	11995	1005000	995000
6	11990	1010000	990000
7	11992	1008000	992000
8	11986	1014000	986000
9	11982	1018000	982000
10	11981	1019000	981000
11	11973	1027000	973000
12	11965	1035000	965000
13	11952	1048000	952000
14	11930	1070000	930000
15	11921	1079000	921000
16	11906	1094000	906000
17	11885	1115000	885000
18	11880	1120000	880000
19	11900	1100000	900000
20	11925	1075000	925000
21	11935	1065000	935000
22	11968	1032000	968000
23	11990	1010000	990000
24	12020	980000	1020000
25	12047	953000	1047000
26	12125	875000	1125000
27	12158	842000	1158000
28	12204	796000	1204000
29	12210	790000	1210000
30	12190	810000	1190000

As it is seen from above table, when prices for titanium fell, seller benefitted (margin account balance was above 1000000 TMT!) because he sold titanium for higher price (he is on green) but to the end of trade prices rise sharply (very volatile market!) and buyer benefitted because he bought asset for cheaper price (now seller is on red and buyer is on green, see margin account balance of buyer from 24th day, above 1000000 TMT which shows that he is doing great!). Simply by looking at the margin account balances at the end of the trade deal one can say easily who is winner and who is loser. Look at the margin account balance of seller, it is 810000 TMT but trade started with 1000000 TMT so seller is 1000000-810000=190000 TMT on loss. Somebody's loss is somebody's gain: buyer is 190000 TMT on green.

Seller's total loss:

$$(P_f - P_i)*Q_{\text{underlying asset}}$$

(12000-12190)*1000= -190000 TMT

Revenue-Cost=Profit

-190000-1000 (clearing house fee) -1200 (0.01% commission fee)

= -192200 TMT

Buyer's total profit:

$$(P_f - P_i)*Q_{\text{underlying asset}}$$

(12190-12000)*1000= 190000 TMT

Revenue-Cost=Profit

190000-1000 (clearing house fee) =189000 TMT

Now, let's conclude this analysis. How much seller will be paid? He will be paid 12000 TMT per ton for 1000 ton of titanium as per contract, thus he will be paid 12 MLN TMT. Did he lose anything? He was paid exact amount which was agreed on, so how come he is in loss? The thing is, the contract was signed a month before the "real trade" and prices were "old" prices. After a month market prices changed, but he was paid in "old" prices. That is why seller incurred a loss. Actually nothing comes out of his pocket, he received all money agreed on but he lost in this trade because he sold titanium well below the market equilibrium price. Seller incurred a "loss of income" not an accounting loss.

The Beauty of Daily Settlement Mechanism

The Daily Settlement Mechanism transfers daily gains and losses from margin accounts of traders on daily basis. This has many benefits and first of all to the seller. In our first exercise 1000 ton of beef meat was sold initially for 10000 TMT per ton by seller. On

20^{th} day Buyer 1 sold this futures contract to Buyer 2 for 9986 TMT per ton. But was not the price 10000 TMT agreed between seller and buyer initially? How can Buyer 1 sell futures contract for cheaper price without consent of Seller? He does not need consent of Seller because due to Daily Settlement Mechanism of futures exchanges market, gains and losses are transferred on daily basis. Seller already earned all from falling prices. At the end of the trade deal prices were 9992 TMT per ton. Buyer 2 purchased contract for 9986 TMT per ton. On that day Seller already was on green on 14000 TMT as we showed. At the end of the trade seller will be paid 9986 TMT per ton, not 10000 TMT per ton but this does not matter for seller, because he is going to earn still 10000 TMT per ton. (14000 TMT he already earned and was transferred from margin account of Buyer 1 to Seller's margin account + 9986000 TMT will be paid by Buyer 2 at the end of trade then 9986000+14000=1000000 TMT!) We showed in that example that seller's revenue was 8000 TMT. Due to Daily Settlement Mechanism of market no matter how prices changes, no matter how many hands will contract change, seller will earn initially agreed price for his asset. I actually showed one very good calculation in previous chapter where I showed that only **initial** and **final** price on market matters for sellers and buyers:

In secondary market, shares of the companies are purchased and sold. Majority of traders in stock exchange are speculators: those who want to make profit in differences of prices. Speculators purchase shares and then they sell when prices are high enough. Prices might go either way depending on company situation that is why there are no guarantees that there will be any profit at all. Assume that you have bought K amount of shares with price of P_1 per each share. During a day price changed and market closed with price P_2, difference will be either your profit or loss:

$$(P_1 - P_2)*K$$

Next day markets open with price P_2 and closes with P_3. Assume you kept shares for 50 days, what will be your gain or loss from this investment? Answer:

$$(P_1 - P_2)*K + (P_2 - P_3)*K + (P_3 - P_4) *K \dots (P_{50} - P_{51})*K$$

We can take K's out:

$$[(P_1 - P_2) + (P_2 - P_3) + (P_3 - P_4) + \dots + (P_{50} - P_{51})]*K$$

$$[P_1 - P_2 + P_2 - P_3 + P_3 - P_4 \dots P_{50} - P_{51}]*K$$

After cancellations we will have only:

$$(P_1 - P_{51})*K$$

Thus, your gain or loss from investment will depend only on your purchase price and sales day price, other fluctuations does not matter!

Forwards

Forward contracts have two differences from futures contracts and first one is that they are traded only in OTC (Over-the-Counter) markets and second is that they do not have Daily Settlement Mechanism, thus settlement occurs at the end of the contract. Details of forward contracts generally are known only to seller and buyer that is why until the end both sides will not know who is winner and who is loser. Other than that, both futures and forward contracts have the same objective fixing prices and trade of future today.

Exercise 4: *1000 shares of "Angry Ram" company for 1000 TMT per share is agreed to be sold 90 days later. Contract size is 1000*1000=1000000 TMT. 90 days later market price of share of "Angry Ram" company is to 1082 TMT per share. Who won seller of forward contract or buyer of forward contract?*

Let's use our favorite math for finding seller's revenue:

$$(P_f - P_i)*Q_{\text{underlying asset}}$$

(1000-1082)*1000= -82000 TMT

Since there is no clearing house in OTC market, commission and clearing services will be arranged in between traders. As it can be seen from above, seller will lose because he sold shares 1000 TMT while share prices rose to 1082 TMT. Seller incurred an income loss. Buyer won because he purchased shares cheaper than market price. His profit equals to loss amount of seller.

IMPORTANT!

Financial assets (shares, stocks, bonds, currencies, etc.) are same as commodity assets (cotton, gold, gasoline, gold, house, etc.) they are traded in stock exchanges, futures exchanges, OTC markets, and they are even used as collateral for bank loans.

Benefits of futures and forwards: Futures and forward contracts help to stabilize prices in markets. When hundreds of thousands and millions of traders sign futures and forward contracts they pretty much make "unpredictable" future predictable. Suppose hundreds of cotton producers signing futures contract this will

make future prices of cotton pretty much "exact" thing rather than "expected". Futures and forwards make future a little bit transparent especially when it comes to prices. Producers, manufacturers, dealers, and others feel more confident when they have information regarding future input prices. Of course if nothing external intervenes like war, conflicts, epidemics, catastrophes or cataclysm. No economy is guaranteed from things like that. Even though recent global financial crisis is thought to be happened due to derivatives, I truly believe that derivatives are excellent financial instruments and have more benefit than negatives. **The problem is not in derivatives, it is in misusing of them.**

Options

An **option** is a contract which allows trader to exercise the right: to buy or not to buy, to sell or not to sell. Futures and forward contracts do not leave choice (option) for traders, they both must comply the agreement. **Options** give choice to traders, and these choice have a price it is called **option price.** There are two types of **option contracts:**

1) **Call option:** gives holder (the buyer of call option) **the right** to buy an asset for specified price at specified time.

2) **Put option:** gives holder (the buyer of put option) **the right** to sell an asset for specified price at specified time.

These **rights** are not for free of course, and buyer actually buys that right.

Exercise 5: *Jeren wants to sell her house and she is pricing her house at 2 MLN TMT. Akjemal is her best friend and she wants to buy Jeren's house but she does not have all amount of money right now and she needs a time to raise funds (she probably will ask money from her parents and friends as always). Since house is in good condition and at excellent location Akjemal knows that Jeren will have no problem in selling this house immediately if she puts that house on sale. Seller (Jeren) offers a contract to buyer (Akjemal) where she promises to sell this house to her for 2 MLN TMT a month later if buyer agrees to pay her 2000 TMT now for "withholding the house for one month period". This is a* ***call option*** *contract, and Akjemal immediately signs it and pays Jeren 2000 TMT, this payment is called* ***option price.*** *Now, Akjemal just bought herself a* ***right to buy*** *this house one month later for 2 MLN TMT and Jeren is obliged to keep this house for buyer for one month as per contract. Jeren got paid for her waiting. If Akjemal would not be able to find funds at a given time, within one month, Jeren will have a right to sell house to anybody else. But during that one month, nobody but only Akjemal has a right to buy this*

house. If Akjemal will not be able to find funds than her 2000 TMT payment will also be gone. So, if Akjemal exercises her right and buys this house, then this house will cost her 2002000 TMT. If she does not exercise her right to buy this house, then she will lose 2000 TMT. If Akjemal exercises her right to buy this house total Jeren's earnings will be 2002000 TMT, if Akjemal does not exercise her right then Jeren's earning will be 2000 TMT and right to put the house on public sale. Call option gives Akjemal (buyer) the right to exercise or not to exercise her rights within specific time.

Exercise 6: *A farmer is growing tomatoes and this year she is expecting a harvest of 1000 tons in four months and she is already on a long term sales contract for 900 tons with local ketchup making company. She is expecting to have 100 tons of tomatoes in excess and she is afraid she will not be able to sell them on time. Ketchup company owner knows farmer's problem and offers her a contract where he promises to buy 100 tons of her excess tomatoes for 2000 TMT per ton. Ketchup Company offers this contract to farmer for 10000 TMT. The price of tomato in contract is a little below the market price (assume market price is 2500 TMT) but farmer is happy to purchase this contract because this will guarantee that her 100 ton of tomato will not be wasted and she will not incur loss. Little is better than nothing says she and purchases this contract for 10000 TMT. Farmer just bought a* ***put option****, right to sell 100 ton of tomato for 2000 TMT to ketchup making company and ketchup making company is obliged to buy tomatoes. Ketchup Company paid for this 10000 TMT upfront,* ***option price.*** *Four month later farmer indeed harvested 1000 ton of tomatoes and per her long term contract obligation she sold 900 tons to local ketchup making company. She had 100 ton of tomatoes in excess and she had a put option ready to be exercised. Imagine market prices of tomatoes that year had risen up to 4000 TMT per ton. For farmer it is better to sell the excess 100 ton in market rather than to ketchup making company because if she sells it to Ketchup Company she will earn 100*2000-10000 (price of option)=190000 TMT. If she sells 100 ton in market then 100*4000-10000 (this money will be gone even if she exercises her right or not!)=390000 TMT. A farmer is a smart women and she sold her tomatoes in market and earned 390000 TMT. She did not want to exercise her* ***right to sell.***

There are two types of options when it comes to exercising: **American option** which gives holder of an option a right to exercise the right any time during the specified time, and **European option** which gives the holder a right to exercise this option only in the end of contract specified time. If Jeren had

agreed to exercise her right to buy a house any time within one month period this would have been **American option.** If her contract said that she can only exercise her right in the end of month, this would have been **European option.** One more thing, a house price of 2 MLNT TMT and tomato price of 2000 TMT per ton is called a **strike price** in financial world. **Strike price is the price of asset agreed in contract.**

Options are traded in exchange markets, stock exchange markets and options markets which are specifically designed for options trading. They also have clearing houses they called Option Clearing Houses (OCC) and function the same ways as exchange clearing houses: they play a role of guarantor for both traders for certain fee. Like forwards, there are also Over-the-Counter options markets where options are traded with same privacy rules as for forwards.

Four sayings and its meaning:

a) **Being long on call option (long call)** means buying call option
b) **Being short on call option (short call)** means selling call option
c) **Being long on put option (long put)** means buying put option
d) **Being short on put option (short put)** means selling put option

Exercise 7: *Suppose 3 month option price of a European call option to buy a share of company for 1000 TMT is 10 TMT per share. There are 100 shares in bundle and call option is sold by bundles. When call option buyer will make a profit?*

*Now, let's first calculate the option price. Since call options are sold in bundles and in one bundle there are 100 shares and each share is 10 TMT then option price is 100*10=1000 TMT. Buyer will pay 1000 TMT for call option which will give him right to buy 100 shares for 1000 TMT at the end of the 3 months (do not forget it is European option). Now, let's assume that in a 3 month period share prices increased by 2%, then now market price of shares are not 1000 TMT but 1020 TMT. At this price, buyer of call option can exercise his right and purchase 100 shares for 1000 TMT and sell them for 1020 TMT each making in total of 102000-100000=2000 TMT and total profit of trader will be 2000-1000 TMT (option price)=1000 TMT. Long word short we can say this by calculations:*

$$\text{Profit} = \text{Revenue} - \text{Total cost}$$
$$\text{Profit} = 100 * P_{shares} - (100*1000 + 1000\text{TMT})$$
$$0 = 100*P - 101000\text{TMT}$$
$$P = 1010\ \text{TMT}$$

If price of share is above 1010 TMT then buyer better exercise his right and purchase bundle and resell it because he can make profit. When price is exactly 1010 TMT it is also better to exercise the right to buy because buyer will not make a profit, but also he will not make a loss. When price drops to 1009TMT per share buyer still better exercise his right and buy call option and resell shares for 1009 TMT because he will make 100900-101000=-100 TMT loss only. If he will not exercise his right, then he will lose 1000 TMT do not forget about that. ***Option price is a sunk cost and will be gone anyway no matter the right is exercised or not!*** *When share prices fall to 1008 buyer better exercise his right and resell shares because loss will be 200 TMT. It is better than 1000 TMT. Long word short, when prices of shares fall below 1000 TMT buyer better not to exercise his right and lose 1000 TMT because if he exercises his loss will be bigger. Assume price per share fell to 990 TMT then total loss of call option buyer if he exercises his right will be 99000-101000=-2000 TMT.*

We can summarize our analysis with logical conclusion that both call option buyer and put option buyer must exercise their rights until **strike price=market price** because they will incur minimum cost which is equals to **option price.** Call and put option buyer will break-even when **strike price + option price=market price.** Put option buyer will start making profits as **strike price + option price > market price.** Call option buyer will start making profits when **strike price + option price < market price.**

Exercise 8: *Suppose farmer has a long put on 1000 ton cotton deal for a strike price of 300 TMT per ton and option price of 10 TMT per ton. What is farmer's maximum loss and maximum profit?*

*Let's first calculate option price which is 10 TMT*1000=10000 TMT. Now, put options will be exercised when prices fall. Assume price fall to 280 TMT per ton farmer will exercise his right to sell and will sell 1000 ton for 300 TMT per ton and will make 1000*300=300000-280000TMT= 20000 – 10000 TMT (option price) = 10000 TMT above market profit. Otherwise, if he did not purchase put option he would have sold his cottons for 280 TMT per ton. So, as market prices fall, profit of put option buyer will increase. Put option buyer will make biggest profit as price falls to zero (which will never happen but theoretically true!). What is put options maximum loss then? His maximum loss will be equal to option price. Assume price of cotton rose to 330 TMT per ton. Farmer will not exercise his right to sell because he can sell for higher price in market. If he does not exercise then sunk cost will be 10000 TMT which is option price.*

Maximum profit = 300000 TMT (when prices fall to 0)

Maximum loss =10000 TMT (when right to sell was not exercised

Benefits of options: Options are like "**insurances against price fluctuations**" or "**insurance against losses**". Buyers of call options are afraid that prices of assets will rise and they want to be "sure" that seller will not change prices. Call option buyers are buying "insurance against price rise". Buyers of put options are afraid that prices will fall, and they want prices be high enough. Put option buyers are buying "insurance against price fall". **Option prices are like insurance premiums.** Options give security feeling for traders, manufacturers, businessmen and others regarding prices. Options serve the same function as insurance; the only difference is that insurance is used for property, health and life while options are for prices only.

Swaps

Swaps are agreements between two companies to exchange future cash flows. Swaps are only traded in Over-the-counter markets, between two companies and clearing is done by Central Counterparties (CCP) not for free of course. There are many types of swaps but we will learn here the main three of them: **Interest rate swaps**, **Currency swaps** and **Credit Default Swaps (CDS).**

Interest rate swaps

Suppose there are two companies and both of them wants to borrow money from a bank. Since credit ratings of two companies are not equal, they will get different rates. Companies with high credit rating will get funds cheaper than companies with bad credit ratings. Especially in developed economies there are two types of rates: **floating** and **fixed. Floating** interest rates are LIBOR (London Interbank Borrowing Rates) which are average rates for banks with AA ratings. I must point out that **floating** rates might be different in each economy, but we will use LIBOR as a standard floating rate in this book. **Floating** rates are non-stable rates of borrowing which will depend on national economy. As we have learned in previous chapters that as money supply increases interest rate decreases: Interest rate of borrowing rates. So, when economy is expanding, floating rates will decrease and as economy shrinks floating rates will increase. Company that borrowed funds in floating rates will have "good" times when economy grows and "bad" times when economy actually shrinks. When experts predict economic growth, companies prefer floating rates when borrowing. When experts predict recession or economic downturn companies prefer fixed rates. **Fixed** rate is familiar rate to us, as name says it all, borrowing with fixed rate. This rate will not change whatever

happens to economy. Assume one of companies borrowed in floating rate wants to swap this with fixed rates. Since banks will not change initial contract, company must find another company with fixed interest rates and swap with him the loan deals. This is called a swap. Swap of interest rates to be exact.

Exercise 9: *Suppose there are two companies, Company A and Company B want to borrow 10 MLN TMT for 7 years.*

	Fixed	**Floating**
Company A	4.1%	LIBOR+0.1%
Company B	5.2%	LIBOR +0.5%

Let's assume that Company A management believes that economy will expand that is why they want to borrow in floating rate while Company B believes that economy will shrink, that is why they prefer fixed rate. Now, these two companies go for CCP to help arrange swap deal between companies. Delegations from both companies come together and with the help of CCP come to conclusion regarding interest rates and make a deal. In all our examples we will assume that two companies will make fair deal, thus split the benefit in half. Let's get into technical detail how they split the benefit now. I have to remind you that this method was used firstly here. ***Solution of this problem can be done in three steps.***

First step: *we must first find is there any opportunity for swap at all? To answer this question we must find difference of borrowing rates of two companies in both fixed and floating rates:*

$$5.2\% - 4.1\% = 1.1\%$$

$$\text{LIBOR}+0.5\% - (\text{LIBOR} - 0.1\%) = \text{LIBOR} + 0.5\% - \text{LIBOR} + 0.1\% = 0.6\%$$

And then find difference of these rates in total, thus:

$$1.1\% \text{ (fixed rate)} - 0.6\% \text{ (floating rate)} = 0.5\%$$

The bigger the difference, bigger the benefits from interest rate swap! *0.5% is the total benefit of this swap but this must be split to three: Company A, Company B and CCP commission for clearing services until the swap deal ends. Suppose CCP agrees 0.1% commission and two companies must pay their own portion, thus each company will pay 0.05% to CCP. Now, from total benefit of 0.5-0.1=0.4% is left and it is split in between companies equally to 0.2%. Each company's benefit from this swap deal will be equal to 0.2% and if this number seems very small imagine loan amount in millions and billions. 0.2% of those amounts are not a small amount at all. We finished the first step.*

*In **second step**, we will find out how swap deal must be arranged: technical part of question. Company A wanted floating rates and Company B wanted fixed rates. Company B will have to borrow in floating rate and Company A must borrow in fixed rate and they swap. Now, let's construct equation of Company B:*

$$(Y\text{-LIBOR-}0.5\%) + (5.2\% - X) - 0.05\% = 0.2\%$$

Company B will borrow with floating rate of LIBOR+0.5% and transfer this to Company A with rate (Y) which must be below LIBOR+0.5%. First parenthesis will have negative result. Company A will borrow with 4.1% and transfer it to Company B with a little bit higher rate but lower than 5.2% rate, second parenthesis will be positive. 0.05% is commission of clearing counterparty and in total this equation must make 0.2%, total benefit of Company B. This equation makes calculation easy and I constructed it for exactly that purpose. Let's solve equation now:

$$(Y\text{-LIBOR-}0.5\%) + (5.2\% - X) - 0.05\% = 0.2\%$$
$$Y\text{-LIBOR-}0.5\% + 5.2\% - X = 0.25\%$$
$$Y\text{-LIBOR+}4.7\% - X = 0.25\%$$
$$Y\text{-}X=\text{LIBOR-}4.45\%$$

***Third step:** Now, this equation might seem as giving nothing to us, but let's look closer: (Y) is the rate that we transferred to Company A, and this rate is smaller than original loan rate of LIBOR+0.5%. (X) is the fixed rate that was transferred to Company B and it is bigger than 4.1% but smaller than 5.2%. Only thing left is just putting numbers. Assume Company B transferred floating rate of LIBOR +0.1% to Company A, thus Y=LIBOR+0.1% then put this into equation:*

$$Y\text{-}X=\text{LIBOR-}4.45\%$$
$$\text{LIBOR+}0.1\%\text{-}X=\text{LIBOR-}4.45\%$$
$$X=4.55\%$$

(X) is fixed transferred rate to Company B. Mechanism works like this. Company B borrows from bank from floating rate of LIBOR+0.5% and transfers this money to Company A with floating rate of LIBOR+0.1%. Now, Company A borrows with fixed rate of 4.1% and transfers money to Company B with fixed rate of 4.55%. Company B made a gain from fixed rate of 5.2-4.55=0.65% (he got fixed rate loan cheaper than 5.2%) and makes little loss on floating rate because (LIBOR +0.1%-LIBOR-0.5% = -0.4%). Total gain from swapping for Company B is 0.65%-0.4%=0.25% of which 0.05% will be paid to clearing counterparty. Total gain from swap is 0.2% for Company B. Let's do the same for Company A; borrowed with fixed rate of 4.1% and transfers to Company B with 4.55%. Company A makes a gain of (4.55-4.1=0.45%). Company A gets a floating rate transfer from Company B of LIBOR+0.1%

which is higher than LIBOR-0.1% and makes slight loss of (LIBOR-0.1%- (LIBOR+0.1%)=-0.2%. Total gain from swap deal is 0.45%-0.2%=0.25% and clearing counterparty commission is 0.25-0.05=0.2%. Both companies earned 0.2% of benefit from swapping deal. 0.2% of 10 MLN TMT is 20000 TMT, better than nothing I will say.

REMARK: Other combination of rates might also satisfy above equation, for example:

(Y=LIBOR and X=4.45%)
(Y=LIBOR+0.2% and X=4.65%)
(Y=LIBOR-0.1% and X= 4.35%)
(Y=LIBOR-0.2% and X= 4.25%)
(Y=LIBOR-0.3% and X= 4.15%)
Etc.

It depends on negotiations of two companies which rate they will choose all of them will earn 0.2% benefit for both parties of this swap deal.

Exercise 10: *Suppose there are two companies, Company A and Company B want to borrow 1 BLN TMT for 8 years. Company A prefers floating rate and Company B prefers fixed rate. Clearing Counterparty charges 0.1% for each side of this swap deal. Find the fair rates and five different combinations how those rates could be arranged.*

	Fixed	**Floating**
Company A	8%	LIBOR
Company B	10%	LIBOR +0.8%

First step: *Finding opportunity for swap and total benefit:*

Fixed rate differences=10-8=2%
Floating rate differences=LIBOR+0.8%-LIBOR=0.8%
Total difference=2%-0.8%=1.2%
Total commission paid to CCP=1.2%-0.2% (0.1% from each) =1%
Fair split between parties=1%/2=0.5%

Second step: *(from Company B perspective):*

(Y-LIBOR-0.8%) + (10% - X) - 0.1% = 0.5%
Y-LIBOR-0.8% + 10% - X = 0.6%
Y-LIBOR+9.2% - X = 0.6%
Y - X = LIBOR-8.6%

***Third step**: Put the numbers. I strongly recommend starting from* Y=LIBOR*, then:*

$$Y - X = LIBOR - 8.6\%$$
$$LIBOR - X = LIBOR - 8.6\%$$
$$X = 8.6\%$$

Company B will borrow from bank for floating rate of LIBOR+0.8% and transfer it to Company A by floating LIBOR rate. Company A will borrow with fixed rate of 8% and transfer funds by 8.6% to Company B.

Company B will lose 0.8% in floating rate and win 1.4% from fixed rate (10%-8.6%=1.4%). Total gain of Company B from swap is 1.4%-0.8%=0.6% and CCP commission 0.6%-0.1%=0.5%.

Company A will lose nothing from floating rate LIBOR and win from fixed rate 0.6% (8.6%-8%=0.6%). CCP commission 0.6-0.1%=0.5%. Fair deal! The rest combinations are:

(Y=LIBOR-0.1% and X=8.5%)
(Y=LIBOR+0.1% and X=8.7%)
(Y=LIBOR+0.2% and X= 8.8%)
(Y=LIBOR-0.2% and X= 8.4%)
(Y=LIBOR-0.3% and X= 8.3%)
(Y=LIBOR+0.3% and X= 8.9%)
(Y=LIBOR+0.4% and X= 9%)
(Y=LIBOR-0.4% and X= 8.2%)
(Y=LIBOR-0.5% and X= 8.1%)
(Y=LIBOR-0.6% and X= 8%)
(Y=LIBOR+0.5% and X= 9.1%)
(Y=LIBOR+0.6% and X= 9.2%)
(Y=LIBOR+0.7% and X= 9.3%)
(Y=LIBOR+0.8% and X= 9.4%)
(Y=LIBOR+0.9% and X= 9.5%)
(Y=LIBOR+1% and X= 9.6%)
(Y=LIBOR+1.1% and X= 9.7%)
(Y=LIBOR+1.2% and X= 9.8%)
(Y=LIBOR+0.35% and X= 8.95%)
Etc.

Currency swaps

There are hundreds of national currencies in the world and there are millions of companies doing business internationally. Monetary policies of nations are designed in such way that each

nation promotes its own national currency. In China, borrowing in Chinese Yuan is cheaper than borrowing in USD. China wants domestic businesses to borrow in Yuan that is why they try to keep Yuan borrowing rates cheaper than other currencies. This is a monetary strategy and companies have to comply with that. Assume Chinese company needs USD and a domestic rate of borrowing in USD is 8% annual and borrowing in Yuan cost only 4% annual. Now, assume US based company also doing international business. For US company borrowing in USD is 5% annual and borrowing in Yuan is 9% annual. Chinese company needs USD and US based company needs Yuan. Can they make a deal? Yes, these companies can borrow the currencies in which they have domestic advantage and transfer to other party while receiving foreign currency for much cheaper than domestic rates. This is called **currency swap.**

Exercise 11: *Let's take above example and solve it (suppose CCP commission to be 0.1% from each party):*

	Borrowing in Yuan	**Borrowing in USD**
Chinese company	4%	8%
American company	9%	5%

First step *finding swap opportunity: I recommend solving it this way:*

$$(Y - 5\%) + (9\% - X) - 0.1\% = a$$
$$(8\% - Y) + (X - 4\%) - 0.1\% = a$$

Add these two equations:

$$(Y - 5\%) + (9\% - X) - 0.1\% + (8\% - Y) + (X - 4\%) - 0.1\% = 2a$$
$$Y - 5\% + 9\% - X - 0.1\% + 8\% - Y + X - 4\% - 0.1\% = 2a$$
$$7.8\% = 2a$$
$$3.9\% = a$$

Each company can earn 3.9% benefit if they agree on a swap.

Second step*: Now, take any of equation above and solve it:*

$$(Y - 5\%) + (9\% - X) - 0.1\% = a$$
$$(Y - 5\%) + (9\% - X) - 0.1\% = 3.9\%$$
$$Y - 5\% + 9\% - X = 4\%$$
$$Y = X$$

In order this swap deal to be fair, transfer rate of USD (Y) must be equal to transfer rate of Yuan.

Third step: *Let's try minimum USD (Y) transferable rate of 5%, Y=5%. American company borrows in USD for 5% and transfers funds to Chinese company for 5%. Chinese company borrows in Yuan for 4% and transfers funds to American company for 5% (Y=X).*

Swap analysis: American company borrowed USD for 5% and transferred it to Chinese company; in return Chinese company borrowed in Yuan and transferred to US company for 5% rate. American company made 9%-5%=4% benefit because otherwise it would have been borrowed Yuan by 9%. CCP commission is 4%-0.1% = 3.9% total benefit of American company from swap deal.

Chinese company borrowed Yuan for 4% and transferred it to American company for 5% and made 1% benefit there. It also received USD for 5% and made 8-5=3% benefit in there. Otherwise Chinese company would have borrowed USD for 8%. Total profit (benefit) from swap deal of Chinese company is 3+1-0.1 (CCP commission) =3.9%.

NOTICE: In the end of swap deal all borrowed currency is returned to original lender: Chinese Yuan will be returned to Chinese company and company will close bank loan. USD will be returned to American company and American company will close its bank loan. Chinese company will pay its 5% USD borrowing rate to American company and American company will directly pay it to close its interest on loan. American company will pay 5% for Yuan to Chinese company and Chinese company will pay 4% its interest on borrowing Yuan leaving 1% to itself. All in all both companies will borrow 3.9% cheaper than otherwise they would have to thanks to swap. That is the reason I used the term benefit, rather than profit.

Exercise 12: *Swiss company and Russian company need loan around same amount. Swiss company needs Rubles and Russian company needs Franks. Below is borrowing rates (suppose CCP commission to be 0.1% from each party):*

	Borrowing in Frank	**Borrowing in Ruble**
Swiss company	5.2%	8.1%
Russian company	8.8%	6%

First step *finding swap opportunity:*

$$(Y - 6\%) + (8.8\% - X) - 0.1\% = a$$
$$(8.1\% - Y) + (X - 5.2\%) - 0.1\% = a$$

Add these two equations:

$$(Y - 6\%) + (8.8\% - X) – 0.1\% + (8.1\% - Y)+(X – 5.2\%)–0.1\% =2a$$
$$Y - 6\% + 8.8\% - X – 0.1\% + 8.1\% - Y+X – 5.2\%–0.1\% =2a$$
$$5.5\% = 2a$$
$$2.75\% = a$$

Each company can earn 2.75% benefit if they agree to swap.

***Second step**: Now, take any of equation above and solve it:*

$$(Y - 6\%) + (8.8\% - X) – 0.1\% = a$$
$$(Y - 6\%) + (8.8\% - X) – 0.1\% = 2.75\%$$
$$Y - 6\% + 8.8\% - X= 2.85\%$$
$$Y = X-0.05\%$$

In order this swap deal to be fair, transfer rate of Ruble (Y) must be 0.05% lower than transfer rate of Frank (X).

***Third step:** Let's try minimum Ruble (Y) transferable rate of 6%, Y=6%. Russian company borrows in Ruble for 6% and transfers funds to Swiss company for 6%. Swiss company borrows in Franks for 5.2% and transfers funds to Russian company for 5.25% (Y=X-0.05%). Other possible combinations are:*

(Y=6.1% and X=6.15%)
(Y=6.5% and X=6.55%)
(Y=7% and X=7.05%)
(Y=7.25% and X=7.3%)
Etc.

Credit Default Swaps (CDS)

Credit Default Swaps became extremely popular from beginning of 21st century. This is how it works: Company A let's say have funds for investment but Board of Directors of that company only allows investment to companies with credit scores of higher than or equal to AA. The problem is there is no much companies with AA credit ratings and they pay low returns because they are considered not risky at all. As we all know, lower risk means low returns. If Company A wants to earn more he must invest into riskier assets but this is prohibited by company's policy. There is only one way of getting around that company policy: Company A needs a guarantor! Let's say if one of highly rated banks BankAAA for example or any other company could give guarantees that for example Company BBB will not default and if defaults then, BankAAA will pay off the debt, then Company A will happily invest. Assume Company BBB with credit rating of BBB (not bad) seeking funds and ready to pay 7% annual dividend

on investment. Company A wants to invest but cannot because of company policy, but here comes BankAAA who knows Company BBB and ready to give guarantees if Company A will pay 1% of earned returns to Bank AAA. Company A agrees and three of them sign a contract. Company BBB pays 7% return to Company A and Company A sends 1% to Bank AAA leaving itself rest 6%. This is called Credit Default Swap.

Securities

In the second half of the 20th century with attaining relative peace, most economies witnessed economic boom. Demand was increasing faster than supply, especially in money market. Banks always finance their loans with deposits. In "booming" times banks simply did not have enough deposits to fund their loans. This created money scarcity increasing interest rates. Banks based in US found a good solution for this problem but they needed a guarantor. Their solution was this: They create portfolios from all loans, especially mortgage loans which are backed by property. (This means that collateral for this loan is a house) and according to interest they bring (cash flow) they sell them. The only thing is that, since these portfolios are risky, banks needed a "neutral judge" who would rate them and give guarantees that this is a "secure" financial instrument. This is called a **securitization** and portfolios with cash flows are called a **security.** US government agreed with that bankers plan because at those times it coincided with national housing project. Still in US majority bulk of population does not own their own house. US saw this offer from bankers as an opportunity and created Government National Mortgage Association (shortly known as **Ginnie Mae**). Ginnie Mae was the government authority "securitizing" loan portfolios. The process kick started. Result was amazing at first: banks were creating portfolios from old loans (package) and sending them to Ginnie Mae for approval. Ginnie Mae approved them and banks could legally sell securities. Banks earned more income and thus they could give more loans for housing. Win-win situation (kind of). Many types of securities were created: MBS (Mortgage backed security), ABS (Asset backed security, the ones that are backed by cars and other assets), etc. Unfortunately later on Ginnie Mae trusted banks too much and became "lazy". Securitization opened giant opportunities in front of banks, now they were giving loans to everybody even if you are not creditworthy, because after securitization, this was considered "safe" investment and banks easily sold it to other banks, transferring default risk to other banks. These securities were traded globally. These securitized "bad loans" which are called Subprime Mortgage (word subprime means "not the best", it was referred to not-creditworthy client of banks) eventually led to global financial crisis of 2008.

Homework:

I. You are a farmer and expecting 5000 tons of potato harvest within 6 month period and current prices of 5 TMT per KG fits you well but you are afraid prices might change. What can you do about it?

II. If there were no maintenance margin mechanism in futures exchanges, what kind of risk would have been prevailing?

III. I need 2 MLN tons of chemicals in 7 month and current chemical prices are good for me and I am afraid that prices will rise. What can I do about it?

IV. Gold futures contract (Pi = 18000 TMT per ounce and 1000 ounces in total per contract) is traded in exchange with 1% of initial margin and 0.6% maintenance margin. How much price per ounce of gold must rise so that one of the traders must have to raise funds to close maintenance margin? Which trader must have to do that, buyer or seller?

V. In above question, how much prices must fall so that one of traders must have to raise funds to close maintenance margin? Which trader must have to do that?

VI. Initial price of futures contract is 4500 TMT per ton and final prices is 4621 TMT per ton. If initial contract size was 9 MLN TMT, what is final price of contract and who won the trade and by how much?

VII. Jahan has purchased put option for 100 ton of vegetable oil at a strike price of 3000 TMT per ton maturing in the end of the year. Price of put option is 100 TMT per ton. What is Jahan profit or loss when price of vegetable oil per ton falls to 2870 TMT per ton?

VIII. What is Jahan's profit or loss if price of vegetable oil rises to 3180 TMT per ton?

IX. Jemal bought call option on 1000 ton oranges at strike price of 5000 TMT per ton maturing in 6 month. Option price is 50 TMT per ton. What must be the price of oranges during 6 month so that Jemal makes 0 profits in the end?

X. You were planning buying house in certain location of a city and you heard that new development project is starting nearby which might increase prices of property in that location. You already talked with property owner but you are afraid he will not keep his word. You know that you will find needed money

in three months and you do not have them right now. What can you do about it?

XI. You are CCP and charge 0.1% for each company for negotiating swap deal. Below are two companies that came to you, negotiate swap deal that will benefit all of you:

	Fixed	**Floating**
Company A	4.5%	LIBOR
Company B	6.7%	LIBOR +1%

XII. You are CCP and charge 0.1% for each company for negotiating swap deal. Below are two companies came to you and Company A wants a swap deal with floating rate of LIBOR only. Can you arrange it?

	Fixed	**Floating**
Company A	7%	LIBOR+0.5
Company B	9.6%	LIBOR +1.7%

XIII. You are CCP and charge 0.2% for each company for negotiating swap deal. Below are two companies come to you and Company B wants a swap deal with fixed rate of 7% only. Can you arrange it?

	Fixed	**Floating**
Company A	6.5%	LIBOR
Company B	9.8%	LIBOR +1.1%

XIV. You are CCP and charge 0.25% for each company for negotiating swap deal. Below are two companies come to you for negotiating a swap deal? Can you arrange it?

	Fixed	**Floating**
Company A	5.5%	LIBOR+1
Company B	7.5%	LIBOR +2%

XV. You are CCP and charge 0.2% for each company for negotiating swap deal. Below are two companies come to you for negotiating a swap deal? Can you arrange it?

	Borrowing in TENGE	**Borrowing in TMT**
Kazakh company	5.5%	8%
Turkmen company	8.7%	3%

XVI. You are CCP and charge 0.2% for each company for negotiating swap deal. Below are two companies come to you for negotiating a swap deal? Can you arrange it?

	Borrowing in PESO	**Borrowing in DIRHAM**
Mexican company	4%	9.7%
Saudi company	10%	5%

XVII. You are CCP and charge 0.3% for each company for negotiating swap deal. Below are two companies come to you for negotiating a swap deal? Can you arrange it?

	Borrowing in YEN	**Borrowing in LIRA**
Turkish company	8.8%	6%
Japanese company	2%	7%

XVIII. You are CCP and charge 0.25% for each company for negotiating swap deal. Below are two companies come to you for negotiating a swap deal? Turkish company wants to borrow YEN by 4.4%, can you arrange it?

	Borrowing in YEN	**Borrowing in LIRA**
Turkish company	8.8%	6%
Japanese company	2%	7%

XIX. You are CCP and charge 0.2% for each company for negotiating swap deal. Below are two companies come to you for negotiating a swap deal? Can you arrange it?

	Borrowing in BAHT	**Borrowing in EURO**
Spanish company	9.1%	4.7%
Thai company	3.3%	8%

XX. Pension fund “Youngmoney” has 20 BLN TMT for investment and investment bank “Moneygone” needs capital and ready to pay return up to 7% yearly. “Youngmoney” wants to invest into “Moneygone” but it has a policy to invest only entities with Moody’s credit rating of BBB+ and above, “Moneygone” has credit rating of BBB only. Can you recommend a solution for this problem?

XXI. You have a stock of shares of auto manufacturing company (assume 1000 shares) which is rumored having a management issues which will definitely impact share prices. Your boss wants to keep those shares regardless of any price changes.

Recommend your boss some solution how to protect company from price falls of shares. (Buying a put option)

XXII. You bought a call option for buying 1000 ton of potatoes for 3000 TMT per ton for 50000 TMT. What is your maximum loss? What is your maximum profit?

XXIII. You bought a put option for selling 1000000 LITRES of milk for 8 TMT per LITER for 100000 TMT. What is your maximum loss? What is your maximum profit?

XXIV. You are CCP and charge 0.2% for each company for negotiating swap deal. Below are two companies come to you for negotiating a swap deal? You negotiated a deal where both companies maximum can benefit from this swap deal by 4% each. Find Y

	Borrowing in BAHT	**Borrowing in EURO**
Spanish company	10%	4%
Thai company	5%	Y

XXV. You are CCP and charge 0.1% for each company for negotiating swap deal. Below are two companies come to you for negotiating a swap deal? You negotiated a deal where both companies maximum can benefit from this swap deal by 1.8% each. Find Y and X

	Borrowing in BAHT	**Borrowing in EURO**
Spanish company	X	3%
Thai company	3%	Y

XXVI. You are CCP and charge 0.2% for each company for negotiating swap deal. You have calculated that maximum benefit each company can have from this swap deal is 1% each. Find Y

	Fixed	**Floating**
Company A	6%	LIBOR
Company B	9%	Y

XXVII. You are CCP and charge 0.25% for each company for negotiating swap deal. You have calculated that maximum benefit each company can have from this swap deal is 0.9% each. Find X

	Fixed	Floating
Company A	X	LIBOR-0.1%
Company B	9%	LIBOR+0.3%

XXVIII. I had a European call option with maturity of 6 month for buying 1000 ton of cement for 5000 TMT per ton which I paid 100000 TMT for. At the end maturity date I made pure profit of 400000 TMT. What was the price at that time and calculate my total cost.

XXIX. I had a European put option with maturity of 6 month for buying 1000 ton of sugar for 10000 TMT per ton which I paid 300000 TMT for. At the end maturity date I made pure profit of 1000000 TMT. What was the price at that time and calculate my total cost.

XXX. I have long call on chemicals which I paid 50000 TMT. Chemical prices plummeted deeply, what is my loss from futures contract?

XXXI. I have short put on 1 MLN ton of gasoline per 2000 TMT per ton which I sold for 20 MLN TMT. What will be my total loss if prices drop by 10%?

XXXII. I have long put on 50000 ton of corn for 4000 TMT per ton which I paid for 10 MLN TMT. Prices rose to 4090 TMT per ton. What is my total profit and show me total cost.

XXXIII. I have short call on 90000 ton of rice per 10000 TMT per ton which I sold for 40 MLN TMT. Prices of rice plummeted to 9000 TMT per ton. What is my total profit?

XXXIV. Securitization is bad or good?

Solutions:

I. If I am afraid of price fall, to insure myself against it I will buy a put option or the seller can sign a future contract. In case if market price within 6 months will be below the strike price, I will sell my tomatoes for 5 TMT /kg. If market price is above strike price, then I will sell my tomatoes in the market.

II. The aim of maintenance margin is to show that buyer (or seller) is able to pay for the trade deal. If there are were no a maintenance margin in future exchange markets, there would have been a default risk.

III. If I am afraid of price jump, to insure myself against it I will buy a call option. In case if market price rises in 7 months, I could buy chemicals at a strike price.

IV. P_i = 18000 TMT per ounce
1% initial margin
0.6% maintenance margin
Total amount of contract =18000*1000=18000000 TMT
18000000*0.01=180000 TMT initial margin
18000000*0.0006=108000 TMT maintenance margin
If price rises, loss will be from sellers' side, because they will sell at lower than market price.
$180000-1000*(P_f-P_i) < 108000$
$180000-108000<1000*(P_f-180000)$
$72000<1000*(P_f-180000)$
$72<P_f-180000$
$180072<P_f$
If price will be higher than 180072 TMT per ounce, seller must raise funds in balance up to initial margin amount.

V. If price falls, loss will be from buyers' side, because they will buy at higher than market price.
$180000-1000*(P_f-P_i) < 108000$
$180000-108000<1000*(180000-P_f)$
$72000<1000*(180000-P_f)$
$72<180000-P_f$
$P_f<179928$ TMT
If market price will fall below 179928 TMT per ounce, buyer must raise funds up to initial margin.

VI. P_i=4500 TMT per ton

P_f=4621 TMT per ton

Initial contract size= 9 MLN TMT

$\frac{9000000}{4500}$=2000 tons total

2000*4621=9242000 TMT is final price of the contract.

Buyer won by 9242000-9000000=242000 TMT.

VII. 100*100=10000 TMT is Jahan's option price

100*3000=300000 TMT (she can sell worth of 300000 TMT vegetable oil)

Even if prices fall, Jahan can sell them at strike price, so her profit will be:

100*3000=300000 TMT

300000-10000(for put option) = 290000 TMT

VIII. If prices rise, Jahan can sell her vegetable oil in the market and not exercise the options she bought. Then her profit will be:

100*3180 =318000 TMT

318000-10000 (option price will be a sunk cost) = 308000 TMT is her profit.

IX. 1000*50=50000 TMT is her call option price

P_i=5000

Profit = Revenue - Total cost

Profit = $(P_f - P_i)*Q_{\text{underlying asset}}$ - (call option price)

0 = $(P_f - 5000)*1000 - 50000$

50000= $(P_f - 5000)*1000$

50= $P_f - 5000$

P_f=5050 TMT per ton

X. I should buy a call option from a seller of the house, so that he will keep his house for three months. Until I find money. Thus by selling me a call option, he will promise me and keep his word on not selling his house to anybody else.

XI.

	Fixed	**Floating**
Company A	4.5%	LIBOR
Company B	6.7%	LIBOR +1%

First step:

Difference of fixed rates: 6.7%-4.5%=2.2%

Difference of floating rates:
LIBOR+ 1% - (LIBOR) =LIBOR+1%-LIBOR=1%
Total difference: 2.2%-1%=1.2%
Total commission paid to CCP: 1.2%-0.1%*2=1%
$\frac{1\%}{2}$=0.5% for each company.
Second step: (From company B prospective)
(Y-LIBOR-1%) + (6.7%-X)-0.1%=0.5%
Y-LIBOR-1%+6.7%-X=0.6%
Y-X=LIBOR-5.1%
Third step: (Y=LIBOR)
LIBOR-X=LIBOR-5.1%
X=5.1%
Company B will borrow from the bank with LIBOR+1% and transfer to company A with LIBOR, while company A will borrow from the bank with fixed rate 4.5% and transfer to company B with 5.1%.

XII.

	Fixed	**Floating**
Company A	7%	LIBOR+0.5%
Company B	9.6%	LIBOR +1.7%

First step:
Difference of fixed rates: 9.6%-7%=2.6%
Difference of floating rates:
LIBOR+1.7% - (LIBOR+0.5%) =LIBOR+1.7%-LIBOR-0.5%=1.2%
Total difference: 2.6%-1.2%=1.4%
Total commission paid to CCP: 1.4%-0.1%*2=1.2%
$\frac{1.2\%}{2}$=0.6% for each company.
Second step: (From company B prospective)
(Y-LIBOR-1.7%) + (9.6%-X)-0.1%=0.6%
Y-LIBOR-1.7%+9.6%-X=0.7%
Y-X=LIBOR-7.2%
Third step: (Y=LIBOR+0.5%)
LIBOR+0.5%-X=LIBOR-7.2%
X=7.7%
Company A will borrow from the bank with fixed rate 7% and transfer to company B with 7.7%. Company B will borrow from the bank with LIBOR +1.7% and transfer to Company A with LIBOR+0.5%.

But company A wants to borrow with LIBOR rate let's check the combinations:
Y=LIBOR; X=7.2% (yes, we can arrange it)
In this case company B will take from the bank with LIBOR+1.7% and give to company B with LIBOR, but borrow the loan from company A with fixed rate of 7.2%.

XIII.

	Fixed	**Floating**
Company A	6.5%	LIBOR
Company B	9.8%	LIBOR +1.1%

First step:
Difference of fixed rates: 9.8%-6.5%=3.3%
Difference of floating rates:
LIBOR+1.1% - (LIBOR) =LIBOR+1.1%-LIBOR=1.1%
Total difference: 3.3%-1.1%=2.2%
Total commission paid to CCP: 2.2%-0.2%*2=1.8%
$\frac{1.8\%}{2}$=0.9% for each company.
Second step: *(From company B prospective)*
(Y-LIBOR-1.1%) + (9.8%-X)-0.2%=0.9%
Y-LIBOR-1.1%+9.8%-X=1.1%
Y-X=LIBOR-7.6%
Third step: (Y=LIBOR)
LIBOR-X=LIBOR-7.6 %
X=7.6%
Company A will borrow from bank with 6.5% and transfer to company B with 7.6%. Company B will borrow from bank with LIBOR +1.1% and give to Company A with LIBOR.

But company B wants to borrow with fixed rate of 7%, let's check the combinations:
When Y=LIBOR-0.6%, X=7%. (Yes, we can arrange it)
In this case company B will borrow from the bank with LIBOR+1.1% and give to company B with LIBOR-0.6%, but take the loan from company A with fixed rate of 7%.

XIV.

	Fixed	**Floating**
Company A	5.5%	LIBOR+1
Company B	7.5%	LIBOR +2%

First step:
Difference of fixed rates: 7.5%-5.5%=2%
Difference of floating rates:
LIBOR+2% - (LIBOR+1%) =LIBOR+2%-LIBOR-1%=1%
Total difference: 2%-1%=1%
Total commission paid to CCP: 1%-0.25%*2=0.5%
$\frac{0.5\%}{2}$=0.25% for each company.
Second step: *(From company B prospective)*
(Y-LIBOR-2%) + (7.5%-X)-0.25%=0.25%
Y-LIBOR-2%+7.5%-X=0.5%
Y-X=LIBOR-5%
Third step: (Y=LIBOR+1%)
LIBOR+1%-X=LIBOR-5%
X=6%
Company A will take from bank with fixed rate 5.5% and transfer to company B with 6%, and company B will take from bank with floating rate LIBOR+2% and transfer to company A with LIBOR+1%.

XV.

	Borrowing in TENGE	**Borrowing in TMT**
Kazakh company	5.5%	8%
Turkmen company	8.7%	3%

First step finding swap opportunity
(Y – 3%) + (8.7% - X) – 0.2% = a
(8% - Y) + (X – 5.5%) – 0.2% = a
(Y – 3%) + (8.7% - X) – 0.2% + (8% - Y) + (X – 5.5%) – 0.2%=2a
Y – 3% + 8.7% - X – 0.2% + 8% - Y + X – 5.5% – 0.2%=2a
7.8% = 2a
3.9% = a
Each company can earn 3.9% benefit if they agree on a swap.
Second step: Now, take any of equation above and solve it:
(Y - 3%) + (8.7% - X) – 0.2% = a
(Y - 3%) + (8.7% - X) – 0.2% = 3.9%
Y - 3% + 8.7% - X = 4.1%
Y = X-1.6%
In order this swap deal to be fair, transfer rate of TMT (Y) must be 1.6% lower than transfer rate of TENGE (X).
Third step: Let's try minimum TMT (Y) transferable rate of 3%, Y=3%. Turkmen company borrows in TMT for 3% and transfers for 3%, Kazakh Company borrows in TENGE from its

bank for 5.5% and transfers for 4.6% to Turkmen company, when Y=3%, X=4.6% (Y = X-1.6%).

XVI.

	Borrowing in PESO	**Borrowing in DIRHAM**
Mexican company	4%	9.7%
Saudi company	10%	5%

First step finding swap opportunity
(Y – 5%) + (10% - X) – 0.2% = a
(9.7% - Y) + (X – 4%) – 0.2% = a
(Y – 5%) + (10% - X) – 0.2% + (9.7% - Y) + (X – 4%) – 0.2%=2a
Y – 5% + 10% - X – 0.2% +9.7% - Y + X – 4% – 0.2%=2a
10.3% = 2a
5.15% = a
Each company can earn 5.15% benefit if they agree on a swap.
Second step: Now, take any of equation above and solve it:
(Y - 5%) + (10% - X) – 0.2% = a
(Y - 5%) + (10% - X) – 0.2% = 5.15%
Y - 5% + 10% - X = 5.35%
Y = X+0.35%
In order this swap deal to be fair, transfer rate of DIRHAM (Y) must be 0.35% higher than transfer rate of PESO (X).
Third step: Let's try minimum DIRHAM (Y) transferable rate of 5%, Y=5%. Saudi company borrows in DIRHAM for 5% and transfers funds to Mexican Company for 5%. Mexican company borrows in PESO for 4% and transfers funds to Saudi company for 4.65%. Y=5%, 4.65%=X. (Y=X+0.35%).

XVII.

	Borrowing in YEN	**Borrowing in LIRA**
Turkish company	8.8%	6%
Japanese company	2%	7%

First step finding swap opportunity
(7% - Y) + (X – 2%) – 0.3% = a
(Y-6%) + (8.8%-X)-0.3%=a
(Y – 6%) + (8.8% - X) – 0.3% + (7% - Y) + (X – 2%) – 0.3%=2a
Y – 6% + 8.8% - X – 0.3% +7% - Y + X – 2% – 0.3%=2a
7.2% = 2a
3.6% = a
Each company can earn 3.6% benefit if they agree on a swap.

Second step: Now, take any of equation above and solve it:
(Y-6%) + (8.8%-X)-0.3%=a
(Y - 6%) + (8.8% - X) – 0.3% = 3.6%
Y - 6% + 8.8% - X = 3.9%
Y = X-1.1%
In order this swap deal to be fair, transfer rate of LIRA (Y) must be 1.1% lower than transfer rate of YEN (X).
Third step: Let's try minimum LIRA (Y) transferable rate of 6%, Y=6%. Turkish company borrows in LIRA with 6% and transfers funds to Japanese Company with 6%. Japanese company borrows in YEN with 2% and transfers funds to Turkish company for 4.9%. X=6%, Y=4.9 % (Y=X-1.1%).

XVIII.

	Borrowing in YEN	Borrowing in LIRA
Turkish company	8.8%	6%
Japanese company	2%	7%

First step finding swap opportunity
(7% - Y) + (X – 2%) – 0.25% = a
(Y – 6%) + (8.8% - X) – 0.25% = a
(Y – 6%) + (8.8% - X) – 0.25% + (7% - Y) + (X – 2%) – 0.25%=2a
Y – 6% + 8.8% - X – 0.25% +7% - Y + X – 2% – 0.25%=2a
7.3% = 2a
3.65% = a
Each company can earn 3.65% benefit if they agree on a swap.
Second step: Now, take any of equation above and solve it:
(Y - 6%) + (8.8% - X) – 0.25% = a
(Y - 6%) + (8.8% - X) – 0.25% = 3.65%
Y - 6% + 8.8% - X = 3.9%
Y = X+1.1%
In order this swap deal to be fair, transfer rate of LIRA (Y) must be 1.1% lower than transfer rate of YEN (X).
Third step: Let's try minimum LIRA (Y) transferable rate of 6%, Y=6%. Turkish company borrows in LIRA for 6% and transfers funds to Japanese Company for 6%. Japanese company borrows in YEN for 2% and transfers funds to Turkish company for 4.9%. When Y=6%, X=4.9%. (Y=X-1.1%).
But Turkish company wants to borrow YEN for 4%. We combine the rates:

When X=4.4%, Y=5.1%. In this case Turkish Company borrows YEN with 4.4% from Japanese company and gives LIRA with 5.5% to Japanese company.

XIX.

	Borrowing in BAHT	**Borrowing in EURO**
Spanish company	9.1%	4.7%
Thai company	3.3%	8%

First step finding swap opportunity
(Y – 4.7%) + (9.1% - X) – 0.2% = a
(8% - Y) + (X – 3.3%) – 0.2% = a
(Y – 4.7%) + (9.1% - X) – 0.2% + (8% - Y) + (X – 3.3%) – 0.2%=2a
Y – 4.7% + 9.1% - X – 0.2% +8% - Y + X – 3.3% – 0.2%=2a
8.7% = 2a
4.35% = a
Each company can earn 4.35% benefit if they agree on a swap.
Second step: Now, take any of equation above and solve it;
(Y – 4.7%) + (9.1% - X) – 0.2% = a
(Y -4.7%) + (9.1% - X) – 0.2% = 4.35%
Y – 4.7% + 9.1% - X = 4.55%
Y = X+0.15%
In order this swap deal to be fair, transfer rate of EURO (Y) must be 0.15% higher than transfer rate of BAHT (X).
Third step: L t's try m n mum URO (Y) transferable rate of 4.7%, Y=4.7%. Spanish company borrows in EURO for 4.7% and transfers funds to Thai Company for 4.7%. Thai company borrows in BAHT for 3.3% and transfers funds to Spanish company for 4.55%. When Y=4.7%, X=4.55% (Y=X-0.15%).

XX. CDS (Credit Default Swap) helps us to solve this kind of problems. Pension fund "Youngmoney" will find a company with Moody's credit rating of BBB+ or above, and arrange a deal with that company to be a guarantor between "Youngmoney" and "Moneygone" in return for fee, say, 1% from return rate. In the end, "Moneygone" from total return rate 7%, pays 1% to the guarantor company (BBB+), and the rest 6% to the "Youngmoney" pension fund.

XXI. To insure the company from price falls of shares, my Boss should buy a put option (right to sell) for 1000 shares. If after a certain time period price of shares fall below strike price, my boss can sell those shares at a strike price. And if price of

shares will be above strike price, he can sell his shares at a market price.

XXII. If prices fall below 3000 TMT per ton, my maximum loss will be 50000 TMT (price of call option will be a sunk cost). If price will be above 3000 TMT per ton and cover price of call option I will start making profit:

$0<1000*(P_f-3000)$ -50000

$50000<1000*(P_f-3000)$

$50<P_f-3000$

$P_f>3050$ TMT per ton

If final price in the market will be above 3050 TMT per ton I will start making profit.

XXIII. If price in the market will be above 8 TMT per liter, price of put option 100000 TMT will be my sunk cost, my loss.
If final market price will be below 8 TMT per liter, and also covers the price of put option, then I will start making profit:

0<1000000*(8-Pf) -100000

100000<1000000*(8-Pf)

0.1 <8-Pf

Pf<7.9 TMT per liter

If market price falls below 7.9 TMT per liter, I will start making profit.

XXIV.

	Borrowing in BAHT	**Borrowing in EURO**
Spanish company	10%	4%
Thai company	5%	Y

First step finding swap opportunity
a=4%
Each company can earn 4% benefit.
Second step: Now, take any of equation above and solve it;
(10% - X)+ (Y– 4%) – 0.2% = 4%
(X-5%) + (Y1% – Y) – 0.2% = 4%
10%-X+ Y-4%-0.2%+X-5%+Y1-Y-0.2%=8%
Y = 0.6%
If Y1=7.4%

Now, take any of equation above and solve it:

(X-5%) + (7.4% – Y) – 0.2% = 4%
X-5%+7.4%-Y=4%+0.2%
Y=X-1.8%

In order this swap deal to be fair, transfer rate of EURO (Y) must be 1.8%% higher than transfer rate of BAHT (Y).

Third step: Let's try minimum URO (Y) transferable rate of 4%, Y=4%. Spanish company borrows in EURO for 4% and transfers funds to Thai Company for 4%. Thai company borrows in BAHT for 5% and transfers funds to Spanish company for 5.8%. When Y=4%, X=5.8% (Y=X-1.8%).

XXV.

	Borrowing in BAHT	**Borrowing in EURO**
Spanish company	X	3%
Thai company	3%	Y

First step finding swap opportunity
$(Y-3\%)+(X_1-X)-0.1\%=1.8\%$
$(Y_1-Y)+(X-3\%)-0.1\%=1.8\%$
$Y-3\%+X_1-X-0.1+Y_1-Y+X-3\%-0.1\%=3.6\%$
$X_1+Y_1-6.2\%=3.6\%$
$X_1+Y_1=9.8\%$
If both companies have same rate (Y=X), then X_1+Y_1:
$X_1=4.9\%$
$Y_1=4.9\%$
Second step: Now, take any of equation above and solve it:
(Y – 3%) + (4.9% - X) – 0.1% = 1.8%
Y – 3% + 4.9% - X = 1.9%
Y = X
In order this swap deal to be fair, transfer rate of EURO (Y) must be equal to the transfer rate of BAHT (X).
Third step: Let's try minimum EURO (Y) transferable rate of 3%, Y=3%. Spanish company borrows in EURO for 3% and transfers funds to Thai Company for 3%. Thai company borrows in BAHT for 3% and transfers funds to Spanish company for 3%.

XXVI.

	Fixed	**Floating**
Company A	6%	LIBOR
Company B	9%	Y

First step:
a=1% for each company.
$(Y-Y_1)+(9\%-X)-0.2\%=1\%$
$(X-6\%)+(LIBOR-Y)-0.2\%=1\%$
$Y-Y_1+9\%-X-0.2\%+X-6\%+LIBOR-Y-0.2\%=2\%$
$LIBOR-Y_1+2.6\%=2\%$
$Y_1=LIBOR+0.6\%$
Second step: (From company B prospective)
$(Y-LIBOR-0.6\%)+(9\%-X)-0.2\%=1\%$
$Y-LIBOR-0.6\%+9\%-X=1.2\%$
$Y-X=LIBOR-7.2\%$
$(Y=LIBOR)$
$X=7.2\%$
Third step:
Company A will take from bank with fixed rate 6% and transfer to company B with 7.2%, and company B will take from bank with floating rate LIBOR+6% and transfer to company A with LIBOR.

XXVII.

	Fixed	Floating
Company A	X	LIBOR-0.1%
Company B	9%	LIBOR+0.3%

First step:
a=0.9% for each company.
$(Y-LIBOR-0.3\%) + (9\%-X)-0.25\%=0.9\%$
$(X-X_1) + (LIBOR-0.1\%-Y)-0.25\%=0.9\%$
$Y-LIBOR-0.3\%+9\%-X-0.25\%+X-X_1+LIBOR-0.1\%-Y-0.25\%=1.8\%$
$8.1-X_1=1.8\%$
$X1=6.3\%$
Second step: (From company B prospective)
$(Y-LIBOR-0.3\%) + (9\%-X)-0.25\%=0.9\%$
$Y-LIBOR-0.3\%+9\%-X=1.15\%$
$Y-X=LIBOR-7.55\%$
$(Y=LIBOR-0.1\%)$
$LIBOR-0.1\%-X=LIBOR-7.55\%$
$X=7.45\%$
Third step:
Company A will take from bank with fixed rate 6.3% and transfer to company B with 7.45%, and company B will take from the bank with floating rate LIBOR+3% and transfer to company A with LIBOR-0.1%.

XXVIII. Profit = $(P_f - P_i)*Q_{underlying\ asset}$ -(call option price)
400000= $(P_f$-5000)*1000-100000
500000= $(P_f$-5000)*1000
500= P_f-5000
P_f=5500 TMT per ton of cement.
My total cost is call option price.

XXIX. Profit = $(P_f - P_i)*Q_{underlying\ asset}$ -(call option price)
1000000= (10000-P_f)*1000-300000
1300= (10000- P_f)
1300=10000- P_f
P_f=8700 TMT per ton of sugar.
My total cost is call option price.

XXX. If price of chemicals plummeted, my loss will be the difference between final price and strike price, and price of the call option.
Revenue-Cost=Profit
Profit = $(P_f - P_i)*Q_{underlying\ asset}$-50000TMT

XXXI. If prices drop by 10%: 1000000(1800-2000) = -200000000 TMT
20000000(option price)-200000000=-180000000 is my total loss.

XXXII. Profit= 4090*50000-10000000(sunk cost of put option)=194500000 TMT
My cost is cost of the put option: 10 MLN TMT.

XXXIII. Profit = 40 MLN (call option price)

XXXIV. As well as all financial instruments like bonds, stocks, swaps and others, securitized loans are also one of the wonderful financial instruments. As any other financial instrument, securitization helps to raise funds in banking sector. If using securities properly, securitization can be an appropriate tool for transferring risk efficiently, while misusing them can lead to bad consequences like global financial crisis in 2008.

Chapter 11: Accounting and Audit

Finances love order. Accounting and audit are fundamental for order! Professional must know how much is "in" and how much is "out". No order, no growth! Without efficient accounting and audit system economy will never grow. That is why all economies have national accounting and audit standards. Thousands of years of experience helped humanity develop the most efficient accounting standards which are pretty much the same in all economies except some minor differences. Accounting principles are the same though. This chapter will not focus on accounting and audit principles and rules, there are more than enough literature about that. Rather, this chapter will focus on the consequences of inefficiency of accounting and audit systems of economies. Let's start from accounting.

Accounting

Accounting is a registration of financial activities of entity in legally standardized order. The word legally is important here because accounting standard and principles of each economy is protected by law. Nobody is allowed to do accounting the way they wish but only legally supported way. French company branch doing business in Thailand will follow accounting rules of Thailand. Japanese company branch doing business in Russia will be allowed to do accounting only the way Russian laws allow. Technique of accounting goes this way: accounts are divided into three categories: **Assets**, **Liabilities** and **Equity** accounts. Asset accounts consist of all asset accounts that belong to entity: Cash, equipment, inventory, machinery, property, accounts receivable, investments (stock, bonds, futures, options, etc.) and others. Each of these assets has its own account and all financial activity related to those assets must be recorded in their own account. Equipment was sold, recorded in equipment account. Cash received, recorded in cash account. Property depreciated, recorded to property account. Each of those asset accounts also might have its sub-account for example Cash account may have sub-accounts like USD cash account, EURO cash account or TMT cash account. Property account may have many sub-accounts for each property. Investment accounts may have many sub-accounts like GM shares account, BMW shares account, US Treasury Bonds account or Stock options account where each investment activity is recorded separately. **Liabilities** consist of all liability accounts such as bank loans, accounts payable, and others. Each of those accounts may have sub-accounts where financial activity related with them is recorded such as bank loans account may have USD loans account and EURO loans account or Swiss Bank loan account and Senagat bank loan account, etc. Accounts payable may have sub-accounts

like Samsung payables or Office supplies payable where all payables for companies are recorded. **Equity** accounts will consist of accounts that directly affect the equity of entity such as expense account, revenue account, shareholders account, dividend accounts and each of those accounts may have sub-accounts where transactions related with them is recorded. Expense account may have sub-accounts like salary expenses, office expenses, rental expenses, etc. Revenue account may have sub-accounts such as petrol sales revenue, gas sales revenue, and machinery sales revenue or consulting services revenue, it all depends on business scope of entity. Since entity's assets are attained through using equity capital (initial owner's capital) and loans (liability), total assets must be equal to total liabilities plus equity and this principle is fundamental in accounting. Financial activities are recorded through **debiting** and **crediting** related accounts, thus **double entry** system used. In asset accounts, when assets increase or add value, it is debited and when assets decrease or lose value, related asset accounts are credited. Liability and Equity accounts recorded reverse, when liabilities or equity increase or add value then related liability accounts are credited. When liabilities decrease or lose value, related liability and equity account is debited. Financial activity of entity may lead transactions recorded in different account categories or within same account categories.

***Exercise 1**: Entity paid 300000 TMT bank loan. This transaction is recorded in asset accounts and liability accounts. TMT Cash account is credited (cash decreased) and Bank loan account (in liabilities) is debited by 300000 TMT (bank loan decreased). This transaction recorded in two different account categories: assets and liability accounts.*

***Exercise 2**: Entity bought a tractor for 2 MLN TMT. This transaction is recorded in asset accounts only. TMT Cash account is credited (cash decreased) and Machinery account (also in asset accounts category) is debited (asset increased).*

***Exercise 3:** Company received a payment of 20000 TMT for provided service. TMT Cash account is debited (cash increased) and Consulting Revenue account is credited (equity increased).*

At the end of the business day accountants will check if all accounts are balanced. **Balanced accounting book** means that **debit side of asset** accounts category **must be equal** to **credit side of liability and equity** accounts. Ledger that all accounting records are kept is called **accounting book**. **Accounting book must be balanced at all times!** National accounting standards will contain rules regarding amortization expenses, depreciation rates, financial

reporting, numbering of accounts and sub-accounts, legal and illegal transactions, tax discounts, applied fees and others which might change from country to country **but** fundamental accounting principles such as **Assets = Liabilities + Equity**, **debiting and crediting, balanced book** and **double entry** remains the same in all corners of the world.

Audit

Audit is vital for healthy financial system. Objective external audit used as assurance by investors in making investment decisions. Capital markets demand transparency and nothing boosts confidence of capital owners' more than independent, transparent and objective audit report. Lead by professional skepticism, independent auditors play important role in obtaining and evaluating evidences regarding assertions about financial actions and their level of compliance with established criteria. Efficient audit system is the main bridge in turning savings into investments in economy that is the reason why all developed economies try to and spend huge effort in building that "efficient, independent and transparent" audit system. **Audit means trust!** No trust, no investment! Backbone of audit is laws related to economy and finance. Weapon of auditors are laws and regulations. Blurry written laws and not exactly defined actions in regulations leave huge opportunity for non-compliance of entities which they happily use. Good auditing system starts from well written financial laws and regulations. When I say "well written" I mean "complete, without open end, with exactly defined actions and crimes, with clear cut meaning". When we talk about financial world we must be extremely clear about every point we are talking about because as I have mentioned many times before, financial world is extremely volatile, lucrative and sensitive. Laws and regulations must not be of huge numbers, they must be few but very sharply written. Constantly amending laws and regulations is also not a good thing to do for achieving stable economy. Financial world loves stability, sharpness and transparency. National audit standards will contain methods and process of legal audit main objective of which will be assertion of legality of financial actions of an entity. Again, like in accounting standards, fundamental principles of audit assertions are the same almost in all corner of the world. It is:

A. **Independency of an auditor!** Auditor (both licensed entity and person) and entity being audited must not have any kind of **current** (they might have had financial ties long ago but during an audit there must be none!) financial ties so that **conflict of interest** issue will not arise.

B. **Professional skepticism of an auditor!** Auditor must not trust words; professional skepticism must be present at all times.

Auditor must have that "gut feeling" or "sixth sense" as some people says. This "gut feeling" must only calm down when evidences (legal documents, receipts, orders, etc.) assure that "all is good". An auditor is a guardian of capital and never must let his/her "guards down".

C. **Audit report.** Auditing is completed with independent audit report (evidences submitted) which is given to interested parties: investors, state agency, public, etc. and audited entity itself of course. This audit report will say it all, if entity's compliance is at required level or not.

Why economies need efficient accounting system, tax administration and audit standards?

i. **Taxes.** Budget revenue is the first reason why economies must strive for improvement of accounting and audit standards. State authorities must have strict control over tax and audit compliance of entities and first step is to criminalize it legally. Especially in developed economies fight against tax evasion is continues non-stop and penalties are very harsh. Accountants, state inspectors and auditors are the main "ears and eyes" of government on fighting tax evaders. Not only tax evaders harm economy, but tax havens too. Tax havens are locations, sovereign states, jurisdictions with extremely low taxes (or none at all) and with high banking secrecy. International organizations such as International Monetary Fund (IMF) and Organization of European Countries for Development (OECD) estimate that around 10 TRLN USD is hidden in tax havens. Entities try to pay less tax that is why they look for places with minimum tax rate and with highest bank secrecy regulations. Technology of "profit shifting" has evolved greatly. Nowadays, countries that associated with "profit shifting" are divided into two categories. First ones are classic Offshore Financial Centers (OFCs) where corporate profits are "vanished" from government supervision:

Top Offshore Financial Centers
1. Anguilla
2. Antigua and Barbuda
3. Aruba
4. Bahamas
5. Belize
6. British Virgin Islands
7. Cayman Islands
8. Cook Islands
9. Costa Rica
10. Cyprus
11. Dominica
12. Grenada

13. Lebanon
14. Liechtenstein
15. Marshall Islands
16. Mauritius
17. Montserrat
18. Nauru
19. Netherlands Antilles
20. Niue
21. Panama
22. Palau
23. Samoa
24. Seychelles
25. St. Kitts and Nevis
26. St. Lucia
27. St. Vincent and the Grenadines
28. Turks and Caicos Islands
29. Vanuatu

First group mainly consist of island nations where tax rates are almost non-existent or minimal.

Second group consist of countries whose legal system allows channeling funds (not for free of course) from other jurisdictions to OFCs. Hefty portion of all "profit shifting and profit hiding" done globally through these five countries, which assist channeling (hiding) profits of corporates to OFCs. These countries are: **Netherlands, United Kingdom, Switzerland, Ireland** and **Singapore**. **All five of them have tax treaties (or other diplomatic ties) with island nations which are considered as "offshores" and above five nations use their treaties assisting non-resident companies to shift profits (move capital from withholding tax).** How do they do that? Here are couples of examples:

Exercise 4: US based IT company develops a software program which cost company (US1) a 100 USD (example), and this cost is below market cost. US based company sells intellectual property of this newly developed program to its wholly owned subsidiary in Bahamas (BAH 1) at its cost. BAH1 re-evaluates software program to its market price of 1000 USD and sells intellectual property to its wholly owned subsidiary in Ireland (IRE1) for 1000 USD. IRE1 sells program in Europe and Asia for 1100 USD for example. Now, assume IRE1 sold 1000 copies of software program at a price of 1100 USD. According to regulations corporate profit tax is paid from cost deducted revenue. Thus, IRE1 sold programs for 1100 USD (which is market price) but IRE1 has to pay royalty to BAH1 for program (intellectual property rights!). From each program sold IRE1 transfers 1000 USD to BAH1 without any tax imposed

because that is a royalty payment, exempt from any taxes. BAH1 collects all royalty payments (1000*1000=1MLN USD in our case from sales of 1000 programs) and lends to US1 whenever company needs it. Since lending is exempt from taxes BAH1 will lend to US1 all 1 MLN USD and US1 can use this money as company wishes it without any taxes imposed. Technique of "profit shifting" using intellectual property rights (royalty payments) through Ireland is called **Double Irish. Double Irish** is extensively used by IT, visual and musical industry and software development companies because those companies have huge intellectual property assets (royalty rights). Ireland is key player in this scheme because Ireland has many tax treaties with most of offshore island countries which other countries does not have due to political or legal reasons.

Exercise 5: A little variation of **Double Irish** is a **Single Malt** (Malta) where instead of offshore island (Bahamas in our case) is changed with Malta, island in European continent which has a little legal advantage over offshore islands. **Single Malt** can be a little pricier intellectual property based profit shifting tool though. Effect of Ireland turning into hub of the profit shifting channel to economy is immense. Little example: according to Ireland's labor code all subsidiaries located in Ireland (especially US company subsidiaries) must have offices and hire representatives (pay salaries, etc.). Only this has huge positive impact on unemployment rate of Ireland alone.

Top companies using Double Irish
Apple Inc. (first started shifting profits)
Abbott Laboratories
Adobe Systems
Facebook
Forest Laboratories
General Electric
Gilead
Google
IBM
Johnson & Johnson
Medtronic Inc.
Microsoft
News Corp
Oracle Corp.
Pfizer Inc.
Starbucks
Yahoo!

Exercise 6: Using another subsidiary in Netherlands between subsidiaries in Ireland for shifting profits is called a **Dutch sandwich** (Dutch subsidiary is sandwiched between Irish subsidiaries) later on this was cut off due to exaggerated shifting. The main target of all these base erosion and profit shifting tools using subsidiaries in Ireland, Netherlands and offshore islands is to avoid incurring withholding tax from profits from sales in Europe. Actively used by US based companies. **Double Irish, Dutch sandwich** and **Single Malt** are all called **Base erosion and profit shifting (BEPS)** tools which used by companies to erode tax bases (profits, revenues) and shift them to jurisdictions where low taxes are implemented.

Exercise 7: Tax avoidance using restructuring of entity by replacing parent company (which is located in "normal tax jurisdiction") with foreign parent effectively moving tax residency (entity now becomes subsidiary and is not under tax jurisdiction of "normal tax jurisdiction") to low tax jurisdictions is called **tax inversion.** Company changes its tax residence by changing parent company that is it. The main object of **tax inversion (**entities choose company restructuring) is to reduce tax payments.

Economies must fight against BEPS and the only way they can succeed is to have strict accounting standards where all non-business related transactions are kept under strict control, opening of offshore accounts are not allowed and transactions to Netherlands, Switzerland, Singapore, Ireland and United Kingdom is monitored by state agencies. Erosion of tax base will cause revenue reduction which will lead to economic problems in future. Auditors and accounts must be warned about strict measures and criminal punishment must be harsh enough to stop all this "hide and seek" games. Tax administration of a country must work hand in hand with independent auditors to reduce the risk of capital flights.

Financial Secrecy Index of top 100		
1	Switzerland	1,589.57
2	United States	1,298.47
3	Cayman Islands	1,267.68
4	Hong Kong	1,243.68
5	Singapore	1,081.98
6	Luxembourg	975.92
7	Germany	768.95
8	Taiwan	743.38
9	United Arab Emirates	661.15
10	Guernsey	658.92
11	Lebanon	644.41

12	Panama	625.84
13	Japan	623.92
14	Netherlands	598.81
15	Thailand	550.6
16	British Virgin Islands	502.76
17	Bahrain	490.71
18	Jersey	438.22
19	Bahamas	429
20	Malta	426.31
21	Canada	425.84
22	Macao	424.92
23	United Kingdom	423.76
24	Cyprus	404.44
25	France	404.18
26	Ireland	387.94
27	Kenya	378.35
28	China	372.58
29	Russia	361.16
30	Turkey	353.89
31	Malaysia	335.11
32	India	316.62
33	South Korea	314.06
34	Israel	313.55
35	Austria	310.41
36	Bermuda	281.83
37	Saudi Arabia	278.58
38	Liberia	277.29
39	Marshall Islands	275.29
40	Philippines	269.81
41	Italy	254.14
42	Isle of Man	248.68
43	Ukraine	246.25
44	Australia	244.36
45	Norway	242.85
46	Liechtenstein	240.86
47	Romania	232.3
48	Barbados	230.95
49	Mauritius	223.47
50	South Africa	216.44
51	Poland	215.4
52	Spain	213.89
53	Belgium	212.97
54	Sweden	203.55
55	Latvia	195.65
56	Anguilla	195.04
57	Indonesia	188.79
58	New Zealand	178.56
59	Costa Rica	168.78
60	Chile	168.64

61	Denmark	166.12
62	Paraguay	158.52
63	St. Kitts and Nevis	152.55
64	Portugal	151.63
65	Puerto Rico	151.06
66	Vanuatu	149.27
67	Uruguay	148.2
68	Aruba	148.05
69	Dominican Republic	147.09
70	Czech Republic	145.1
71	Finland	142.23
72	Iceland	139.69
73	Brazil	138
74	Hungary	132.73
75	Tanzania	128.92
76	Slovak Republic	127.89
77	Seychelles	125.26
78	Guatemala	123.63
79	Croatia	119.36
80	Greece	118.58
81	Samoa	115.9
82	Mexico	107.57
83	Gibraltar	107.44
84	Curacao	105.66
85	Venezuela	105.03
86	US Virgin Islands	101.89
87	Turks and Caicos Islands	98.08
88	Bolivia	94.82
89	Bulgaria	91.38
90	Belize	86.3
91	Brunei	85.6
92	Monaco	82.93
93	Estonia	79.47
94	Maldives	74.87
95	Ghana	68.85
96	Dominica	62.02
97	Lithuania	58.75
98	Antigua and Barbuda	54.53
99	Montenegro	52.64
100	Cook Islands	44.97

ii. **Money laundering and terrorist financing:** An intergovernmental organization Financial Action Task Force (FATF) report that above 2 TRLN USD is being laundered annually in the world and trend is unfortunately increasing. Money laundering is process of legalizing illegal earnings through using financial institutions, trade, legal entities and other available tools. Illegal earnings are those earnings that accrued through drug trafficking, human trafficking,

prostitution, corruption, murder, fraud, embezzlement, insider trading, counterfeit, hacking, tax evasion, or any other illegal way. Launderer wants to "clean" that "dirty" money through using available financial and other instruments so he can spend "cleaned" money with no question asked. Financial and other instruments help to hide the source of "dirty" money which if known by authorities probably will be confiscated immediately with arrest of beneficiary. Last couple of decades had been done a lot to stricken financial monitoring by almost every country but still there are a lot to do. Increasing tax avoidance, geopolitical instability, radicalism, drug trafficking, illegal arms sales, human trafficking, refuge migration, etc. all these are predicates of money laundering. There are measures that are considered effective against fighting money laundering and terrorist financing. FATF has 40 recommendations regarding fighting money laundering, here are they:

1) **Assessing risks and applying a risk-based approach:** Each country face money laundering and terrorist financing risks depending on socio-economic, geographical, financial, diplomatic and political factors. Every country must assess the risks its facing and implement financial monitoring program accordingly.

2) **National cooperation and coordination:** All state mechanisms (administrative, law enforcement, local government and center, etc.) must cooperate effectively to fight money laundering and there must be a national plan or target about it.

3) **Money laundering offence:** Money laundering must be a criminal offence. Laws and regulations must be in place for fighting this crime.

4) **Confiscation and provisional measures**: Freeze, arrest and confiscation of assets must be one of tools when fighting money laundering and terrorism financing. Like in any other criminal offences (drug trafficking, fraud, etc.) tools of offence and result of offence is being confiscated by authorities, same must be applied to money laundering and terrorist financing.

5) **Terrorist financing offence:** Terrorism financing must be a criminal offence. Laws and regulations must be in place for fighting this crime.

6) **Targeted financial sanctions related to terrorism and terrorist financing:** United Nations Security Council resolutions regarding organizations related to terrorism must be implemented immediately. (Targeted sanctions of countries, entities and people)

7) **Targeted financial sanctions related to proliferation:** United Nations Security Council resolutions regarding proliferation of

weapons of mass destruction must immediately be implemented.

8) **Non-profit organizations:** Non-profit organizations must be watched closely as they could be used for financing of terrorism or money laundering.

9) **Financial institution secrecy laws:** Business secrecy laws in any country must not be allowed to be used as shields by financial entities. Government authorities must be able to get required information whenever needed.

10) **Customer due diligence:** Anonymity must not be allowed by law especially when dealing with finances. Registration must through using adequate identification documents by law.

11) **Record-keeping:** Financial institutions must be required to keep information regarding transaction (or deals, contracts, etc.) for at least 5 years by law.

12) **Politically exposed persons:** People who held high offices, ranks or any other political weight, and their relatives must be dealt with stricter due diligence by financial institutions due to their positions.

13) **Correspondent banking:** When dealing with other banks, respondents, banks are required to make sure that they are dealing with legal entities with strict compliance rules.

14) **Money or value transfer services:** Money or value transfer services also must follow financial compliance regarding anonymity and keeping information.

15) **New technologies:** Countries must always research and learn emerging technologies if they could be used in criminal offenses.

16) **Wire transfers:** Accurate information on beneficiary and other compliance measures are applicable when transferring funds.

17) **Reliance on third parties:** Financial institutions may use third parties for implementation of compliance measures.

18) **Internal controls and foreign branches and subsidiaries:** Internal control, audit and compliance measures, instructions and periodic training of personnel against money laundering and terrorist financing required from all financial institutions.

19) **Higher-risk countries:** When dealing with those countries that was considered a "high risk" by FATF financial institutions must implement enhanced due diligence.

20) **Reporting of suspicious transactions:** Financial institutions must report suspicious activities of its clients immediately to related authorities.

21) **Tipping-off and confidentiality:** Employees of the financial institutions must be protected law from criminal or civil liability from disclosure of information if they reported activity to financial intelligence units.

22) **Designated non-financial businesses and professions (DNFBPs) customer due diligence:** Due diligence and record keeping requirements must also be followed by casinos, real estate agents, dealers with precious metals, lawyers, notaries, accountants, and trusts.

23) **DNFBPs: other measures:** Suspicious activities must also be required to be reported for all DNFBPs listed above.

24) **Transparency and beneficial ownership of legal persons:** Legal persons must also be required to follow due diligence and information keeping rules of compliance.

25) **Transparency and beneficial ownership of legal arrangements:** Contracts and other legal arrangements must also be required to follow financial compliance rules.

26) **Regulation and supervision of financial institutions:** Financial institutions must be adequately supervised on following FATF recommendations.

27) **Powers of supervisors:** Supervisors must be able to inspect financial institutions and have other powers to ensure compliance of financial institutions in combating money laundering and financing of terrorism.

28) **Regulation and supervision of DNFBPs:** Supervisors must be able to inspect DNFBPs and have other powers to ensure compliance of financial institutions in combating money laundering and financing of terrorism.

29) **Financial intelligence units:** Independent unit must be created for analysis of suspicious activities, and implementing supervisory duties and be used as coordination center for combating money laundering and terrorism financing.

30) **Responsibilities of law enforcement and investigative authorities:** Law enforcement agencies must have responsibilities to investigation money laundering and financing of terrorism crimes within framework of national AML/FT policies.

31) **Powers of law enforcement and investigative authorities:** Law enforcement and investigative authorities must have enough power for full investigation procedures when investigating money laundering and terrorism financing offences.

32) **Cash couriers:** Countries must have measures in place for controlling cross-border transportation of cash and declaration.

33) **Statistics:** Countries should maintain statistical information related AML/FT matters.

34) **Guidance and feedback:** Competent authorities, FIU or any other related agencies must contain and create guidelines for other institutions on financial compliance and combating money laundering and financing of terrorism.

35) **Sanctions:** Countries must implement sanctions (administrative, civil or criminal) to legal persons for non-compliance to national AML/FT measures.

36) **International instruments:** Important international legal instruments, that every country must try to be part of (ratify), for combating money laundering and financing of terrorism:

a) Vienna Convention 1988
b) Palermo Convention 2000
c) United Nations Conventions against Corruption 2003
d) Terrorist financing Convention 1999

37) **Mutual legal assistance:** Countries must assist each other in investigation, prosecution and related proceedings when dealing with money laundering and financing of terrorism.

38) **Mutual legal assistance: freezing and confiscation:** When requested, assist in freezing and confiscation of assets of individuals or entities under investigation of other jurisdictions.

39) **Extradition:** Assist in extradition of individuals for further investigations under framework of AML/FT policy.

40) **Other forms of international cooperation:** Countries must make sure that their competent authorities could rapidly, constructively and effectively cooperate in wide range of areas with competent authorities of other countries under AML/FT policy framework.

There are billions (if not trillions) of financial transactions a day and the hardest part is to find those that are exactly related with money laundering and financing of illegal activities like terrorism. None of those transactions go with real purpose right? Launderer will never transfer payments like “Dirty money transaction” or “Invoice for last amount of drugs” or anything like that. All illegal transactions are conceived under legal ones. Shell banks and

companies are created; shell accounts, false payments and many more instruments are used to conceive real criminal activity. It is very hard to follow the money. It takes weeks and month for one investigation and imagine how many transactions are there and how many investigators and compliance officers are needed for effectively combating money laundering and financing of terrorism? Money laundering and financing of terrorism are one of those crimes that consume a lot of time until investigation is completed. Investigators and compliance officers need to be experts not only in their respected fields but also good at international law, IT specialist, and have extremely creative mind to effectively fight money laundering and financing of terrorism. Money laundering is around 2 TRLN USD worth "business" and it is the largest crime by its "market capitalization". Money laundering and financing of terrorism is such a crime where smart people like lawyers, financiers, economists, consultants, legal persons, accountants, and other related people are involved. Investigators and compliance officers must not forget that they are combating those types of people, which is called a **white collar** by experts. Since these are **white collar crime,** countries need specially trained investigators and compliance officers to fight that crime.

Homework:

I. 22nd, 23rd and 28th recommendations of FATF are concerned with designated non-financial businesses and professions. Tell me how lawyers might be used in money laundering or financing of terrorism?

II. How casinos could be used for money laundering?

III. How real estate agents might be used in money laundering?

IV. How banks could be used in money laundering?

V. How companies could be used in money laundering?

VI. 8th recommendation of FATF is concerned with non-profit organizations. How non-profit organizations could be used in money laundering or in financing of terrorism?

VII. 1st recommendation of FATF says that each country must assess, identify and understand the money laundering and financing of terrorism risk for country and implement measures accordingly. Should not be there universal AML/FT measures that fit all?

VIII. 36th recommendation of FATF mentions international instruments (conventions) which are considered as vital in combating money laundering and financing of terrorism. Why ratification of those documents is vital for every country?

IX. 39th recommendation of FATF touches the topic of extradition. How extradition is important in fighting money laundering and financing of terrorism? Do you think extradition is effective instrument in fighting all types of crimes?

X. 20th recommendation of FATF mentions suspicious transactions that must be reported to competent authorities. What do you consider as suspicious transaction?

XI. 10th recommendation of FATF points out the importance of customer due diligence. What is that? What goes into customer due diligence policy?

XII. Fire caused by accident caused 2 MLN TMT worth of inventories got burned down. How this will be recorded in book?

XIII. Company purchased chemicals for 4 MLN TMT and sold it all earning 5 MLN TMT. How this will be recorded in book?

XIV. Company owner wants to keep foreign currency as an insurance against losses from inflation of local currency. It purchases 30000 USD for 3.5 TMT=1 USD. How this recorded in book?

XV. Suddenly, local currency TMT has increased its value and now price of 1 TMT is equal to 2 USD. How this change will be reflected in book?

XVI. Company bought call option paying option price of 20000 TMT. How this will be recorded in book?

XVII. Company sold put option for 10000 TMT. How this will be recorded in book?

XVIII. Company purchased an office building which was discounted by 10% of its original price. Company paid 1 MLN TMT for this purchase. How this will be recorded in book?

XIX. Company sold call option for 5000 TMT. How this is recorded?

XX. Company bought a futures contract. How this is recorded?

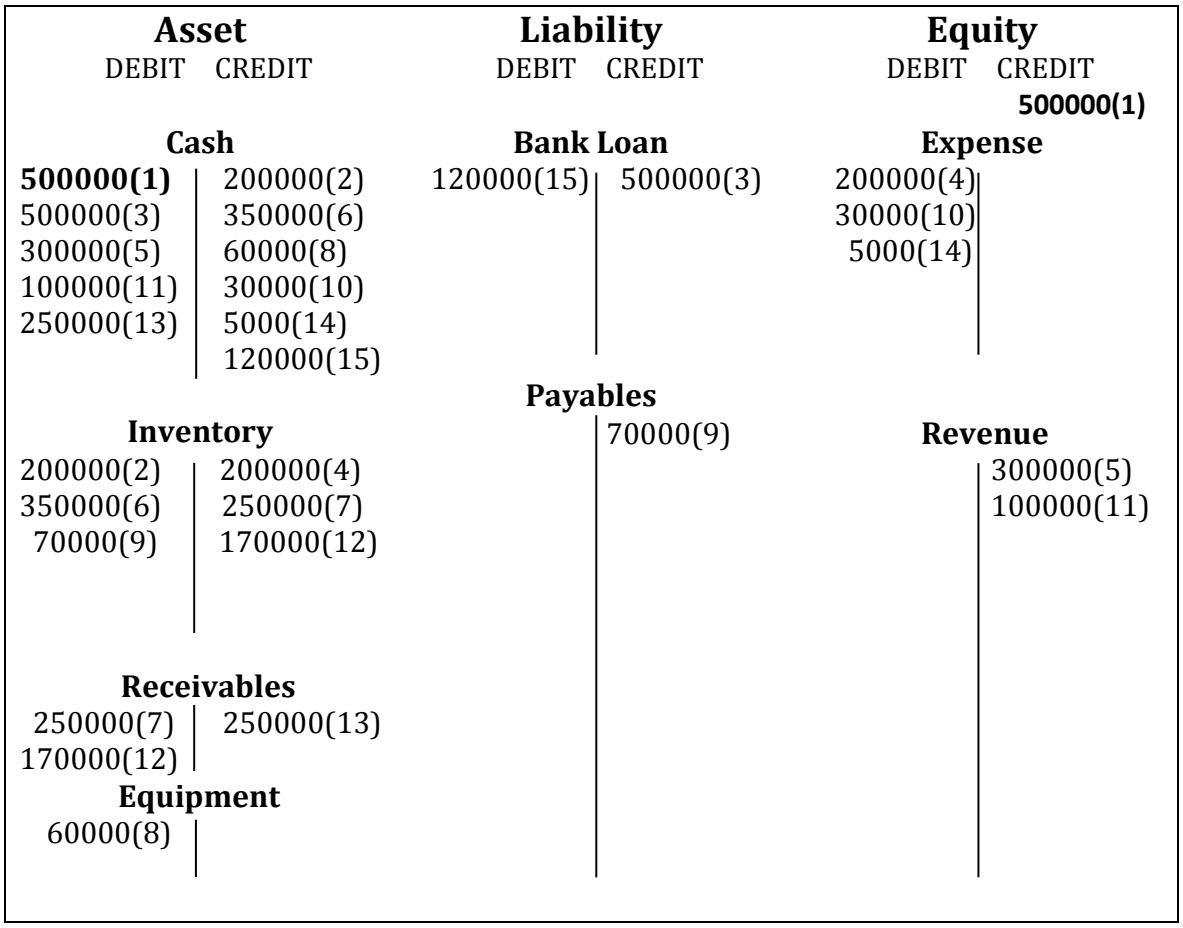
XXI. Company bought 1000 shares of "Angry Ram" company for 220 TMT per share. How this is recorded?

XXII. Company bought 1000 Japanese zero coupon government bonds with face value of 1000 Yen with maturity rate of 6 years. Price paid is 880 Yen per bond. How this recorded in book?

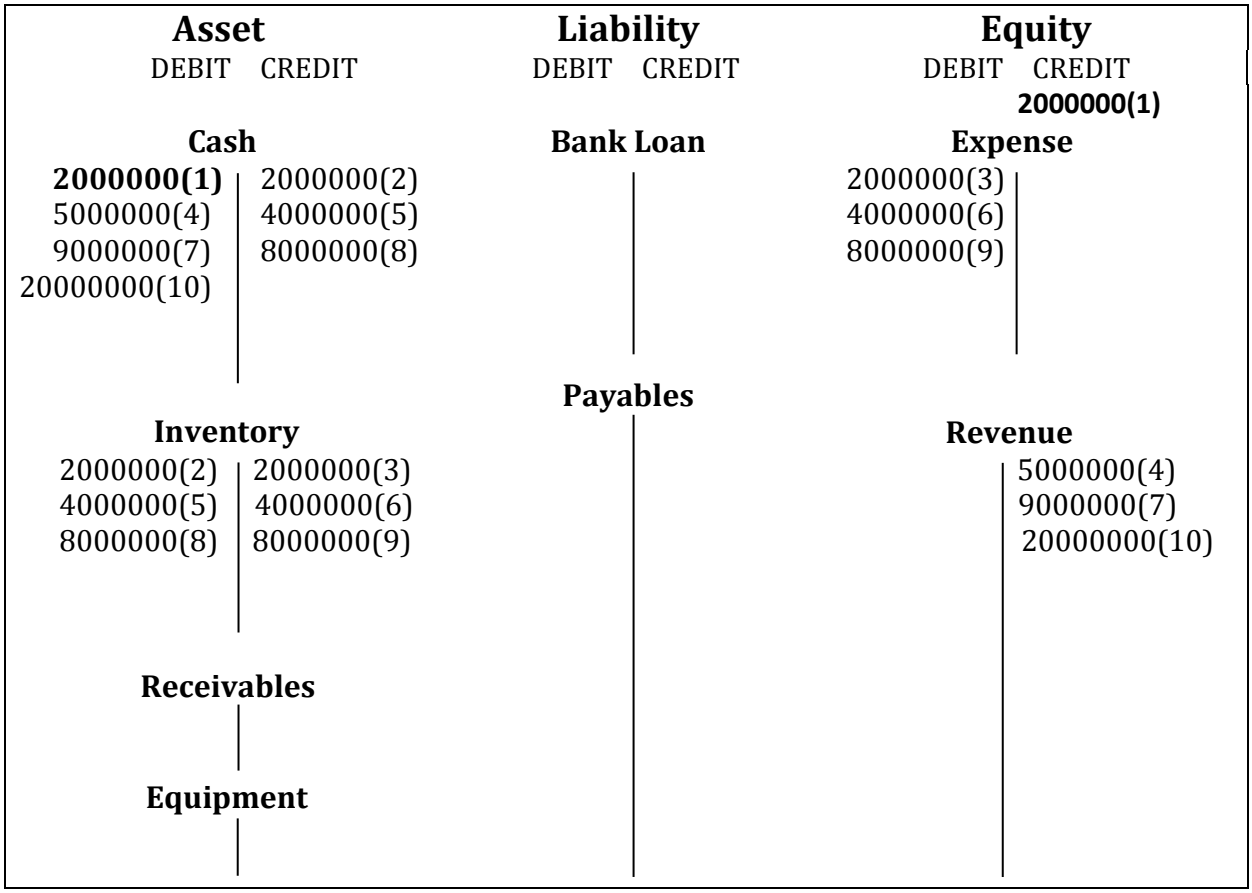
XXIII. Company gave 1000 TMT as a present for each employee for upcoming Oraza Bayram. How this is recorded in book (total of 30 employees)?

XXIV. Two products (each worth a 5000 TMT) were returned and refunded by company due to malfunctioning of those products. How those refunds are recorded?

XXV. Below is the T-ledger of supermarket with transaction records. Please find which transaction is suspicious.



XXVI. Below is T-ledger of manufacturing company. Is there anything wrong going on in this company?



XXVII. Mergen has just got his auditor license and he was immediately being offered an auditing service job from his former employer. Can he accept that contract?

XXVIII. Friend of yours asks to be a partner in his business he already owns and runs for 8 years. He says all good things about past, present and future of the company and requests to invest hefty sum of money into the business. What will be your actions?

XXIX. Public companies are required by law to publish their financial statement reports and independent audit reports annually in all countries. Why is that needed for?

XXX. There are two independent audit reports about one Mexican company: one from local auditor and another one are from Belgian Auditor Company. Which report you will trust more as an investor?

Solutions:

I. In most of the countries lawyers' service fees are not limited. Entities or people of special interest may use this for their benefit. A lawyer may be hired for legal consultation and paid well over average fee for that kind of service. Lawyer must simply transfer the above amount to required parties later. Assume Company A hires a lawyer for consultation and average fee is 20000 TMT for this service. Instead, lawyer is paid 1 MLN TMT with one condition: he must keep 100000 TMT to himself and the rest of money must transfer to individual the Company A wants. How many lawyers would not want the job? Lawyers are very actively used in all kinds of white collar crimes. Giant companies transfer bribe money through legal companies as "consultation" fees. Later those legal companies transfer required amount to targeted person. And the last thing, white collar crimes are done by professionals in the legal, financial and IT sector. Lawyers are professionals in legal system and they know better than others the loopholes in laws and regulations. If not them, then whom to hire for this job?

II. Casinos have huge cash flow and in many countries legal system allows real identification clients to be hid by casinos for safety and security reasons. Client who has just won couple of millions of USD may chose to remain anonymous and request casino to hide his real name from all interested. Huge cash inflows and outflows make casinos excellent place to clean dirty money. Assume Bad guy comes to casino with request to "clean" 3 MLN TMT. Casino owner agrees if Bad guy pays 20% fee, which is 600000 TMT. Bad guy agrees. Bad guy brings 3 MLN TMT to casino owner; he enters it into books as revenue and later writes a winning check to the Bad guy for 3 MLN TMT. Casino owner deducts all needed taxes and remaining money is transferred to Bad Guy's bank account. Voila! Bad Guy can spend this money as he wishes with no question asked. Even if asked, he shows the casino winner check for that amount.

III. Bad Guy comes to real estate agent and asks him to help to "clean" 3 MLN TMT. Real estate agent agrees for the fee of 20% of money. We need a property for this technique: Real estate agent buys very ugly property (poor maintenance) and show a lot of investment for maintenance (on a book only) and sells this house for extremely high price. Actual sales and investment was not done, this deal is on a paper only. On a book he buys the house for 500000 TMT and invests 1 MLN TMT and sells it for 3 MLN TMT. Voila! Later he just lends money to his best friend Bad Guy (of course he takes his portion).

IV. In all around the world, all banks in the world require source of money when deposit exceeds the limits. Assume you want to deposit 300 USD to your deposit account bank will not ask source of money because money is too small. When you go to bank with 300000 USD for example, bank clerk will ask you the source of money. How did you get that cash? If you do not give good answer bank will inform competent authorities. Banks will never allow depositing you money which source was unknown. If they do, their license might be forfeited. Assume Bad Guy comes again to his really good friend who works in bank. If friend who work in bank is corrupted, it is enough just letting Bad Guy to deposit 300000 TMT. Bad Guy will transfer money to other account and money will be cleaned.

V. Assume Bad Guy has a friend who owns a supermarket, where cash flows are immense. Bad Guy can give 5 MLN TMT to his friend and he can slowly enter that amount as revenue (piece by piece). 200000 TMT daily for example. He will be able to clean 5 MLN TMT in 25 days. Later on he can lend that money to Bad Guy and everything will be clean, of course not for free. Companies are used in money laundering because they can pretend earning without actually any trade.

VI. Honestly, non-profit organizations are trusted. Their basic earnings are from donations and many countries allow non-profit organizations to hide the real identity of the donors (at least from public eye) if they want it. “Dirty” money can be shown as “donations” and later on cleaned money returned to launderers as a “charity spending” in book. Principle of good faith can be very false.

VII. Country’s geographical location, neighbors, socio-economic condition, the level of ethnic diversity, cultural difference of its citizens, level of religiousness, level of underground economy, crime rate and unemployment rate, rule of law, corruption rate and many other factors can be potential engines of money laundering and financing of terrorism. Since all these factors are different in each country, the risk level must be assessed independently and tactics of combating money laundering and financing of terrorism must be chosen accordingly.

VIII. All those conventions contain an article where mutual cooperation and assistance is mentioned and other very useful legal points such as extradition and other legal assistances are touched. By ratifying those conventions we join all other who ratified it effectively increasing the “allies” in combating

money laundering and financing of terrorism. This "union" might be very beneficial for country in future.

IX. Many countries have mutual extradition treaties when crime accusations of individuals are fully proven by law. Extradition is extremely important instrument in fighting any type of crime, especially in such crimes like: drug trafficking, human trafficking, terrorism, financing of terrorism, embezzlement, money laundering, organized crime, fraud, ecological damage, murder, etc. because extradition leaves no options (place) for criminals to hide from their eventual punishment.

X. All transactions of big amount, continuous transactions in between non-relative people, continuous but small transactions daily or weekly, strange transactions by companies (construction company buying 1000 horses for example), transactions as "presents", charity transactions of big amount, donations of big amount, lottery winnings, casino winnings of big amount, etc.

XI. Customer due diligence is simple list of basic information about customer such as: name, last name, sex, passport copy and number, living address, signature, and picture copy. If it is an entity then: company name, company legal address, legal status of company, charter of company, registration number, tax code and founders identifications. All financial entities must have this basic information about their clients. All entities must know their clients very well; otherwise it could be very hard to convince competent authorities that entity is not a partner to criminal.

XII. Inventories (in Assets) credited and Expenses (in Equity) is debited for 2 MLN TMT.

XIII. **Purchasing:** Cash (in Assets) credited and Inventory (in Assets) is debited for 4 MLN TMT. **Sales:** Inventory (in Assets) credited and Expenses (in Equity) debited for 4 MLN TMT. Cash (in Assets) debited and Revenue (in Equity) is credited for 5 MLN TMT.

XIV. TMT Cash account (in Assets) credited for 105000 TMT (30000*3.5=105000 TMT) and USD Cash account (in Assets) is debited for 30000 USD.

XV. Deflation and inflation of currency will not be reflected in books because no transaction (financial activity took place) occurred. Accounting book only records financial activity and transactions.

XVI. Option price is sunk cost (cost that cannot be returned), that is why: Cash account (in Assets) credited and Expense account (in Equity) is debited for 20000 TMT.

XVII. Price of put option is non-refundable, that is why: Cash account (in Assets) is debited and Revenue account (in Equity) is credited for 10000 TMT.

XVIII. Books will go with only actual amount paid (actual amount of transactions), that is why: Cash account (in Assets) is credited and Properties account (in Assets) debited for 1 MLN TMT.

XIX. Call option price is non-refundable, that is why: Cash account (in Assets) is debited and Revenue account (in Equity) is credited for 5000 TMT.

XX. When buying futures and forward contract there is no upfront payments, or down payments, all is done is just depositing money into your margin account which is not considered accounting transaction (cash just changed its place from one account to another account) it is more like bank transaction but not an accounting transaction. Daily settlement will start the next day only, so today, there is no financial transaction occurred to record.

XXI. Shares are assets, they are an investment that is why: Cash account (in Assets) credited and Investment account (in Assets) debited for 220000 TMT.

XXII. Bonds are assets, they are investment that is why: Cash account (in Assets) credited and Investment account (in Assets) debited for 880000 Yen (880*1000=880000).

XXIII. Presents are an expense that is why: Cash account (in Assets) credited and Expenses account (in Equity) is credited for 30000 TMT.

XXIV. Refunds are expense that is why: Cash account (in Assets) credited and Expenses account (in Equity) is credited for 10000 TMT.

XXV. Transaction number 11 (100000 TMT) is suspicious because this is shown as revenue, but it is not clear what was sold (inventory is not credited). Company must show the source of that revenue.

XXVI. Yes, this supermarket is earning “super profits”. “Super profits” and “super losses” are always suspicious!

XXVII. No, he cannot accept the job because “principle of independency of auditor” will be violated. Mergen had financial ties with this company; his objectivity during an audit will be questioned by competent authorities.

XXVIII. Get that company audited!

XXIX. Transparency! It is a public company that is why there should not be any secrets from public anymore.

XXX. Higher the level of independency of an audit, the more trust worthy it is! I will trust the Belgian auditor.

Chapter 12: Economics of Human capital (technology and education)

Technology is a "new drug", "new petrol" and "new natural gas" of our epoch. The only difference is that technology is more expensive. There are technology exporting countries and importing ones. Just look at the top companies by their market capitalizations and by their profits and you will see that IT related companies (communication, social networking, online sales, etc.) are on top of the list and their number is increasing year by year. Now almost everything is hooked up to some device: we cannot work without computers anymore, our car has sensors, our homes are protected by automatic alarm systems, we buy and sell things over the phone, our whole banking systems are online and automated, majority part of manufacturing is done by robots, medical equipment and services, etc. we are dependent on technology and our dependence increases. Works on artificial intelligence are progressing so fast that some years later the most of our socio-economic problems will be solved by humanized robots, or robotized humans (whatever). Alternative energy sources like wind, solar, and many others (experimented) are overpowering the classic energy sources like coal and petrol. Market share of classic energy sources are decreasing, they are losing positions to cleaner and to more efficient alternatives. See, before, world is divided into two: those who were exporting energy and those who were importing it, and let's admit that those who exported energy developed their economy thanks to those energy resources. The most of the resource rich countries are underdeveloped, that is fact (or had been underdeveloped). Now, let's for a second imagine what will happen in future: human capital advancement of developed world led to technological advancement of those nations. Technological advancement led everything to be dependent on technology.

Top supercomputers by country		
1	Fugaku	Japan
2	Summit	US
3	Sierra	US
4	Sunway	China
5	Tianhe	China
6	HPC5	Italy
7	Selene	US
8	Frontera	US
9	Marconi	Italy
10	Piz Daint	Switzerland

In nowadays most of developed nations are technology exporting nations while developing nations are technology

importing. Again in nowadays most developed nations are energy importing nations and the most developing nations are energy exporting. Developed nations closed their current account deficit from energy import by exporting technology to developing nations. They are importing petrol and natural gas but exporting cars, computers, medical equipment, chemicals, medical drugs, software, machinery, etc. **Question is this: How will developing nations close their current account deficit from technology imports if their energy resources are no more traded (needed) anymore?** When developed nations will stop importing energy from developing nations but their export of technologically advanced goods will continue, then how will developing nations close their trade deficit? Developed nations are increasing their independency from classic energy resources by switching to alternative, which is a bad news for technology importing resource rich developing countries. Resource rich Middle Eastern countries understood this long time ago and started economic diversification programs to decrease dependence of economy on energy sector (Saudi Arabia, United Arab Emirates, Qatar, Kuwait, and Iran. Also add other resource rich countries with diversification programs including investment on human capital such as Russia, Malaysia, Indonesia, Venezuela, Kazakhstan, and others) and increase investment on human capital. **Technology and other products are derivation of human capital. Human capital is only asset for all nations; all other assets derive from human capital!**

Top countries by number of supercomputers		
1	China	226
2	United States	114
3	EU	79
4	Japan	29
5	France	19
6	Germany	16
7	Netherlands	15
8	Ireland	14
9	Canada	12
10	United Kingdom	10
11	Italy	7
12	Brazil	4
13	Singapore	4
14	Norway	3
15	Saudi Arabia	3
16	South Korea	3
17	Australia	2
18	Finland	2
19	India	2
20	Russia	2
21	Sweden	2
22	Switzerland	2

23	Taiwan	2
24	United Arab Emirates	2
25	Austria	1
26	Hong Kong	1
27	Poland	1
28	Spain	1
29	Czech Republic	1

Nation without any resources but with high human capital will develop but nation with immense resources but with low human capital will not be able to produce anything of high value! Human capital is enough for growth. When people are smart and provided with opportunities, they will import resources use them to make high end good and resell it. All resources might be imported, turned into product and resold. Switzerland, Israel and Japan are one of the world's poorest nations in terms of natural resources but they compensate it with extremely developed human capital. The fact that all three of those nations are on top of the most developed countries list says it all. The dangerous part is that military technology is developing with scary speed with all drones, ballistic missiles, aircrafts, submarines, tanks, anti-air missile systems, supercomputers, artificial intelligence and etc. which puts technology importing nations' future security and safety at stake.

Military technology exporting nations	
1	United States
2	Russia
3	France
4	Germany
5	Spain
6	South Korea
7	China
8	United Kingdom
9	Israel
10	Italy

Top military manufacturing companies	
1	Lockheed Martin (USA)
2	Boeing (USA)
3	Northrop Grumman (USA)
4	Raytheon Technologies (USA)
5	General Dynamics USA)
6	BAE Systems (UK)
7	Airbus (EU)
8	Leonardo (Italy)
9	Almaz-Antey (Russia)
10	Thales Group (France)

Top technologically advanced countries	
1	Japan
2	United States
3	South Korea
4	Israel
5	Germany
6	Russia
7	Finland
8	United Kingdom
9	Canada
10	Singapore
11	Netherlands
12	China

Emerging technologies in all sectors are just mind boggling. Knowledge and science is cause of all technological leaps, and everything starts from **educational system** (kindergarten, then school, then university, then academia in the end we end up with a human who can create and understand technology).

Top education systems	
1	Finland
2	Japan
3	South Korea
4	Denmark
5	Russia
6	Norway
7	United Kingdom
8	Israel
9	Sweden
10	Hong Kong
11	Netherlands
12	Belgium
13	Germany
14	China
15	Singapore
16	Portugal
17	Hungary
18	Estonia
19	France
20	United States

International Innovation Index	
1	Singapore
2	South Korea
3	Switzerland
4	Iceland

5	Ireland
7	Finland
8	United States
9	Japan
10	Sweden
11	Denmark
12	Netherlands
13	Luxembourg
14	Canada
15	United Kingdom
16	Israel
17	Austria
18	Norway
19	Germany
20	France

Countries which were able to create educational system at the end of which society is created with highly developed human capital (knowledge, skill, discipline, character, etc.) will eventually turn into technology exporting nation. There is a strong positive correlation between level of education and development of economy. Higher average education level positively correlates with higher development, and vice versa.

Top Universities in the world	
1	(United States) Harvard University
2	(United States) Stanford University
3	(United Kingdom) University of Cambridge
4	(United States) Massachusetts Institute of Technology
5	(United States) University of California, Berkeley
6	(United States) Princeton University
7	(United Kingdom) University of Oxford
8	(United States) Columbia University
9	(United States) California Institute of Technology
10	(United States) University of Chicago
11	(United States) University of California, Los Angeles
12	(United States) Yale University
13	(United States) Cornell University
14	(United States) University of Washington
15	(United Kingdom) University College London
16	(United States) Johns Hopkins University
17	(United States) University of Pennsylvania
18	(United States) University of California, San Diego
19	(Switzerland) Swiss Federal Institute of Technology in Zurich
20	(United States) University of California, San Francisco
21	(United States) University of Michigan
22	(United States) Washington University in St. Louis

23	(United Kingdom) Imperial College London
24	(Canada) University of Toronto
25	(Japan) University of Tokyo
26	(Denmark) University of Copenhagen
27	(United States) University of Wisconsin–Madison
28	(United States) Duke University
29	(United States) Northwestern University
30	(United States) New York University (NYU)
31	(United Kingdom) University of Edinburgh
32	(Japan) Kyoto University
33	(United Kingdom) University of Manchester
34	(United States) University of North Carolina at Chapel Hill
35	(United States) Rockefeller University
36	(Canada) University of British Columbia
37	(France) Paris-Saclay University
38	(Sweden) Karolinska Institute
39	(United States) University of Colorado Boulder
40	(United States) University of Illinois at Urbana–Champaign
41	(Australia) University of Melbourne
42	(United States) University of Minnesota
43	(China) Tsinghua University
44	(France) Sorbonne University
45	(United States) University of Texas at Austin
46	(United States) University of Maryland, College Park
47	(Germany) Heidelberg University
48	(United States) University of California, Santa Barbara
49	(United States) University of Texas - Dallas
50	(Netherlands) Utrecht University

Intellectual property ownership Patent applications	
1	China
2	U.S.
3	Japan
4	South Korea
5	Germany
6	India
7	Russian Federation
8	Canada
9	Australia
10	Brazil
11	U.K.
12	Mexico
13	France
14	China, Hong Kong SAR

15	Iran
16	Singapore
17	Italy
18	Indonesia
19	Thailand

Top patent applicants of 2019		
1	Huawei	China
2	Mitsubishi Electric	Japan
3	Samsung Electronics	South Korea
4	Qualcomm	United States
5	Oppo	China
6	Boe Technology	China
7	Ericsson	Sweden
8	Ping An Technology	China
9	Robert Bosch GmbH	Germany
10	LG Electronics	South Korea

Human capital is main factor in transformation of economy from primitive agricultural to manufacturing and later on to tertiary. Without highly developed human capital economy will not be able to transform, and remain at the same agricultural or manufacturing level. Tertiary economy is economy mainly earning from service sector which consists of:

Telecommunication
Hospitality industry/tourism
Mass media
Healthcare/hospitals
Public health
Pharmacy
Information technology
Waste disposal
Consulting
Gambling
Retail sales
Leisure
Franchising
Real estate
Education
Financial services
a) Banking
b) Insurance
c) Investment management
Professional services
a) Accounting
b) Legal services
c) Management consulting
Transportation
Teaching

Service sector income is 70% of US's GDP and for other developed nations this number is well over 50%. People employed in service sector earn more than people employed in agricultural or manufacturing sector because in order to work in above sector employees need developed human capital.

It is well known fact that humanity had just researched and learned very minor portion of our planet. There are still a lot to learn and a lot of innovations to be made. As I pointed out here, even if country has resources, still that country needs technology and human capital to turn this resource into something useful. Countries must not be in search for resources only; rather, they must in search of knowledge and innovations, because that is the only thing that will help development of economy, and humanity. There will be questions regarding how to increase human capital and how to create a technology, I think I have answered to that question in last chapters. My answer is "Just provide a ground for human development and the rest will come". I still stick to my answer. The main target of economy must be human development: the rest will be developed by humans.

Numbers of emerging technologies are immense and honestly breathtaking. I do not want to discuss them here because it will take a lot of time and that is not my field of study. Instead, I want to focus on emerging "financial technologies or instruments as we say" and the most important one is crypto-currency. We know alternative currencies that are still in use in some corners of the world. They fulfill same functions of national currencies and only thing different is with little twist of authenticity.

Alternative currencies used current times	
Brixton Pound	United Kingdom
Bristol Pound	United Kingdom
BerkShares	United States
Calgary Dollar	Canada
Chiemgauer	Germany
Detroit Community Scrip	United States
Eusko	Basque Country, France
Eko	Findhorn Ecovillage, Moray, Scotland
Fureai kippu	Japan
Ithaca Hours	United States
Kelantanese dinar	Malaysia
Ora	Orania, South Africa
Sarafu-Credit	Kenya
Stroud Pound	United Kingdom
Tumin	El Espinal, Veracruz, Mexico

Crypto-currencies are something different and more original than alternative currencies. This must be beginning of something

very big in economics and it worth spending time for learning what is it and its mechanism.

Crypto-currency

Dependence on US dollar in doing business internationally increased immensely after the fall of USSR. With increasing globalization of banking sector and with the help of SWIFT transaction system, this dependence increased even more. Expanding world markets, increasing rates of development of countries, massive trade, new players and ever-increasing appetite for energy resources all contributed to the increase in demand for universally accepted currency. Dollar made transactions easy but at the same time, it increased dependency of the economy to the dollar, which is very dangerous, because of the reasons stated above paragraphs. De-dollarization of the economy was the only solution for this "dependence problem". Question is **how to do it?** Just switching to another fiat currency does not make a sense, because even if it might lower the influence of US on economy, it will still not resolve the dependency issue. World needs fundamental change, not a cosmetic one.

In October of 2008, anonymous author who go by pseudonym Satoshi Nakamoto, published a paper "*Bitcoin: A peer-to-peer electronic cash system*", where he put forward mechanism of first decentralized digital currency, later named cryptocurrency. The main difference of this digital currency from other fiat currencies (dollar included), is in its supply mechanism, as said above, it is decentralized. Blockchain, decentralized transaction and data management technology runs the Bitcoin, where all transactions are recorded to public ledger visible to everyone. Thus, no third party or any other controller is involved in the process. The goal of the Blockchain is to provide anonymity, security, privacy and transparency to all its users. Special algorithm (*protocol*) limits the amount of coins supplied at a given time, which makes supply side of the currency very transparent and only dependent on users' common effort (*proof of work*). Special algorithm, depending on an amount of users, creates certain math equations where computer's processing power is needed to solve it (*mining*). Reward for involving in mining process is, a digital currency. Special digital keys are created (automatically by protocol) for securing the "earned" cryptocurrencies, and they could only be stored in computer (digital currency!). The value (price!) of the digital currency, in terms of fiat currency, depends on market demand, although initial price was set by website New Liberty Standard in 2009, and it was equal to electric energy spent on mining one Bitcoin. This was the beginning of cryptocurrencies era, and ever since, thousands of them were created by various individuals and groups. Although, initially Bitcoin (first digital currency) was widely used for illegal transactions in all sorts of darknets, because

of its convenience (anonymity), and this hugely hampered the reputation of cryptocurrencies, its popularity among legal businesses grows with astonishing speed. Even though hacking of couple of cryptocurrency trading companies scared off the vulnerable market at times, which resulted in sudden drops in cryptocurrency price, the overall confidence of the market to the cryptocurrencies did not suffer. The main reason was that those hacking incidents directly related to the security gaps of trading companies, not directly related to cryptocurrency itself.

The only difference between thousands of cryptocurrencies created are their names, because they all use, up-and-down, same blockchain technology for transactions and have almost the same philosophy: value is decided by market (*miners*), decentralized, open public ledger system (*blockchain*), reward mechanism (*proof of work*), saving and storing (*only digitally*), protocol (*regulated by special algorithm*), security and transparency.

As of 2020 there are close to 2000 crypto-currencies and below are the most popular and active ones		
Release date	**Name**	**Founder**
2009	Bitcoin	Satoshi Nakamoto
2011	Litecoin	Charlie Lee
2011	Namecoin	Vincent Durham
2012	Peercoin	Sunny King (pseudonym)
2013	Dogecoin	Jackson Palmer & Billy Markus
2013	Gridcoin	Rob Hälford
2013	Primecoin	Sunny King (pseudonym)
2013	Ripple	Chris Larsen & Jed McCaleb
2013	Nxt	BCNext (pseudonym)
2014	Auroracoin	Baldur Odinsson (pseudonym)
2014	Dash	Evan Duffield & Kyle Hagan
2014	NEO	Da Hongfei & Erik Zhang
2014	MazaCoin	BTC Oyate Initiative
2014	Monero	Monero Core Team
2014	NEM	UtopianFuture (pseudonym)
2014	PotCoin	Potcoin core dev team
2014	Titcoin	Edward Mansfield & Richard Allen
2014	Verge	Sunerok
2014	Stellar	Jed McCaleb
2014	Vertcoin	David Muller
2015	Ether or "Ethereum"	Vitalik Buterin
2015	Ethereum Classic	Vitalik Buterin
2015	Nano	Colin LeMahieu
2015	Tether	Jan Ludovicus van der Velde
2016	Zcash	Zooko Wilcox
2017	EOS.IO	Dan Larimer

PROS AND CONS: PURE HYPOTHESIS

Cryptocurrencies are new technology, we have to accept it. Our financial system stayed the same for over thousand years, and cryptocurrenices might be "that technology", which will change our lives fundamentally. Do you remember first internet? Only few used it, while majority looked at it very skeptically. In our days, we cannot imagine our lives without internet. It speeded up our advancement, both financially and socially. World became more connected, information more available, and transactions are cheaper and faster.

Pros:

1. ***Competitor.*** Any new player in the market pushes the profits down through increased competition. "Oldies" have to share the market now with the "new guy in town", so to say. This is actually fundamental idea of free market economy, where tough competition will push firms to produce at the level where marginal revenue equals to marginal costs, equalizing profits to zero as a consequence. Market (people, buyers) will benefit from increased competition among non-cooperating suppliers. Cryptocurrencies are main competitor of fiat currencies, and especially reserve currencies (and especially to USD). Those countries, which were financially sanctioned like Russia, Iran or Venezuela, by reserve currency suppliers, use this technology to overcome the sanctions, making them ineffective. Cryptocurrencies will help to decrease the leverage the reserve currency suppliers had (and which was extensively used!) over other countries. Developed cryptocurrency mechanisms and technology will force all countries to act on "more" equal terms, than we see now. Will leverage disappear for all? No, actually, leverage in supplying the cryptocurrency will be in the "hands" of energy abundant (where energy is cheap!) nations, because almost 80% of cost of cryptocurrency (*mining*) is energy.

2. ***Decentralized.*** Central banks of the governments have monopoly over money supply, thus, supply of the money is centralized. It means, money supply is being controlled by very few people. Generally, it depends on their decisions on how much and when will supply increase and when it will decrease. Markets, trade and many other factors can deeply influence their decisions and there are no warranties that their decision will work. There will never be a perfect (flawless) solution when human factor is involved in the process. Cryptocurrencies are not fully independent from human beings, we still write their *protocols*, but thanks to *blockchain* technology, supply of the currency is no more in few people's hands. It is decentralized. Influence over money supply of few people is ended here, effectively decreasing the harm from imperfect money supply decisions (compared to fiat money

supply). *Blockchain* technology controls the time and amount of cryptocurrency supply.

3. ***Digital.*** No more hard currencies! No gold, no paper, nothing, just a couple of bits or bytes (or terabytes, depending on your wealth), that will be your wealth. All you need is just an access to internet to spend it.

4. ***Anonymity.*** Right because of its anonymity, cryptocurrencies became popular among illegal goods traders in various dark nets. Even though its anonymity characteristics might fade over time (because of rapid development in technological sciences, it is very hard to stay anonymous), its anonymity is its main strength for now.

5. ***Speed.*** When we talk about digital world, the speed is the "name of the game". That is all said.

Cons:

1. ***Ponzi-scheme like structure.*** Since the introduction of cryptocurrencies in 2008, its reputation was its number one enemy, due to its widespread popularity among illegal businesses. The main (and maybe the solely!) reason was its anonymity characteristics. Any user could create thousands (millions and billions) of digital wallets and store digital currency in them, making the tracking extremely challenging. As an economist, I strongly believe that one of digital currencies main enemy is not its reputation, but its structure. As I mentioned above, the more *miners* involved in the process, the harder the mathematical problems gets, making the solution even harder. In the language of economists, as amount of *miners* increase, the price of the digital currency gets higher for suppliers of the digital currency. At the same time, as demand for the digital currency increases, it also increases the price of the digital currency. Rapid increase in market demand increased the price of the digital currency, this also, hugely increased the *miners* amount (because there are huge profits to be made), which in return made *mining* even harder, and the prices just rocketed. Reverse is dangerous too, the lesser the miner involved, makes mining cheap, which decreases the price of the digital currency. Making things even worse, decreasing price scares off the market and dropping prices will plummet with even faster rate. This hugely reminds of Ponzi-schemes, where amount of people involved in the process affects the profits and losses.

2. ***High volatility***. Demand and supply process stated above, and real world examples of price fluctuations of the cryptocurrencies witnessed, will definitely bring us to the conclusion that, high volatility of the digital currencies are less likely curable or controllable in near future. Even tiny shocks might create havoc in the market! One way to resolve this issue might be,

trying to keep demand on stable level, regardless of dropping prices (short term shock absorbance level of demand must be very high!). Since supply is decentralized, the only way to influence the prices might be though demand side.

3. ***Hacking.*** The most vulnerable side of cryptocurrencies is its digital nature. Fiat currencies could be robbed, counterfeited or simply destroyed over when stored improperly. Digital currencies will face the danger of being hacked. All currencies have deficiencies related to their nature, cryptocurrenicies have it too.

4. ***Non-storability***. Biggest flaw, to my personal opinion, of cryptocurrencies are in its valueless nature. I can buy gold or diamond and store it somewhere, and tens of years later I can easily sell them to meet my needs. Fiat currencies do not have such a long term values like precious stones and metals, but under normal conditions, life expectancy of fiat currencies will be long enough to save them for long term. Unfortunately, digital currencies volatile and digital nature makes them very bad long term "savings and investment instrument".

CONCLUSION

It is very hard to predict the future of any technology, especially, for the technology that threatens our "a thousand yearlong stable financial system". Cryptocurrencies, might be just a primitive beginning of something revolutionary, or it might be one of many other "genius" historical Ponzi-schemes. One thing is for sure: all governments must keep close eye on this technology and experiment with it, especially those countries, with cheap and abundant energy resources. Our world is becoming more and more digitalized and our financial system must fit the development process. Increasing trade wars, overall unstable geopolitical situation and currency manipulations in the recent decade opened up deficiencies of the current financial system. World needs a change, and this change will occur we want it or not. Nations must be ready for that.

Homework:

I. What do you think: diversification of economy from resource dependence is a good strategy or bad strategy for growth and development?

II. What are the steps of development of economy?

III. What supercomputers are used for?

IV. Why military technology is important?

V. What do we need to increased development of human capital?

Solutions:

I. Diversification of economy is not only good strategy but it is vital strategy for economic growth and development. Historically, resource prices are not stable and they always fluctuate. This creates a huge risk for economies that are dependent on resource exports. Diversification spreads out the risk, like in portfolio management, every financier will recommend you not to invest to only one type of asset, but invest into many type of assets to minimize risk of loss.

II. All economies start from primitive structure of **agricultural economy**: farming, husbandry, fishery, forestry, etc. As human capital increases (education level, skills, discipline, economic and political system, etc.) economies slowly transform into mass production and manufacturing which is a big scale production and called **industrial economy**. Instead of couple of eggs, couple of millions of eggs a day, instead of couple of fishes, couple of hundreds of tons of fishes, instead of candles electric lights, instead of working only from morning to evening, three shift working schedule (non-stop production), etc. Human capital continues its progress and slowly different types of sectors and products are brought to market: communication, medical services, financial services, management services, personal computers, robots, automatized factories, artificial intelligence, high speed internet, supercomputers and everything adds speed on production and efficiency. This level of development of economy is called **digital economy.** Level up of economy is only possible with development and expansion of human capital!

III. Supercomputers are used for **calculations of very complex questions** (Military projects: instead of testing newly developed weapon all data about weapon is entered into computer and all data regarding environment and computer calculates all damage. Real testing would have cost hundreds of times more than just testing in supercomputers), **for analysis of huge data** (Useful for banks, government agencies, insurance companies, stock exchanges and derivative exchanges, etc.), **for estimations and prediction models** (scientists: economists, medical experts, financiers, government officials, and all others) where estimation and prediction models are used for research supercomputers will be extremely useful. Supercomputers can do things in seconds that took months before, this hugely speeds up development and research.

IV. The first thing that all countries must worry is about their defense and security. That is why countries always want to create stealthy fighters, unbeatable tanks, giant howitzers and the most silent submarines and so on. When they heavily work on those projects they open a new methods, new ways and that is exactly a new technology. Almost all technology that we are having today starting from internet, GPS and mobile connection was first developed and used by military people. That is why if you want to measure technological development level of any country just look at its military production (technology) and that will say it all.

V. Same conditions that required for economic development and growth is required for human capital development. We have talked about it in previous chapters but I will happily repeat it again:

xxxix. Peace: First and foremost condition for any kind of development.

xl. Laws are respected and implemented: Having a written law is one thing, how it is implemented is another.

xli. Human rights are respected and protected: Human being must be the most valuable input in any economy. Nothing is more valuable than human being. Labor force is the most valuable asset.

xlii. Less bureaucracy (zero bureaucracy): Bureaucracy is the enemy of progress because bureaucracy seeks wealth without a progress! Bureaucrat wants wealth, he does not want changes. Progress demands hard work and it is hectic, hard and risky; that is all what bureaucrat hates. Zero bureaucracy for maximum growth.

xliii. Corruption is illegal (fully implemented): Corruption is a child of bureaucracy. The more bureaucracy lives, the more children she will have.

xliv. Monopoly is illegal (fully implemented): Monopoly is a business dictatorship. All said.

xlv. Education is accessible: Kindergartens, schools and universities are cradles of civilization. Everything and everyone starts there!

xlvi. Health services are accessible: The longer I live, the longer I am beneficial to society. The longer I live, the more I can produce. The longer I live, the more experienced I will be, and I can transfer my experience to next generation.

xlvii. Competition in all sectors are encouraged and supported: Competition is a driving force for everything. We must not be

afraid of competition; we must love competition, a healthy competition. Losing and winning is a part of game. A sector without a competition is like a game where everyone wins. It is not possible. In a sector where competition does not exist, everyone is in a loss, except the owner of the sector.

xlviii. Discrimination is illegal (fully implemented): There must not be of any kind of discrimination, full stop.

xlix. Government is transparent: Government is a big teacher. Students learn and implement what they were taught. Transparency is a symbol of honesty! Honesty is the first and the most important lesson for every student because honesty is fundamental principle of any kind of relationship: economic, diplomatic, civil, family, etc.

l. Majority of population belongs to middle class: Middle class is the trampoline of growth. The size of middle class in economy will show the level of political and economic stability. Society starts to rot from within when middle class vanishes.

li. Entrepreneurship is encouraged and supported: It is the brave heart of entrepreneurs broke the borders of "dark minds". Entrepreneurship soul is the biggest asset for any economic development.

lii. Investment is encouraged, supported and protected: Investment is the main source of funding of growth. No funding, no growth.

liii. Financial freedom is protected: Financial limitations, barriers and all kinds of prohibitions are going to limit transactions. Transactions are logistics of money. Money will not go to the areas where "roads" are missing.

liv. Property rights are protected: Property is an extension of its owner. Property rights mean human rights.

lv. Economic freedom is protected: Economic limitations, barriers, and all kinds of prohibitions are going to limit economic operations. How would economy grow then?

lvi. Trade freedom is protected: Trade is "energy" of economic growth. Nothing will move without energy, ask Isaac Newton about it.

lvii. Efficient fiscal policy: Taxes must be progressive: rich must pay more, poor must pay less. Otherwise economic injustice will grow. Injustice means unrest.

lviii. Efficient monetary policy: Inflation is the biggest enemy of investment and inflation is the only head ache of Central bankers.

Controlling inflation is the 99% success of efficient monetary policy.

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